

Presented by Daniel Almirall, PhD

Primary Aim Analyses in a SMART Part II





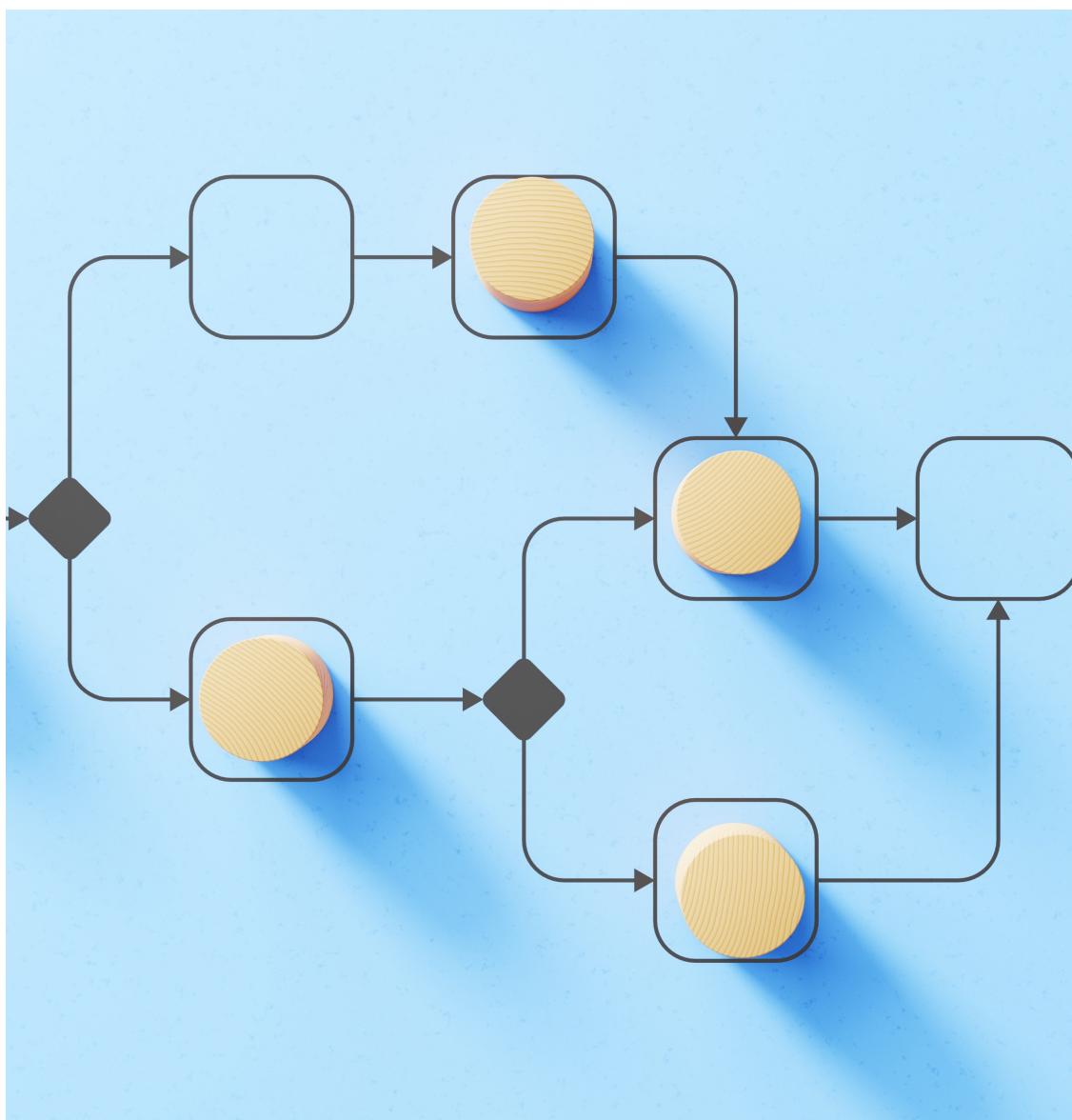


Learning Objectives

You will have a better understanding, and will continue learning how to frame, the typical Primary Aims in a SMART

You will learn about key statistical considerations in Primary Aim analyses in a SMART

You will learn how to interpret the output for the different Primary Aim Analyses in a SMART





Outline

Illustrative Example: ADHD SMART Study (PI: Pelham)

Prepare (again) for a third primary aim analysis by (d): Estimate and compare the mean outcome under two of the embedded AIs using weighted least squares

Use a single weighted-and-replicated least squares regression approach capable of address any/all three primary aims in a SMART





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Illustrative Example: ADHD SMART Study (PI: Pelham)

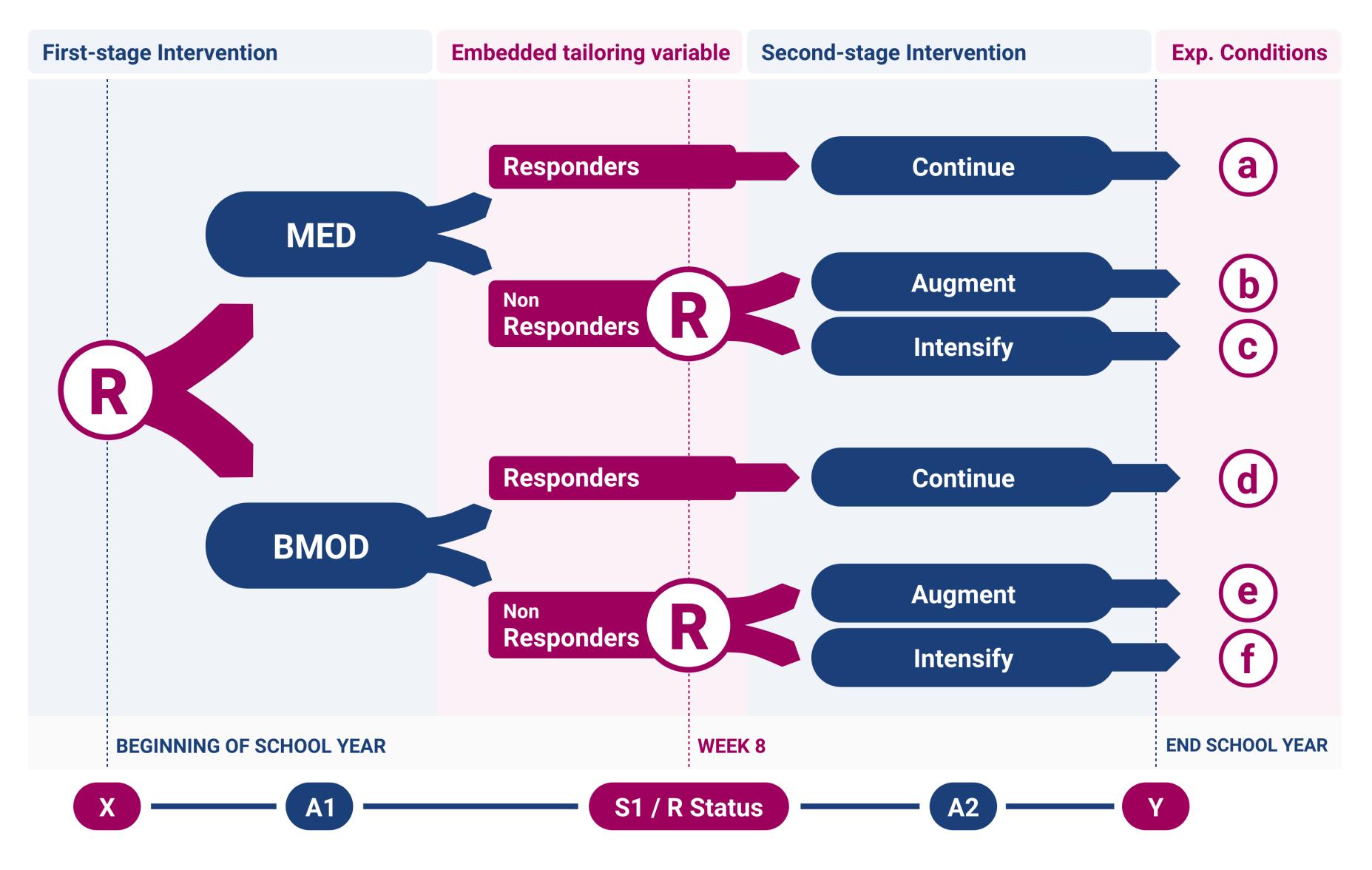
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SMART Example ADHD Study



PI: Pelham





SMART Example 4 Embedded Adaptive Interventions

Adaptive Intervention 1

At the beginning of the school year **Stage 1 = {MED};** then, every month, starting week 8 if response status = {NR}, then, Stage 2 = {AUGMENT}; else if response status = {R}, then, Continue Stage 1

Adaptive Intervention 3

6

```
At the beginning of the school year Stage 1 = {MED};
then, every month, starting week 8
if response status = {NR},
then, Stage 2 = {INTENSIFY};
else if response status = {R},
then, Continue Stage 1
```

PI: Pelham

Adaptive Intervention 2

At the beginning of the school year **Stage 1 = {BMOD};** then, every month, starting week 8 if response status = {NR}, then, Stage 2 = {AUGMENT}; else if response status = {R}, then, Continue Stage 1

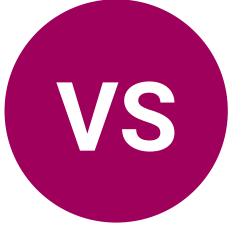
Adaptive Intervention 4

At the beginning of the school year **Stage 1 = {BMOD};** then, every month, starting week 8 if response status = {NR}, then, Stage 2 = {INTENSIFY}; else if response status = {R}, then, Continue Stage 1



Adaptive Intervention 1

At the beginning of the school year **Stage 1 = {MED};** then, every month, starting week 8 if response status = {NR}, then, Stage 2 = {AUGMENT}; else if response status = {R}, then, Continue Stage 1



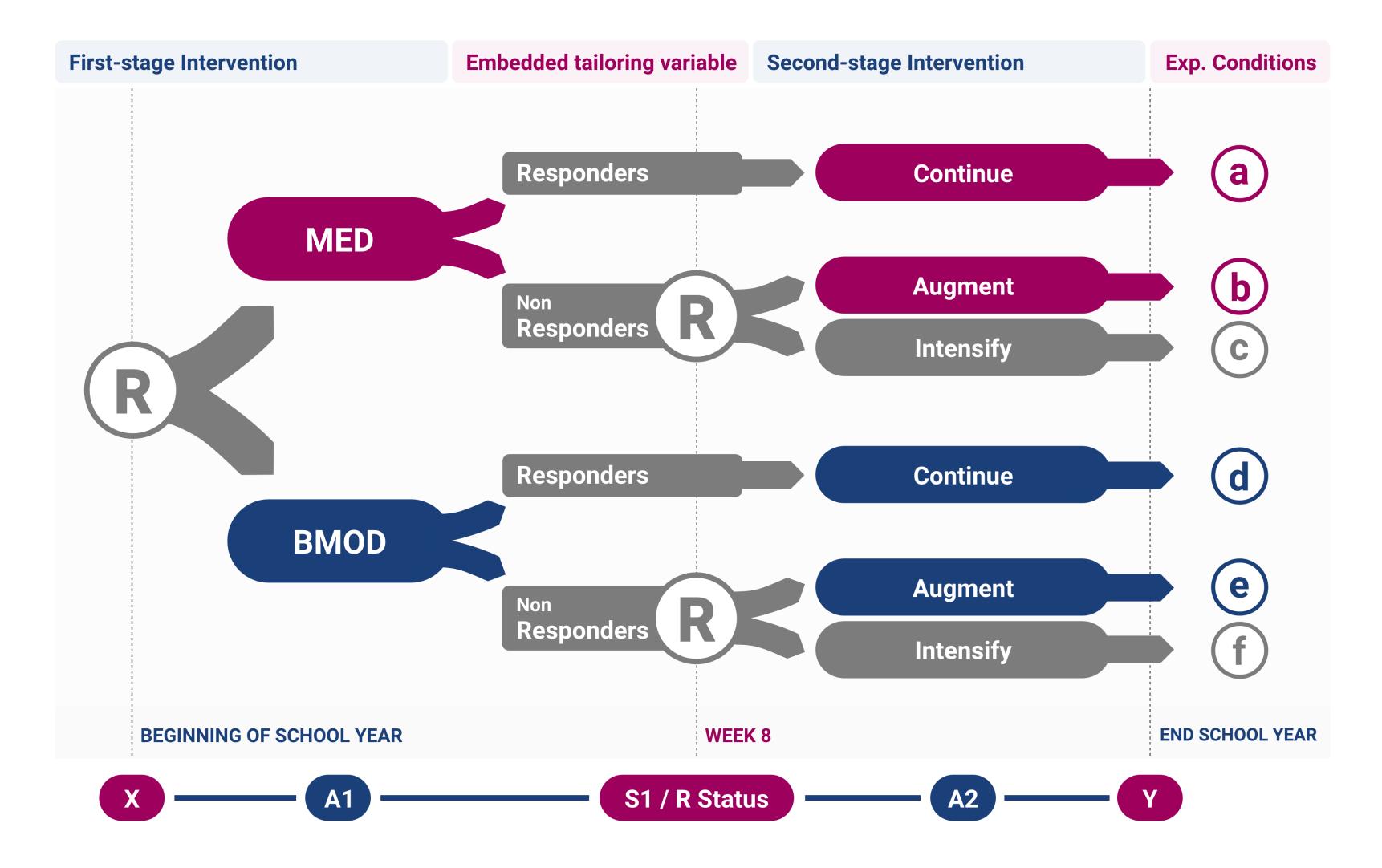
PI: Pelham

Adaptive Intervention 2

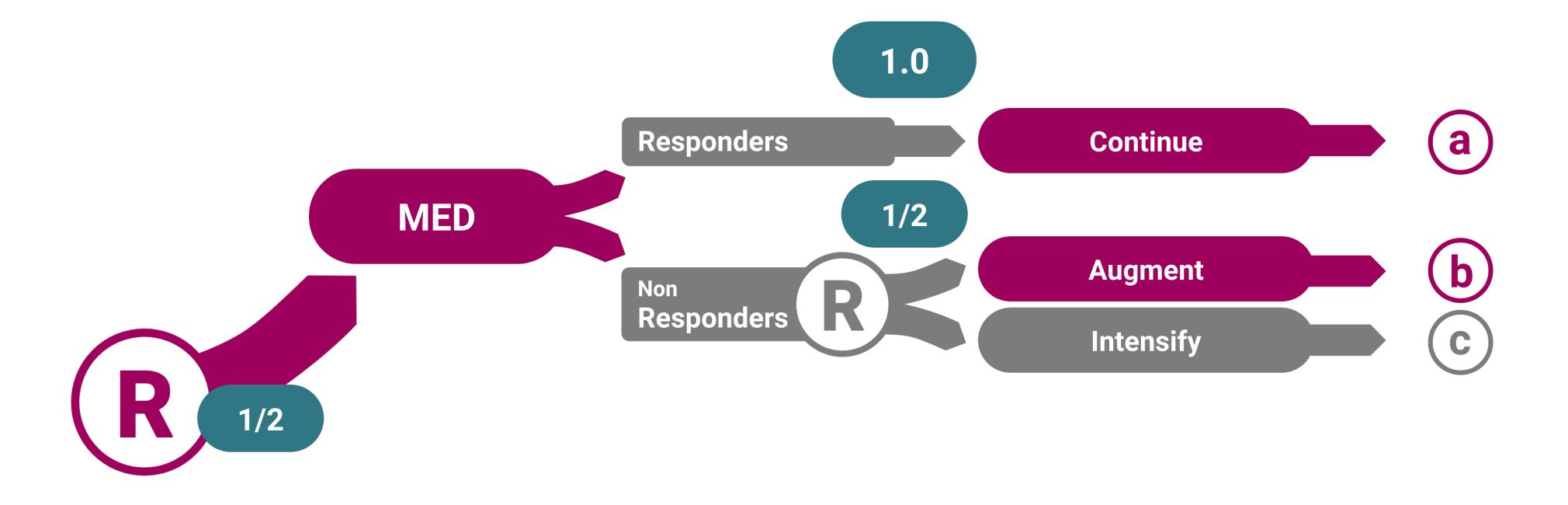
At the beginning of the school year **Stage 1 = {BMOD};** then, every month, starting week 8 if response status = {NR}, then, Stage 2 = {AUGMENT}; else if response status = {R}, then, Continue Stage 1



This Aim is a Comparison of the Mean Outcome under AI#1 vs. the Mean Outcome of AI#2



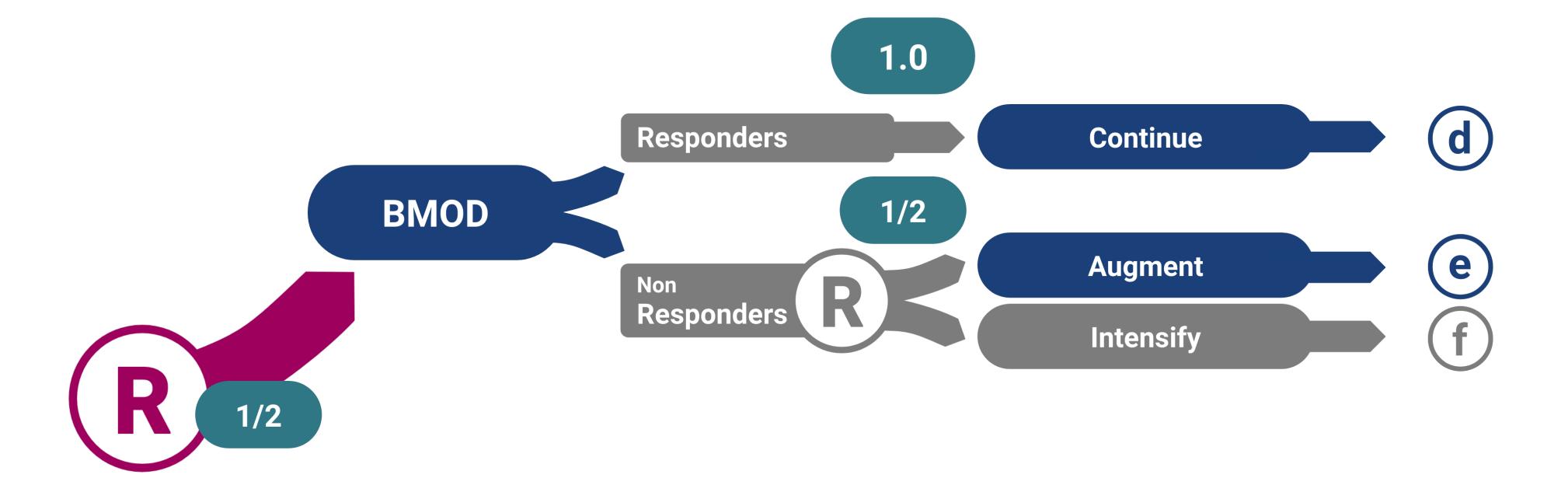
We Know How to Account for Imbalance in Non-Responders Following **AI#1**



• Assign W = weight = 2 to responders to MED $\rightarrow 2* \frac{1}{2} = 1$ • Assign W = weight = 4 to non-responders to MED \rightarrow 4* $\frac{1}{4}$ =1

• Then we take W-weighted mean of sample who ended up in circles A+B.

A Similar Approach (and SAS Code) Can be Used to Obtain Mean Under **AI #2**



- Assign W = weight = 2 to responders to BMOD $\rightarrow 2* \frac{1}{2} = 1$ • Assign W = weight = 4 to non-responders to BMOD \rightarrow 4* $\frac{1}{4}$ =1
- Then we take W-weighted mean of sample who ended up in circles D+E.

Results for Estimated Mean Outcome had All Participants Followed AI#2 (BMOD, AUGMENT)

Analysi	s Of GEE	Parame	eter Esti	mates				
Parameter	Estimate	Standa	ard Error Pr > 2					
Intercept	3.149 0.1477		<.0001					
Z1	0.1477		0.0001					
		Cont	rast Esti	mate Res	ults			
	Mean	95% Co	onfidence L	imits	Standard			
Label		stimate	Lowe	r Upper		Error	Pr > ChiS	þ
Mean Y under (BMOD, AUG	•••	833	3.363	4.303		0.24	<.0001	

Interpretation: The estimated mean school performance score for children consistent with AI #2 is ~3.83 (95% CI: (3.36, 4.30)).



Results are from simulated data.



Outline

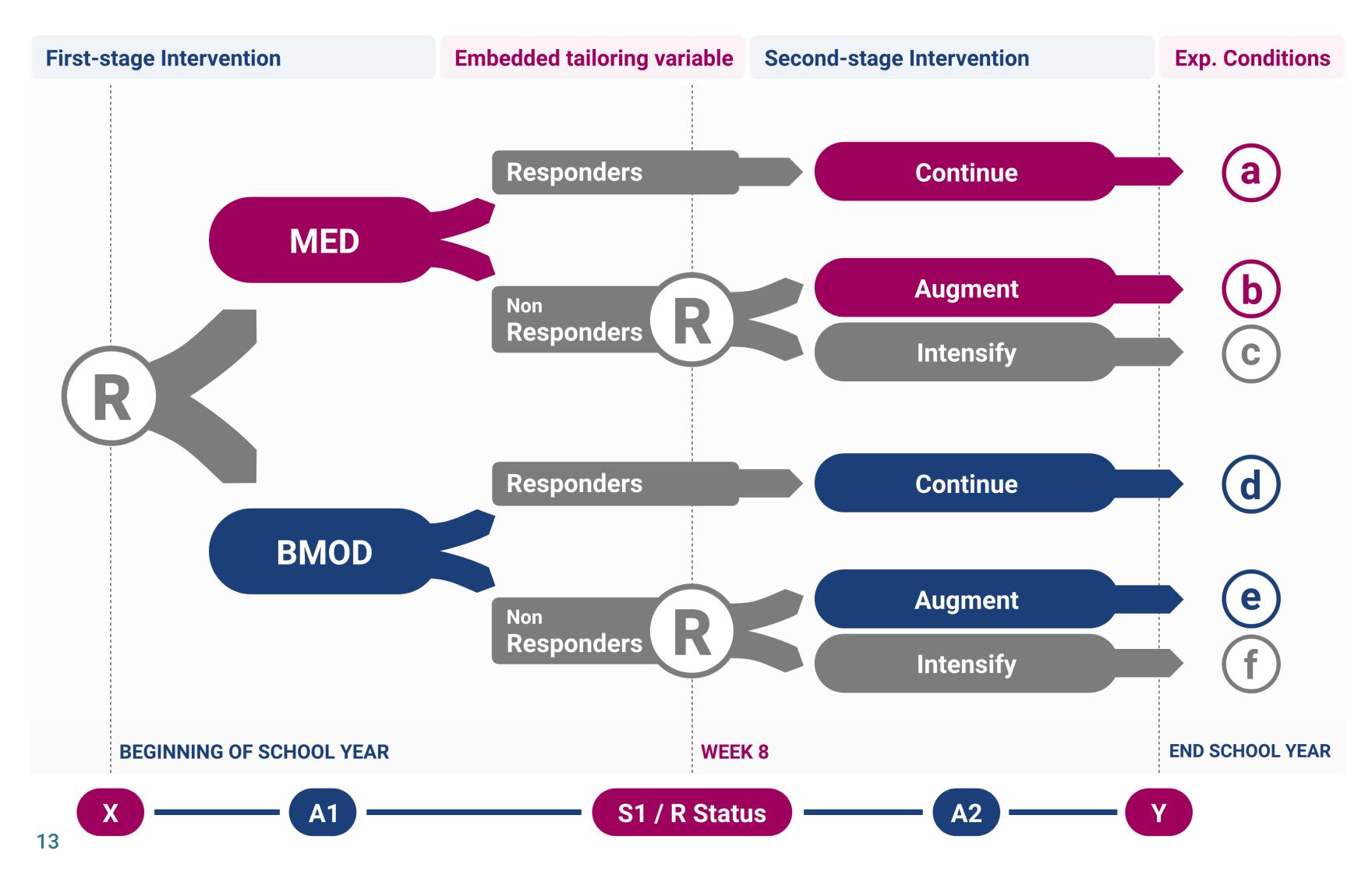
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Use a single weighted-and-replicated least squares regression approach capable of address any/all three primary aims in a SMART









data dat7; set dat1;

Z1=-1;
if A1*R=-1 then Z1=1;
if (1-A1)*(1-R)*A2=-2 then Z1=1;
Z2=-1;
if A1*R= 1 then Z2=1;
if (1+A1)*(1-R)*A2=-2 then Z2=1;
W=2*R + 4*(1-R);

run;

data dat8; set dat7; if Z1=1 or Z2=1; run;

14 This analysis is with simulated data.

Create Z1 —> indicator for whether or not the person is consistent with AI#1



data dat7; set dat1;

Z1=-1; if A1*R=-1 then Z1=1; if (1-A1)*(1-R)*A2=-2 then Z1=1; Z2=-1; ← if A1*R= 1 then Z2=1; if (1+A1)*(1-R)*A2=-2 then Z2=1; W=2*R + 4*(1-R); run;

data dat8; set dat7; if Z1=1 or Z2=1; run;

15 This analysis is with simulated data.

Create Z2 -> indicator for whether or not the person is consistent with AI#2



data dat7; set dat1;

Z1=-**1**;

if A1*R=-1 then Z1=1;
if (1-A1)*(1-R)*A2=-2 then Z1=1;

Z2=-**1**;

if A1*R= 1 then Z2=1;
if (1+A1)*(1-R)*A2=-2 then Z2=1;

W=**2***R + **4***(1-R); ←

run;

data dat8;
 set dat7; if Z1=1 or Z2=1;
run;

16 This analysis is with simulated data.

Assigned Weights



data dat7; set dat1;

Z1=-**1**;

if A1*R=-1 then Z1=1;
if (1-A1)*(1-R)*A2=-2 then Z1=1;

Z2=-**1**;

if A1*R= 1 then Z2=1;
if (1+A1)*(1-R)*A2=-2 then Z2=1;

W=2*R+4*(1-R);

run;

data dat8;
 set dat7; if Z1=1 or Z2=1;
run;

17 This analysis is with simulated data.

Delete data from participants not consistent with either Al#1 or Al#2



- The Regression and Contrast Coding Logic: Recall:
- Z₁ is now an indicator for whether the person is consistent with Al#1 or with Al#2: \rightarrow Z₁ = 1 = Al#1 \rightarrow Z₁ = -1 = AI#2

To compare the 2 Als, we can fit the Model:

Overall Mean Y under Al#1 = $\beta_0 + \beta_1 \times 1$ Overall Mean Y under Al#2 = $\beta_0 + \beta_1 \times -1$ Diff Between Als = $\beta_0 + \beta_1 - (\beta_0 - \beta_1) = 2\beta_1$

This analysis is with simulated data.

 $Y = \beta_0 + \beta_1 Z_1 + e$



proc genmod data = dat8; class id; model Y = Z1;scwgt W; repeated subject = id / type = ind; estimate 'Mean Y AI#1(MED, Add BMOD)' intercept 1 Z1 1; estimate 'Mean Y AI#2(BMOD, Add MED)' intercept **1** Z1 -**1**; estimate 'Diff: Al#1 - Al#2' Z1 **2**; run;

This analysis is with simulated data.

Mean Y under Al#1 = $\beta_0 + \beta_1 \times 1$ Mean Y under Al#2 = $\beta_0 + \beta_1 \times -1$ Diff Between Als $= 2\beta_1$



Analysis Of GEE

Para	ameter E	Estimate	Standard Error	Pr	> Z	
Interce	ept 3.	25	0.1613	<.000)1	
Z1	-0	.583	0.1613	0.000)3	
	Con	trast Es	timate Resul	ts		
	Mean	95% C	onfidence Lim	nits s	tandard	
Label	Estimate	Low	er Uppe		Error	Pr > ChiSq
Mean Y under AI #1 (MED, AUGMENT)	2.666	2.242	3.0892	0.	216	<.0001
Mean Y under AI #2 (BMOD, AUGMENT)	3.833	3.363	4.3028	0.	240	<.0001
Diff: AI#1 – AI#2	-1.167	-1.799	-0.5347	0.	.323	0.0003
This analysis is with simulat	ed data.				↑ ce SE	

An Intuitive Approach to Comparing AI#1 vs. AI#2

proc genmod data = dat8;

class id; model Y = Z1 Y0c oddc; scwgt w; repeated subject = id / type = ind; estimate 'Mean Y AI#1(MED, AUGMENT)' intercept 1 Z1 1; estimate 'Mean Y AI#2(BMOD,AUGMENT)' intercept 1 Z1 -1; estimate 'Diff: AI#1 - AI#2' Z1 2; run;



An Intuitive Approach to Comparing AI#1 vs. AI#2

Analysis Of GEE Parameter Estimates

Parameter	Estimate	Standard Error	Pr > Z
Intercept	3.26	0.1148	<.0001
Z1	-0.45	0.1160	<.0001
Y0	2.13	0.2464	<.0001
oddc	0.09	0.2511	0.715

Contrast Estimate Results

		95% Conf	idence Limit	S	
Label	Mean Estimat	e Lower	Upper	Standard Error	Pr > ChiSq
Mean Y under AI #1	2.806	2.5198	3.0927	0.146	<.0001
Mean Y under AI #2	3.713	3.3491	4.0787	0.186	<.0001
Diff: AI#1 – AI#2	-0.907	-1.3776	-0.4376	0.240	0.0001
	_				
This analysis is with simulate				naller compa rol covariate	



Outline

Illustrative Example: ADHD SMART Study (PI: Pelham)

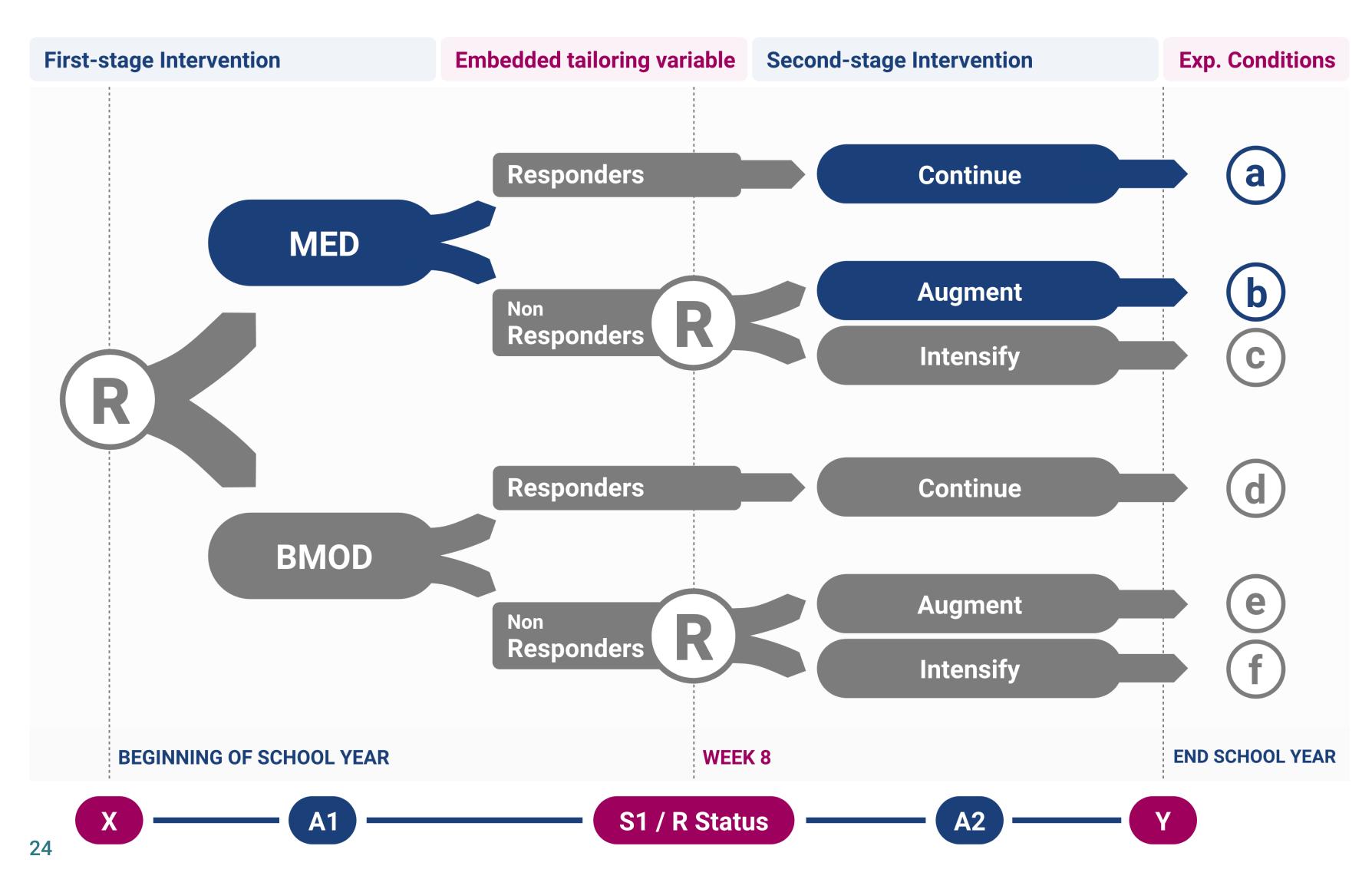
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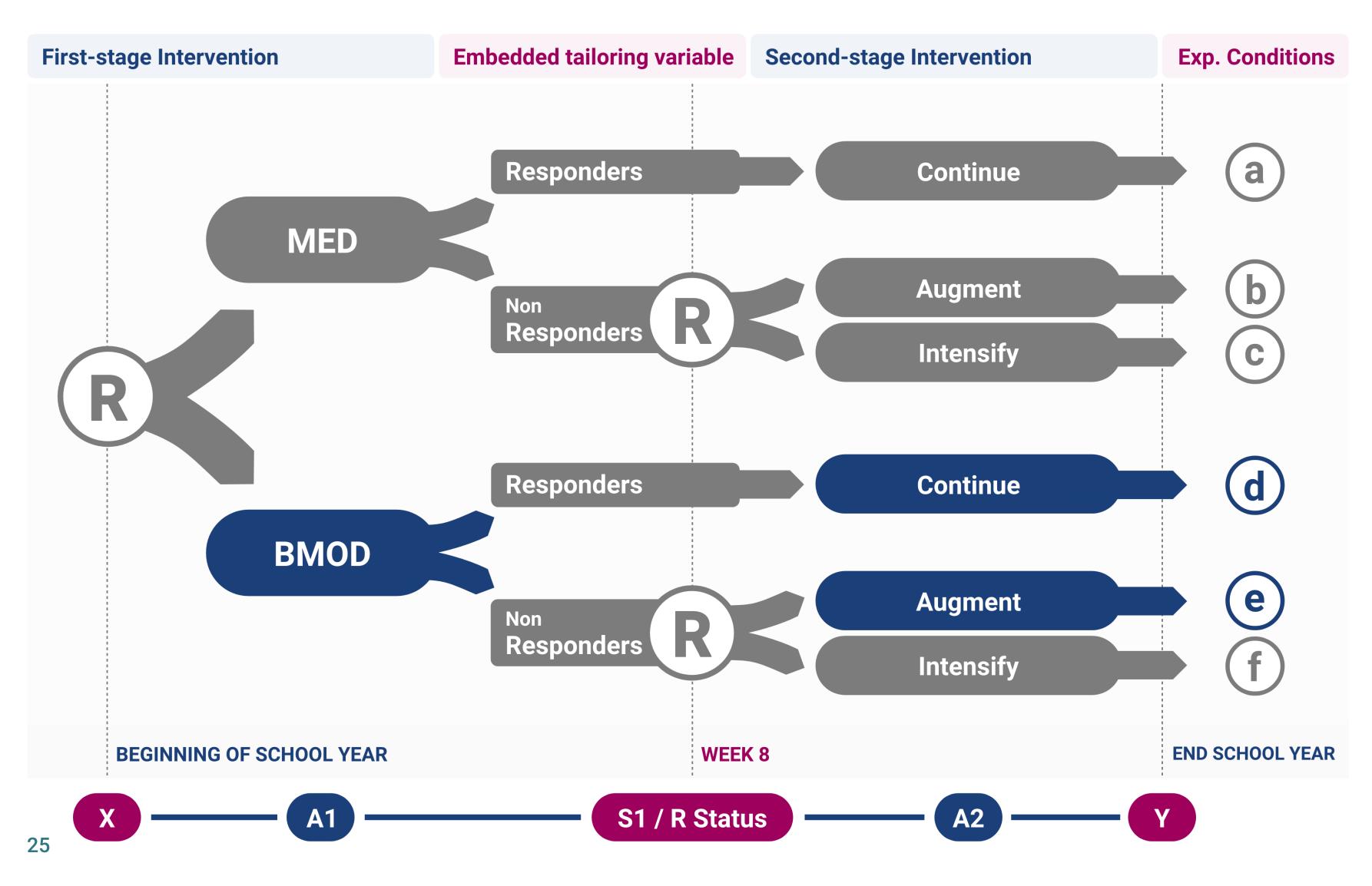
What about a regression to compare AI#1 (MED, add BMOD) vs...





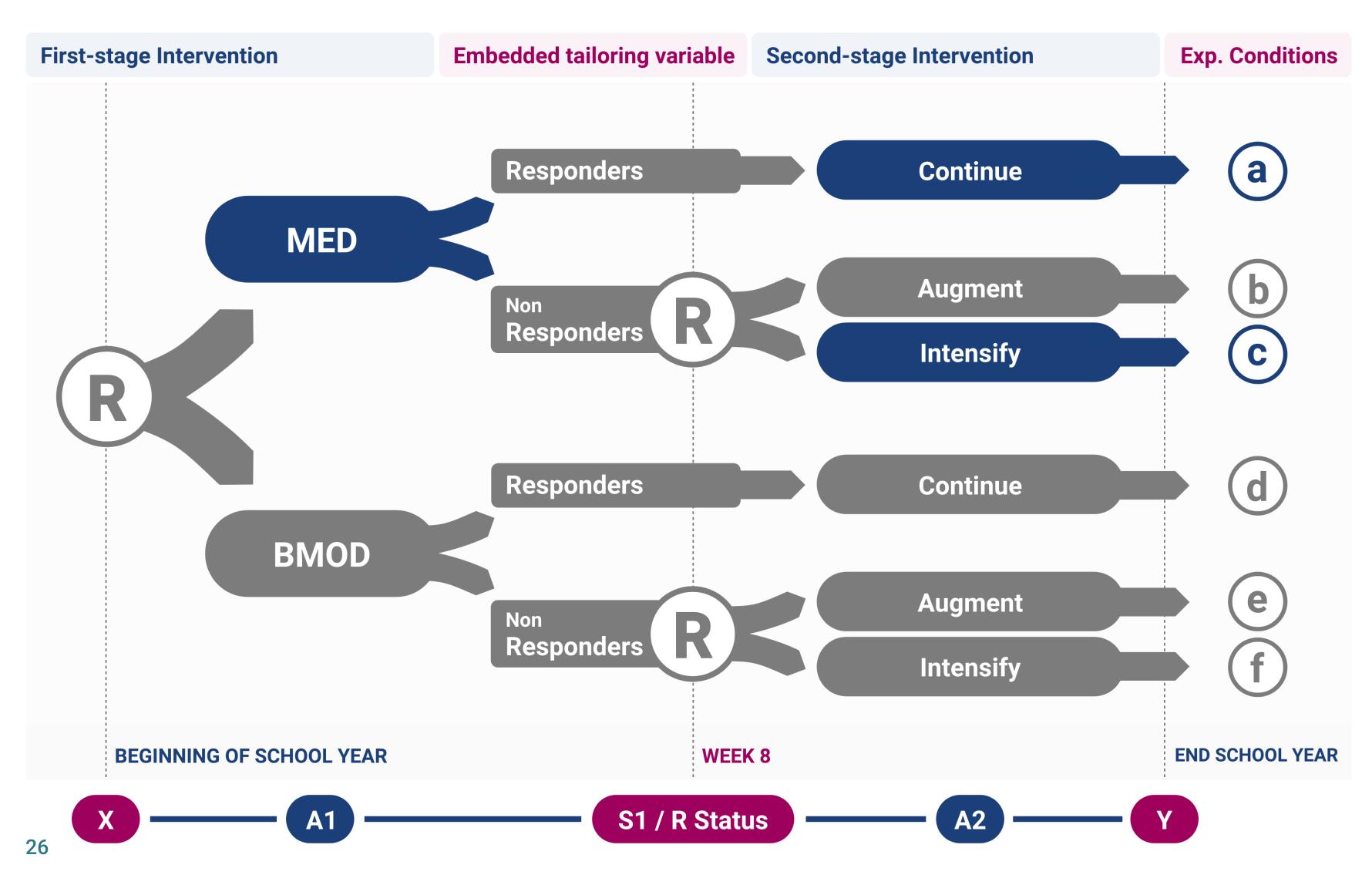


...AI #2 (BMOD, Add MED) vs...



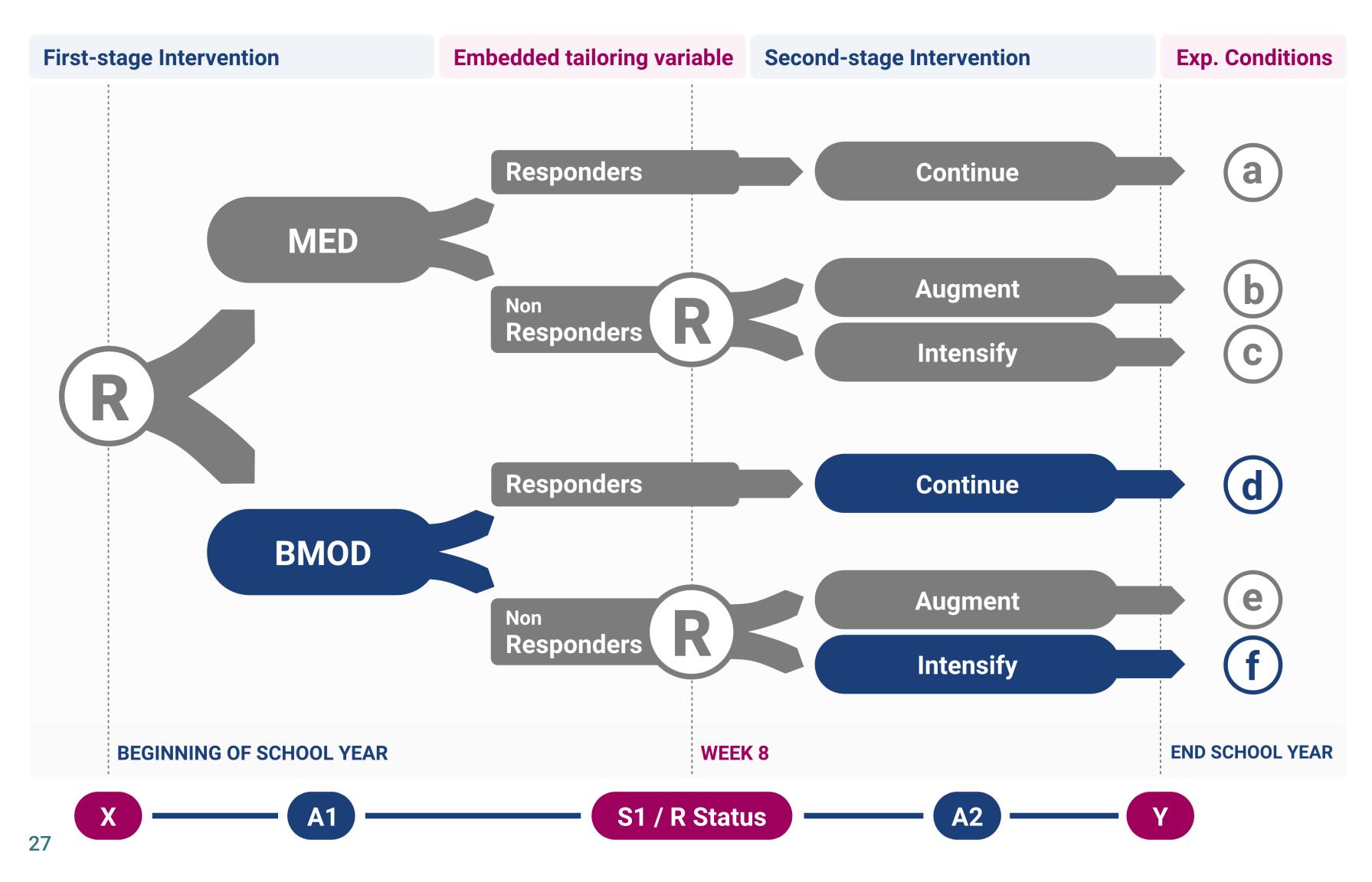


...AI #3 (MED, INTENSIFY) vs...



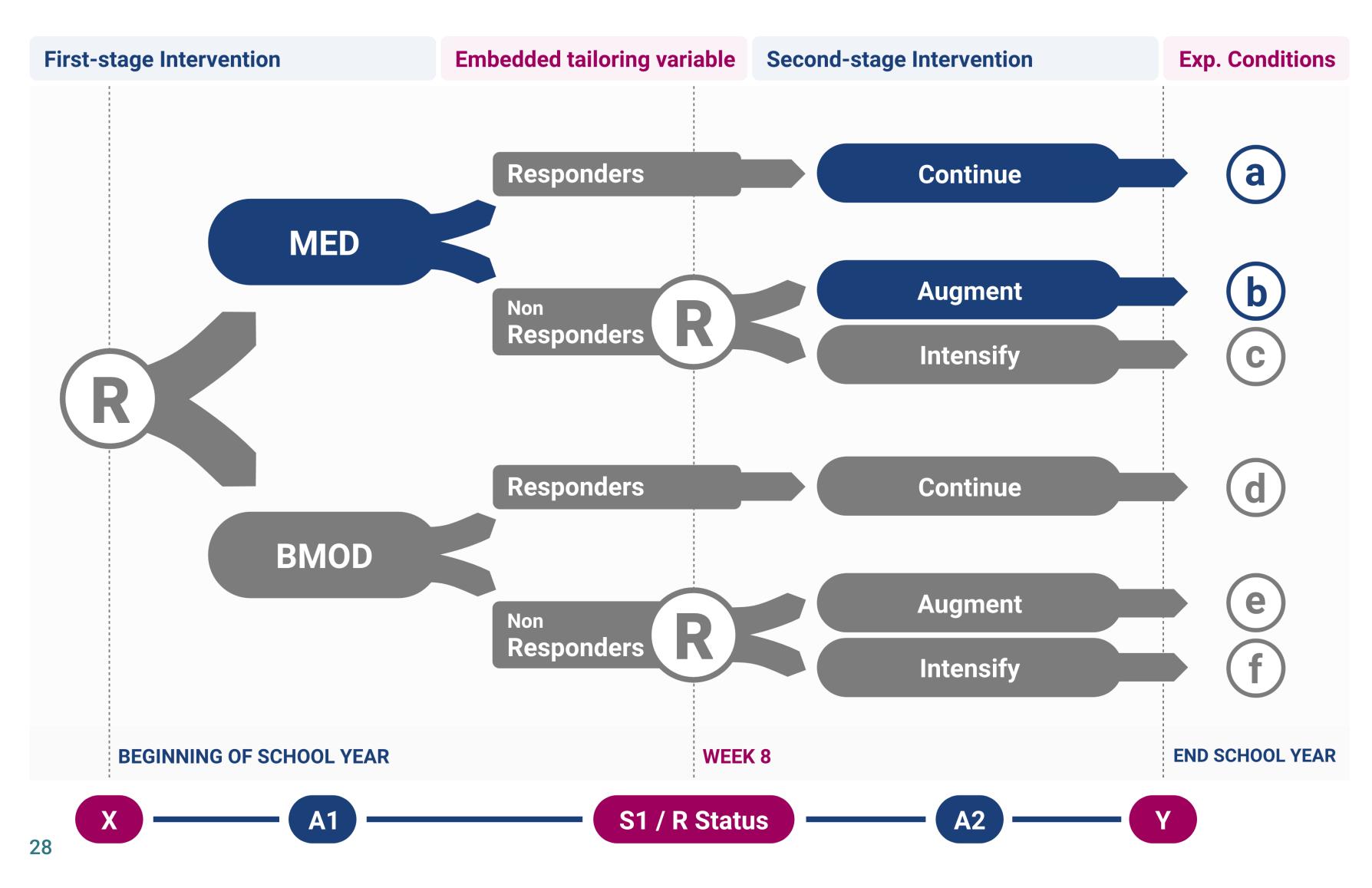


...AI#4 (BMOD, INTENSIFY), all in one swoop!



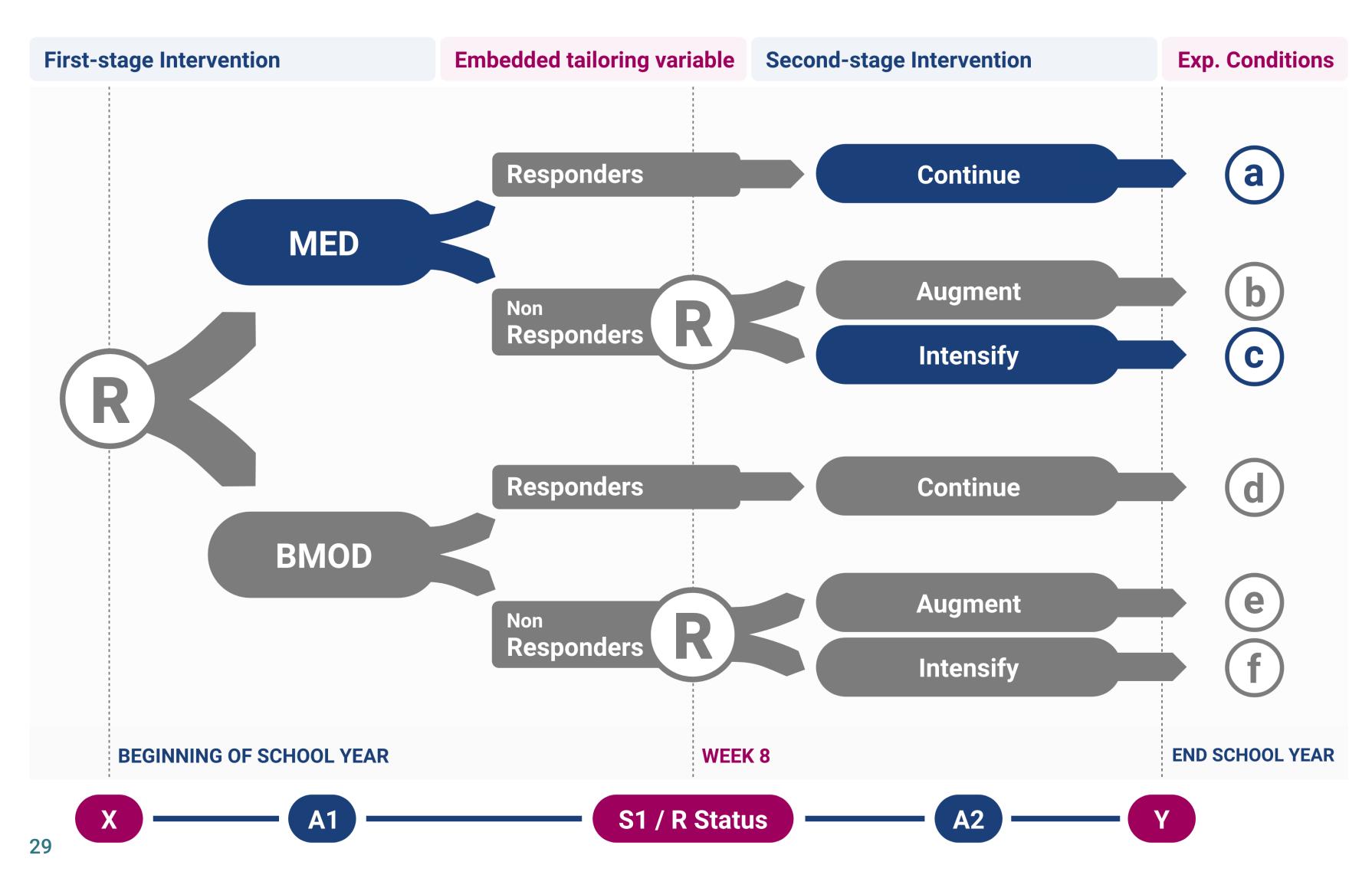


Notice that AI#1 and AI#3 (start MED) share responders (box A)



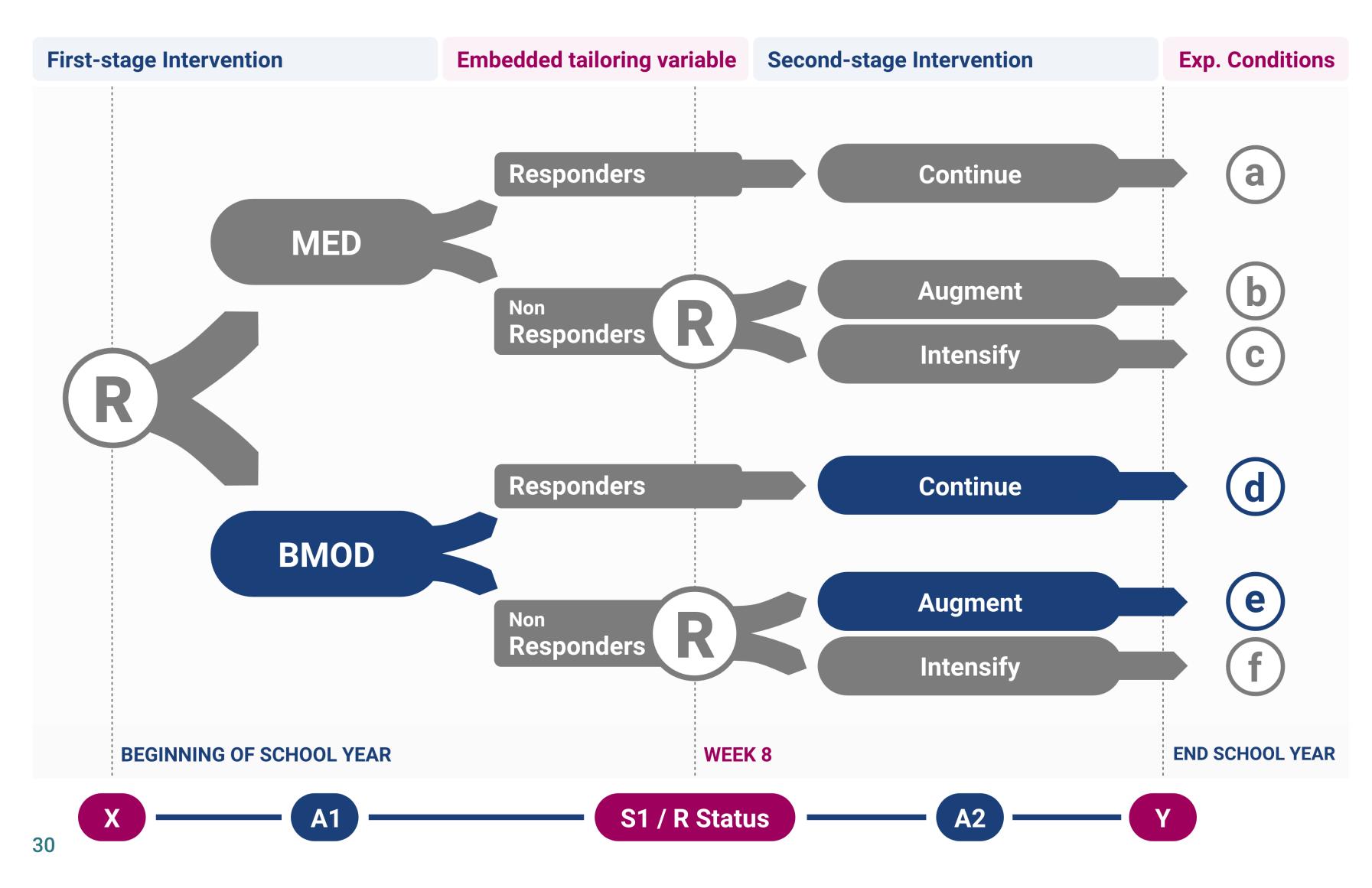


Notice that AI#1 and AI#3 (start MED) share responders (box A)



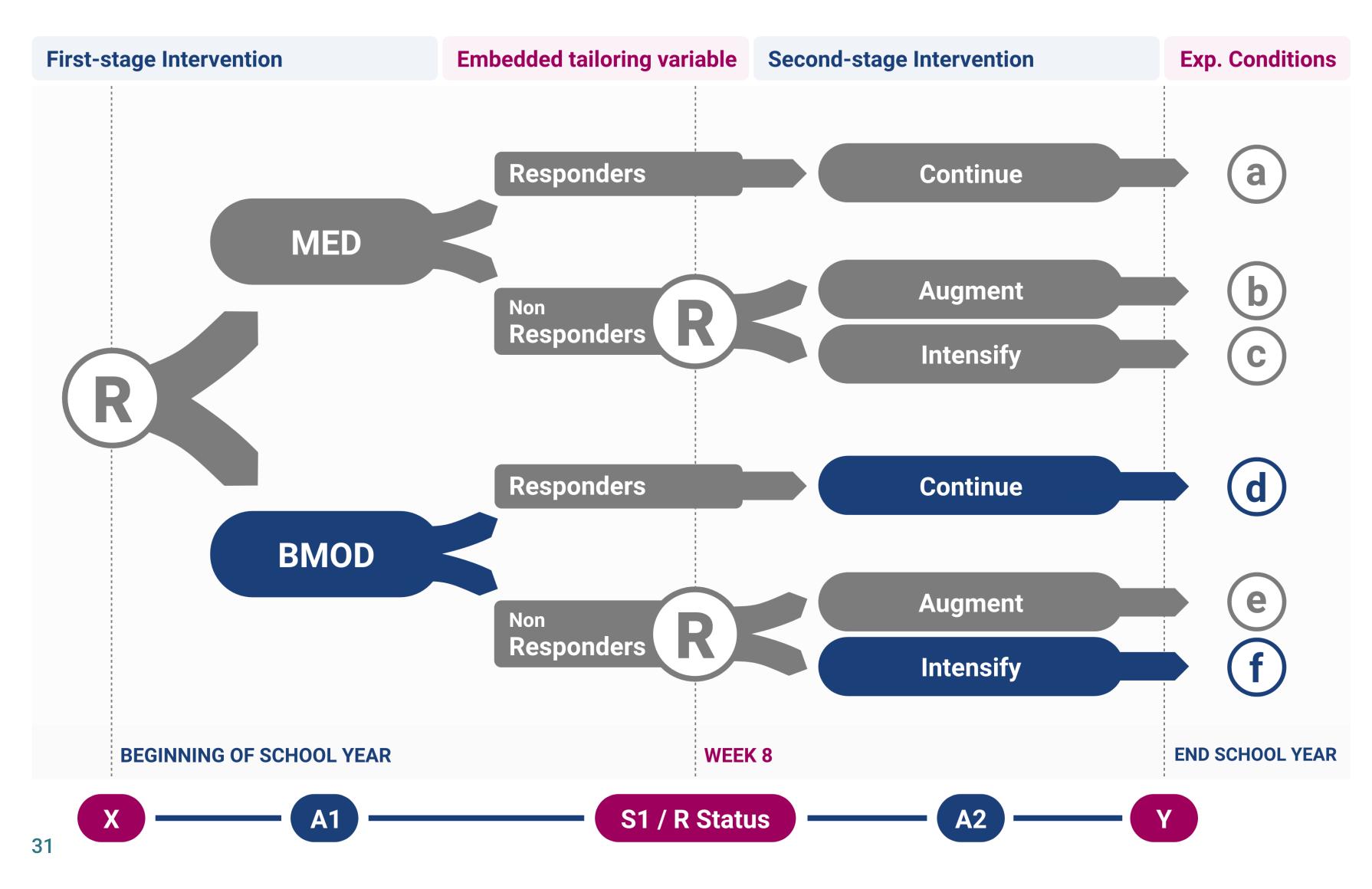


Similarly: Notice that AI#2 and AI#4 (start BMOD) share responders (box D)





Similarly: Notice that AI#2 and AI#4 (start BMOD) share responders (box D)





So, what's going on?

In the ADHD SMART, all responders are consistent with two Als

- Responders to MED are part of AI#1 and AI#3
- Responders to BMOD are part of AI#2 and AI#4

If our goal is to estimate the mean outcome under all Als simultaneously, we must share responders somehow.

• But how?



What do we do?

- We "trick" the software into using the responders twice
- We do this by replicating responders:
 - Create 2 observations for each responder
 - We assign $\frac{1}{2}$ of them A2 = 1, the other $\frac{1}{2}$ A2 = -1
- W=2 to responders and W=4 to non-responders
- Robust standard errors account for weighting and the fact that responders are "re-used". No cheating here!



Weighting and Replicating Serve Different Purposes

Weighting

- Accounts for over/underrepresentation of responders or non-responders
- Because of the randomization scheme

Replicating

- Because participants are consistent with more than one AI

Allows us to use standard software to do simultaneous estimation and comparison



SAS code for Replication-and-Weighting to Compare Means Under All Four Als

```
data dat9; set dat1;
if R=1 then do;
  ob = 1; A2 =-1; weight = 2; output;
  ob = 2; A2 = 1; weight = 2; output;
  end;
 else if R=0 then do;
  ob = 1; weight = 4; output;
  end;
run;
```



Replicated Data

Obs	ID	A 1	R	A 2	Y	o11c	o12c	o13c	o14c	ob	weight
45	32	1	1	-1	5	-0.35333	-2.73889	-0.31333	0.19333	1	2
46	32	1	1	1	5	-0.35333	-2.73889	-0.31333	0.19333	2	2
47	33	1	0	1	3	0.64667	-1.07820	0.68667	0.19333	1	4
48	34	1	0	1	2	0_25222	0.21662	-0.31333	0.19333	1	4
49	35	1	0	-1	5		ders are	-0.31333	0.19333	1	4
50	36	-1	0	1	1	replic	ated!	-0.31333	0.19333	1	4
51	37	-1	1	-1	1	-0.35333	0.99556	-0.31333	0.19333	1	2
52	37	-1	1	1	1	-0.35333	0.99556	-0.31333	0.19333	2	2
53	38	-1	0	-1	3	-0.35333	0.14034	0.68667	-0.80667	1	4
54	39	-1	1	-1	3	0.64667	1.64983	0.68667	0.19333	1	2
55	39	-1	1		3	0.64667	1.64983	0.68667	0.19333	2	2

Non-Responders aren't!



- **The Regression and Contrast Coding Logic:** Recall:
- Our goal is to compare all 4 embedded Als
- We have 2 indicators: A₁, A₂
 - $A_1 = 1 -> BMOD$
 - $A_1 = -1 -> MED$
 - $A_2 = 1 \rightarrow INTENSIFY$
 - $A_2 = -1 \rightarrow AUGMENT$
- To compare all 4 Als, we can fit the following Model:
 - $Y = \beta_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_1 A_2 + e$



The Regression and Contrast Coding Logic:

 $Y = \beta_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_1 A_2 + e$

Mean Y under AI#1 (MED, AUGMENT) = β Mean Y under AI#2 (BMOD, AUGMENT) = β Mean Y under AI#3 (MED, INTENSIFY) = β Mean Y under AI#4 (BMOD, INTENSIFY) =

$$B_{0} + \beta_{1}(-1) + \beta_{2}(-1) + \beta_{3}(-1)(-1)$$

$$B_{0} + \beta_{1}(1) + \beta_{2}(-1) + \beta_{3}(1)(-1)$$

$$B_{0} + \beta_{1}(-1) + \beta_{2}(1) + \beta_{3}(-1)(1)$$

$$B_{0} + \beta_{1}(1) + \beta_{2}(1) + \beta_{3}(1)(1)$$

$$A_1 = 1 -> BMOD$$

 $A_1 = -1 -> MED$
 $A_2 = 1 -> INTENSIFY$
 $A_2 = -1 -> AUGMENT$



The Regression and Contrast Coding Logic:

 $Y = \beta_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_1 A_2 + e$

Mean Y under Al#1 (-1, -1) = $\beta_0 + \beta_1(-1) + \beta_2(-1) + \beta_3(-1)(-1)$ Mean Y under Al#2 (1, -1) = $\beta_0 + \beta_1(1) + \beta_2(-1) + \beta_3(1)(-1)$ Mean Y under Al#3 (-1, 1) = $\beta_0 + \beta_1(-1) + \beta_2(1) + \beta_3(-1)(1)$ Mean Y under Al#4 (1, 1) = $\beta_0 + \beta_1(1) + \beta_2(1) + \beta_3(1)(1)$

$$A_1 = 1 -> BMOD$$

 $A_1 = -1 -> MED$
 $A_2 = 1 -> INTENSIFY$
 $A_2 = -1 -> AUGMENT$



The Regression and Contrast Coding Logic:

 $Y = \beta_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_1 A_2 + e$

Mean Y under Al#1 (-1, -1) = $\beta_0 + \beta_1(-1) + \beta_2(-1) + \beta_3(-1)(-1)$ Mean Y under Al#2 (1, -1) = $\beta_0 + \beta_1(1) + \beta_2(-1) + \beta_3(1)(-1)$ Mean Y under Al#3 (-1, 1) = $\beta_0 + \beta_1(-1) + \beta_2(1) + \beta_3(-1)(1)$ Mean Y under Al#4 (1, 1) = $\beta_0 + \beta_1(1) + \beta_2(1) + \beta_3(1)(1)$

Diff Al#1 – Al#2 = $(\beta_0 - \beta_1 - \beta_2 + \beta_3) - (\beta_0 + \beta_1 - \beta_2 - \beta_3) = -2\beta_1 + 2\beta_3$



proc genmod data = dat9;

class id;

model Y = A1 A2 A1 * A2;

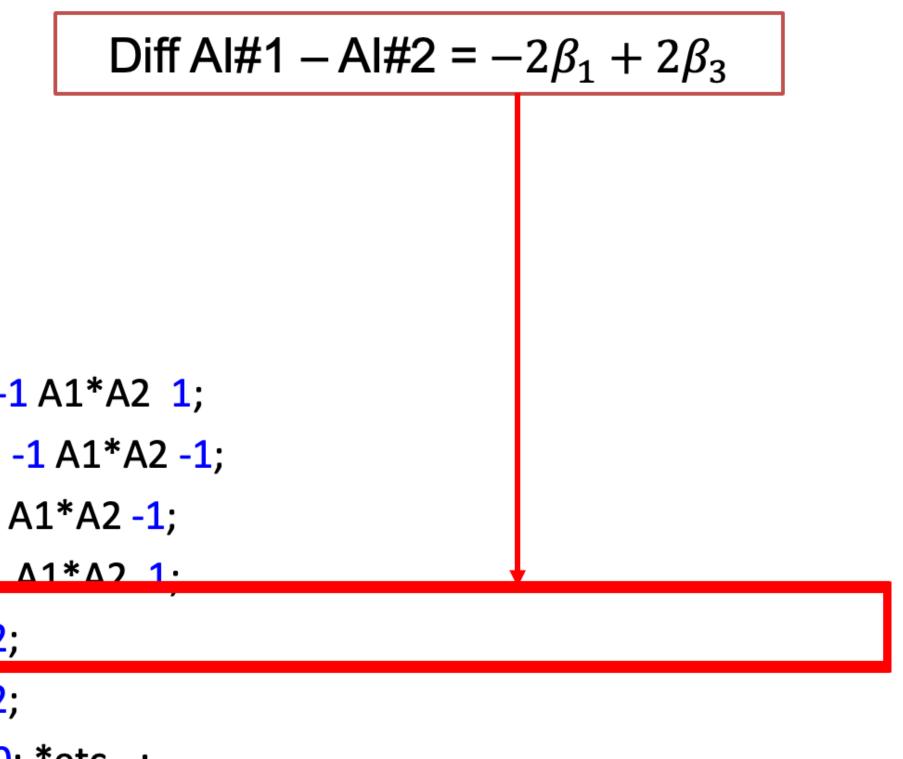
scwgt weight;

repeated subject = id / type = ind;

estimate 'MeanY:AI#1(MED,AUGMENT) ' int 1 A1 -1 A2 -1 A1*A2 1; estimate 'MeanY:AI#2(BMOD,AUGMENT)' int 1 A1 1 A2 -1 A1*A2 -1; estimate 'MeanY:AI#3(MED,INTENSFY)' int 1 A1 -1 A2 1 A1*A2 -1; estimate 'MeanY:AI#4(BMOD,INTNSFY)' int 1 A1 1 A2 1 A1*A2 1; estimate ' Diff: Al#1 - Al#2 ' int 0 A1 -2 A2 0 A1*A2 2; estimate ' Diff: Al#1 - Al#3 ' int 0 A1 0 A2 -2 A1*A2 2; estimate ' Diff: Al#1 - Al#4 ' int 0 A1 -2 A2 -2 A1*A2 0; *etc...; run;



proc genmod data = dat9; class id; model Y = A1 A2 A1 * A2; scwgt weight; repeated subject = id / type = ind; estimate 'MeanY:AI#1(MED,AUGMENT) ' int 1 A1 -1 A2 -1 A1*A2 1; estimate 'MeanY:AI#2(BMOD,AUGMENT)' int 1 A1 1 A2 -1 A1*A2 -1; estimate 'MeanY:AI#3(MED,INTENSFY)' int 1 A1 -1 A2 1 A1*A2 -1; estimate 'MeanV·AI#A(RMOD INITNISEV)' int 1 A1 1 A2 1 A1*A2 1. estimate ' Diff: Al#1 - Al#2 ' int 0 A1 -2 A2 0 A1*A2 2; ' int 0 A1 0 A2 -2 A1*A2 2; estimate 'Diff: Al#1 - Al#3 estimate ' Diff: Al#1 - Al#4 ' int 0 A1 -2 A2 -2 A1*A2 0; *etc...; run;





Comparing Mean Outcomes for All Als Simultaneously

Contrast Estimate Results								
	Mean Estimate	95% Confidence Limits		Standard				
Label		Lower	Upper	Error				
Mean Y under AI #1 (MED, AUGMENT)	2.643	2.5305	3.1992	0.1706				
Mean Y under AI #2 (BMOD, AUGMENT)	3.798	3.1643	3.8490	0.1747				
Mean Y under AI #3 (MED, INTENSIFY)	2.342	2.4644	3.1145	0.1658				
Mean Y under AI #4 (BMOD, INTENSIFY)	3.208	2.2515	3.0552	0.2050				
Diff: AI#1 – AI#2	-1.16	-1.799	-0.5347	0.323				
Diff: $AI#1 - AI#3$	0.0754	-0.3106	0.4614	0.1969				

<u>NOTE</u>: We get the exact same results as before when we compared AI#1 vs AI#2, but now we can simultaneously make inference for all the comparisons.

This analysis is with simulated data.



But wait, there's more...

Weighted-and-replicated regression can improve statistical precision (power)!



Replicated-and-Weighted Regression is More Efficient Statistically

proc genmod data = dat9;

class id;

model Y = A1 A2 A1*A2

YOc oddc;

scwgt weight;

repeated subject = id / type = ind;

estimate 'MeanY:AI#1(MED,AUGMENT) ' int 1 A1 -1 A2 -1 A1*A2 1; estimate 'MeanY:AI#2(BMOD,AUGMENT)' int 1 A1 1 A2 -1 A1*A2 -1; estimate 'MeanY:AI#3(MED,INTENSFY)' int 1 A1 -1 A2 1 A1*A2 -1; estimate 'MeanY:AI#4(BMOD,INTNSFY)' int 1 A1 1 A2 1 A1*A2 1; estimate ' Diff: Al#1 - Al#2 ' int 0 A1 -2 A2 0 A1*A2 2; estimate ' Diff: Al#1 - Al#3 ' int 0 A1 0 A2 -2 A1*A2 2; estimate ' Diff: Al#1 - Al#4 ' int 0 A1 -2 A2 -2 A1*A2 0; *etc...; run;

Improve power: Adjusting for baseline covariates that are associated with outcome leads to more efficient estimates (lower standard error = more power = smaller p-value).



Results for Weighted-and-Replicated Regression: Comparing Mean Outcome for all Als Simultaneously

Improved efficiency: Adjusting for baseline covariates resulted in lower standard error and tighter confidence intervals. Point estimates remained about the same, as expected.

Contrast Estimate Results							
	Mean	95% Confidence Limits		_ Standard			
Label	Estimate	Lower	Upper	Error			
Mean Y under AI #1 (MED, AUGMENT)	2.780	2.5869	3.1733	0.1496			
Mean Y under AI #2 (BMOD, AUGMENT)	3.750	3.0689	3.7018	0.1614			
Mean Y under AI #3 (MED, INTENSIFY)	2.311	2.5163	3.1135	0.1524			
Mean Y under AI #4 (BMOD, INTENSIFY)	3.212	2.3596	3.1081	0.1909			
Diff: AI#1 – AI#2	-0.97	-0.9401	-0.0704	0.2219			
D:ff. 1141 1142	0.0653	0.2011	0.4115	1767			



Results for Weighted-and-Replicated Regression: Comparing Mean Outcome for all Als Simultaneously

<u>Improved efficiency</u>: Adjusting for baseline covariates resulted in lower standard error and tighter confidence intervals. Point estimates remained about the same, as expected.

Contrast Estimate Results							
Mean	95% Confidence Limits		Standard				
Estimate	Lowe	r Upper	Error				
2.8801	2.5869	3.1733	0.1496				
3.3854	3.0689	3,7018	0.1614				
2.8149	2.5163	3.1135	0.1524				
2.7338	2.3596	3.1081	0.1909				
-0.5053	-0.9401	-0.0704	0.2219				
0.0653	0.2911	0.4115	0.1767				
42			0127 07				
g only data							
• •	4 ти	nis analysis is wit	h simulated (
	Mean Estimate 2.8801 3.3854 2.8149 2.7338 -0.5053 0.0000 2 g only data	Mean 95% C Estimate Lowe 2.8801 2.5869 3.3854 3.0689 2.8149 2.5163 2.7338 2.3596 -0.5053 -0.9401 2 11 2 11 2 11 2 11 2 11	Mean 95% Confidence Limits Estimate Lower Upper 2.8801 2.5869 3.1733 3.3854 3.0689 3.7018 2.8149 2.5163 3.1135 2.7338 2.3596 3.1081 -0.5053 -0.9401 -0.0704 2 0.4115 0.4115 g only data -0.5053 -0.9401				

This analysis is with simulated data.



Citations

- Murphy, S. A. (2005). An experimental design for the development of adaptive intervention. Statistics in Medicine, 24, 455-1481.
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