BlockScholes

5TH DECEMBER 2022

Block Scholes Implied Volatility Index (BSIV)

Comments or questions: research@blockscholes.com

SUBSCRIBE TO OUR PLATFORM

Implied Volatility Index (BSIV)

The Block Scholes Implied Volatility (BSIV) indices denote expected volatility of an underlying asset over a time horizon of 30 calendar days, and are presently available for BTC and ETH. It is well-known [1] that this value can be replicated via a portfolio of 30-day out-of-the-money (OTM) and at-the-money (ATM) put and call options over a continuum of strikes. Therefore the value of the BSIV requires only the implied volatility smile of the underlying at the 30-day expiry, and proceeds along two steps:

1. Compute the implied volatility smile for the 30-day expiry; and
2. Use that smile to compute the value of the index.

In the sequel, we elaborate on both these steps.

SMILE CALIBRATION

First, we identify the quoted maturities of interest. Either this is the 30-day expiry if already quoted, in which case we calibrate a SABR model to the market data to obtain the required implied volatility smile.

Most often, the 30-day expiry will itself not be quoted, hence we need to identify the two nearest bracketing expiries, for example, 26 days and 33 days. In this case, and at fixed time intervals (presently one hour, but intended to soon be published once a minute), options market data for these two bracketing expiries are collected from multiple derivatives exchanges (presently Deribit and ByBit), converted into implied volatilities using the Black-Scholes formula, and averaged into a composite volatility that represents the fair value of the market across trading venues.

Then we calibrate two smiles using the SABR model to these composite implied volatilities, one for each of the bracketing expiries. In all our SABR calibrations, we hold $\beta = 1$, and calibrate for the α (at-the-money level), ρ (correlation between spot and volatility) and ν (volatility of volatility) parameters. The forward curve is interpolated assuming piecewise constant forward yields. Using the bracketing smiles, we construct a sample of interpolated volatilities at the 30-day expiry by linearly interpolating cumulative variance along contours of constant forward moneyness of values ranging from 10% to 300% in steps of 10%. We then fit a final SABR model to these interpolated implied volatilities to obtain the 30-day implied volatility smile.

INDEX COMPUTATION

The BSIV index value is defined as

$$V = 100 \times \sqrt{\frac{2e^{RT}}{T}\left(\int_0^F \frac{P(K)}{K^2}dK + \int_F^\infty \frac{C(K)}{K^2}dK\right)}$$

Here, F and R denote the forward price of the asset at a 30-day tenor and the annualised risk free rate (assumed here to be zero) respectively. The parameter T measures time to expiry measure in years, which in the case of the 30-day volatility index is 30/365 = 0.082. For strikes K below the forward price F,

[1] Carr, Madan (1998). "Towards a Theory of Volatility Trading". In "Volatility: New Estimation Techniques for Pricing Derivatives," R. Jarrow (ed.) RISK Publications, London.

we use the put prices P(K), and for strikes above F, we use the call prices C(K) (all options expiring at time T). These option prices are computed using implied volatilities interpolated from the SABR model described earlier. The integrals are computed using numerical quadrature, before the result is square-rooted (to obtain an estimate of expected standard deviation) and multiplied by 100 to express it as a percentage value.

INDEX PUBLISHING

The BSIV index value described above is published once per hour. In the case of poor or corrupted market prices for options received from all exchanges, causing a failure to calibrate a SABR smile at 30-days, the index value is not reported. However, we note that in the three-year history, only 0.0007% of hourly BTC data points and 0.0012% of ETH data points were too poor from both exchanges simultaneously to successfully generate a valid index value.

Interpreting the BSIV Index

The SABR calibration that is used in the construction of the BSIV index describes the volatility market for a given expiry in three parameters: Alpha, Nu, and Rho. These three parameters roughly correspond to the ATM level of implied volatility, the steepness of the smile towards the wings, and the skew of the smile towards OTM calls or puts respectively.

By taking a weighted expectation of the implied volatilities across the strikes of the 30-day interpolated smile, the BSIV index captures each of these features and expresses them in a single summary value.

ATM VOLATILITY

The most important metric of the volatility smile is the volatility implied by an option struck at-the-money. This value controls the height of the volatility smile, and therefore corresponds to the overall level of volatility currently priced by the derivatives market when we ignore the premium assigned to an option for being struck far above or below the spot price. Figure 1 shows how this parameter has responded to several market crashes in 2022, spiking sharply in response to a crisis and falling gradually as the uncertainty dissipates.

The BSIV index value tracks the ATM implied volatility level closely, as the expectation of the implied volatilities of the SABR smile is most dependent on the absolute height of the smile. However, the BSIV value is higher due to the added influence of strikes away from the ATM strike on the expectation calculation.

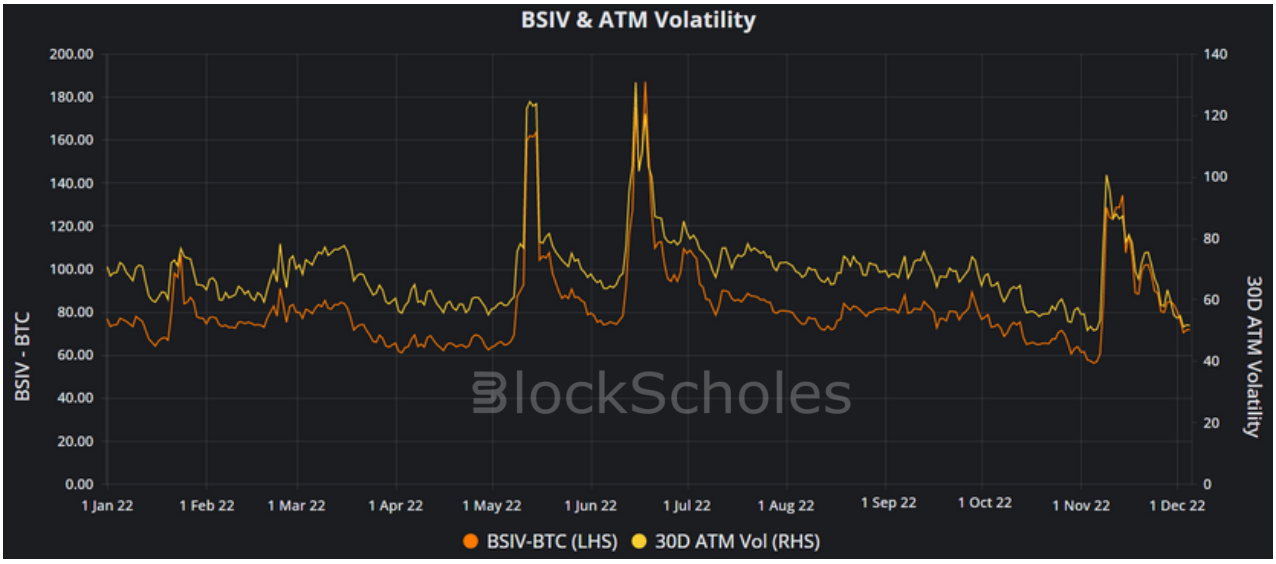

Figure 1 Daily BSIV-BTC index value (orange) and 30D ATM Volatility (yellow) from 1st Jan 22. *Source: Block Scholes*

BUTTERFLY

The 25-delta butterfly is defined by taking the average implied volatility of a 25-delta OTM call and a 25-delta OTM put, and calculating its distance from the implied volatility of an ATM option. This means that it is a robust measure of the steepness of the wings of the smile, ignoring any influence of skew to the left or right. It is a useful analogue to the SABR calibration's "volatility of volatility" parameter, which controls the curvature of the smile into OTM strikes.

Whilst the BSIV index value is most closely determined by the level of ATM volatility, a smile that is steep into the wings will have OTM strikes with implied volatilities much higher than the ATM level. Those strikes will contribute higher values to the expectation integral than a shallow smile, and therefore the difference in implied volatility between the BSIV index value and the smile's ATM level can be largely explained by a high butterfly value.

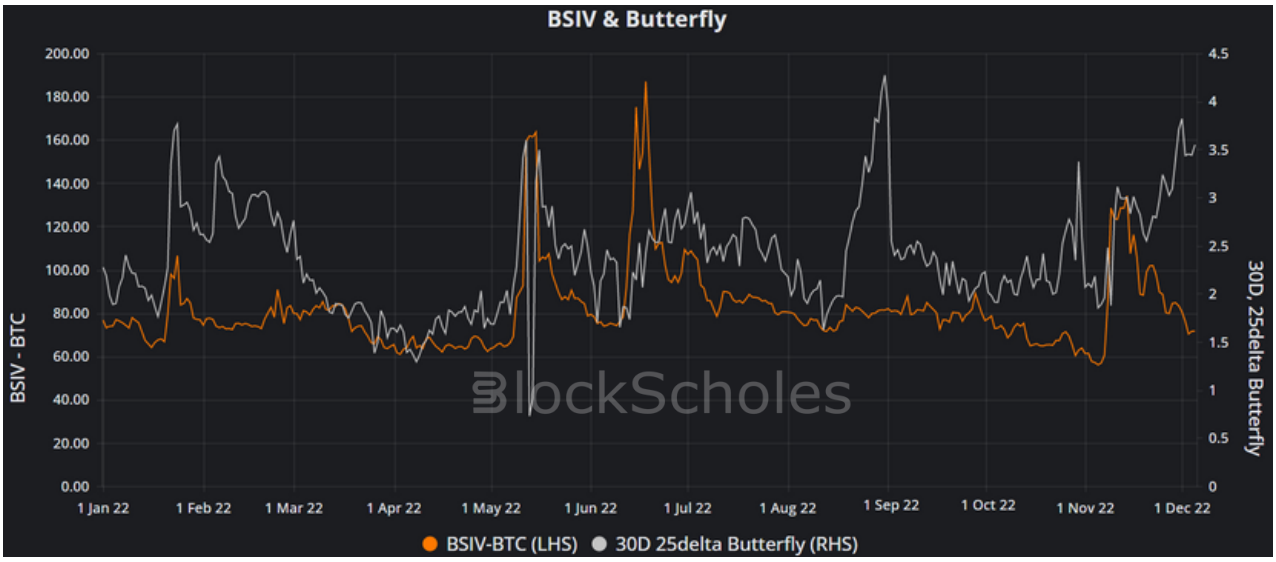

Figure 2 Daily BSIV-BTC index value (orange) and 30D, 25-delta butterfly(grey) from 1st Jan 22. *Source: Block Scholes*

PC SKEW

The final SABR parameter of interest, Rho, controls the skew of the volatility smile towards OTM calls or puts. We measure this in Figure 3 using the 25-delta PC-skew, the difference in implied volatility between a 25-delta put and a 25-delta call.

Throughout 2022, this parameter has moved inversely to the BSIV index. This is largely due to the nature of the spikes in ATM implied volatility levels being caused by market uncertainty. Therefore in times of great market stress, the BSIV index value acts as a measure of fear just as traditional market volatility indices do for equities markets.

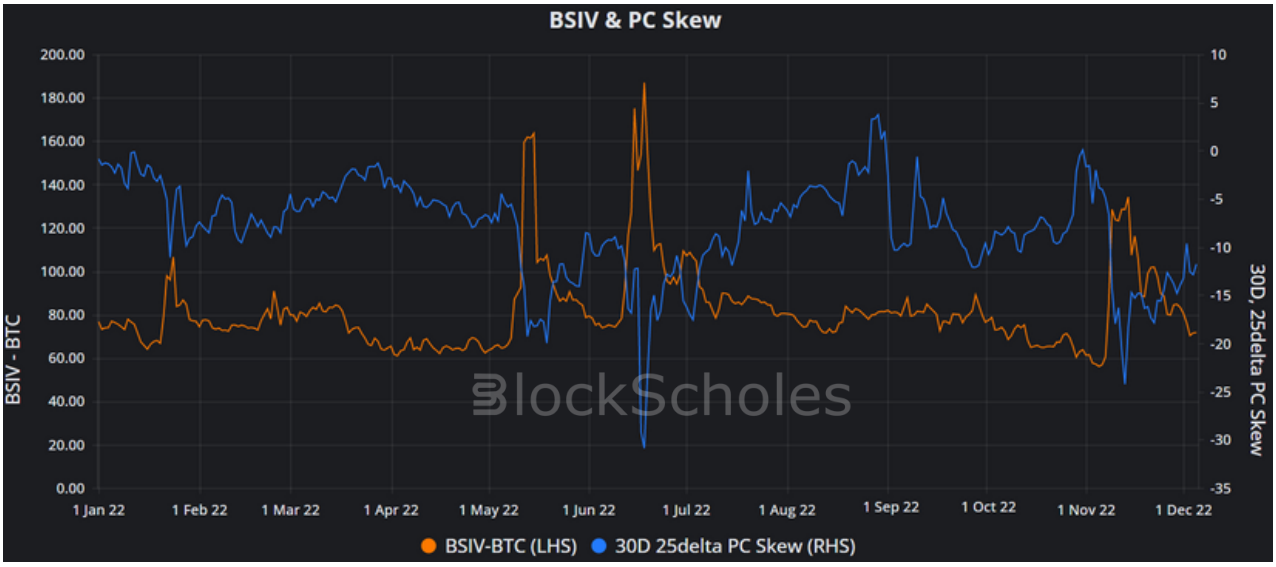

Figure 3 Daily BSIV-BTC index value (orange) and 30D, 25-delta PC skew (blue) from 1st Jan 22. *Source: Block Scholes*

NON-UNIQUENESS

Note that the same BSIV index value is possible from multiple smiles: a low ATM level with a steep smile may have the same expectation as a flat smile with a high ATM-level. This loss of information is natural, as the expectation represents a transformation of a three-dimensional (the three parameters of the SABR calibration) into a one-dimensional index value. In all cases, however, the index value represents the (weighted) average volatility across the smile, considering optionality demand at all strikes.

COMPARISON WITH VIX

Figure 4 Historical daily BSIV-BTC (blue) and VIX (grey) values from 1st Jan 2020.
Source: Block Scholes, Yahoo Finance

Like the VIX, the BSIV index is calibrated to a 30-day forward looking time horizon using bracketing expiries either side of the desired tenor. However, where the VIX is simply a weighted average of the options prices at the two expiries, the BSIV index relies on the calibration of a SABR volatility smile. Despite this, we expect similar functionality from both indices, as displayed in Figure 4.

Both indices peak in response to the COVID market crash in March 2020, and track similar events throughout 2022. However, the crypto-calibrated BSIV index remains much higher than the VIX, owing to the increased market volatility for which crypto-assets have become notorious. This showcases another use case of our index: fair and balanced comparison with other asset classes.

As expected, crypto-specific events such as the FTX collapse in November of 2022 cause divergences between equities and crypto, whereas macro-induced uncertainty (such as those dates around interest rate announcements) are mirrored by both indices. We expect that this index will be a useful measure of the correlation between those asset classes in the months to come.

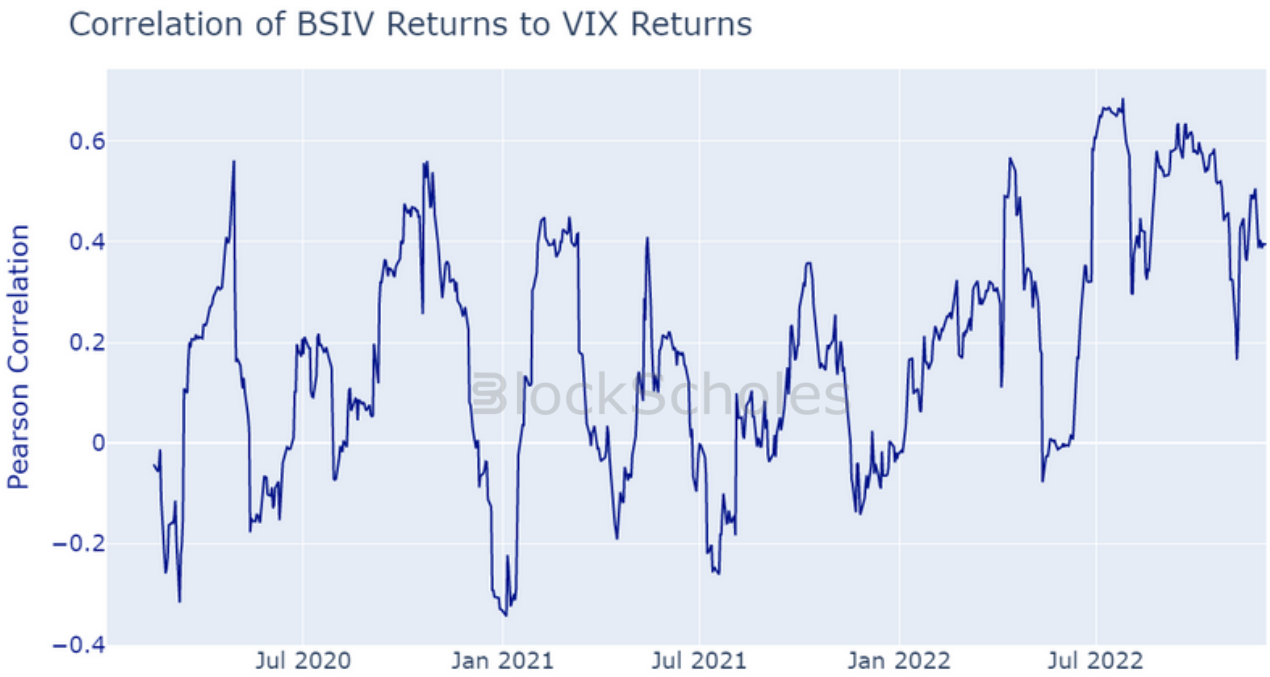

Figure 5 30-day, rolling Pearson correlation estimate of the percentage daily changes of the BSIV-BTC and VIX indices.
Source: Block Scholes, Yahoo Finance

Looking Ahead

In the future, we plan to enhance the composite volatility information by sourcing data from more exchanges, and to create similar indices on other underlyings once they develop a liquid options market. Considering BTC and ETH make up a significant portion of the crypto market, these volatility indices may be used as a broad gauge of market volatility over a 30-day forward looking period.

This also makes the BSIV index ideal for the settlement of tradable volatility products such as futures, swaps, options, and structured products, which are volatility derivatives that can be used to express views on BTC and ETH volatility. In this way, it lays the groundwork for institutional participants to trade volatility as an asset class.

CONTACT US

AUTHORS

ANDREW MELVILLE

Research Analyst

andrew.melville@blockscholes.com

KARAN JESWANI

Quantitative Analyst

karan.jeswani@blockscholes.com

Block Scholes Ltd.

27 Old Gloucester Street
London WC1N 3AX

research@blockscholes.com

SUBSCRIBE TO OUR PLATFORM