

Unit 1

Limits & Continuity

- Order of growth rates from fastest to slowest: $x^x, x!, a^x, x^p, x \ln(x), \ln(x)$
- Methods to algebraically simplify limits if you can't directly plug in: Completing the square, Rationalization, Factoring
- $\lim_{x \rightarrow c} (af(x)) = a \lim_{x \rightarrow c} f(x)$
- $\lim_{x \rightarrow c} (f(x) \pm g(x)) = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x)$
- $\lim_{x \rightarrow c} (f(x)g(x)) = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x)$
- $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$
- $\lim_{x \rightarrow c} f(g(x)) = f(\lim_{x \rightarrow c} g(x))$
- Continuity exists if $f(c) = \lim_{x \rightarrow c} f(x)$
- Intermediate Value Theorem:** Write "Since $f(x)$ is continuous on $[a,b]$ and $f(c)$ is between $f(a)$ and $f(b)$, by the IVT there is a c in (a,b) such that $f(c)=0$ "

Unit 2

Fundamentals of Differentiation

- All differentiable functions are **continuous**, but not all continuous functions are differentiable
- Average Rate of Change = $\frac{f(x+h) - f(x)}{x+h}$
- $f'(x) = \lim_{x \rightarrow h} \frac{f(x+h) - f(x)}{x+h} = \lim_{c \rightarrow x} \frac{f(x) - f(c)}{c-x}$
- $\frac{dy}{dx} x^n = nx^{n-1}$
- Power Rule:** $\frac{dy}{dx} x^n = nx^{n-1}$
- Sum/Difference Rule:** $\frac{dy}{dx} f(x) \pm g(x) = f'(x) \pm g'(x)$
- Product Rule:** $\frac{dy}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule:** $\frac{dy}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
- Other Differentiation Formulas:
 - $\frac{dy}{dx} c = 0$
 - $\frac{dy}{dx} kf(x) = kf'(x)$
 - $\frac{dy}{dx} e^x = e^x$
 - $\frac{dy}{dx} \ln(x) = \frac{1}{x}$
 - $\frac{dy}{dx} \sin(x) = \cos(x)$
 - $\frac{dy}{dx} \cos(x) = -\sin(x)$
 - $\frac{dy}{dx} \tan(x) = \sec^2(x)$
 - $\frac{dy}{dx} \cot(x) = -\csc^2(x)$
 - $\frac{dy}{dx} \sec(x) = \sec(x)\tan(x)$
 - $\frac{dy}{dx} \csc(x) = -\csc(x)\cot(x)$

Unit 3

Composite, Implicit, & Inverse Functions

- Chain Rule:** $\frac{dy}{dx} f(g(x)) = g'(x) \cdot f'(g(x))$
- $\frac{dy}{dx} a^x = a^x \ln(a)$
- $\frac{dy}{dx} \log_a(x) = \frac{1}{\ln(a)x}$
- Implicit Differentiation:** Differentiate each term with respect to the individual variables
- Derivatives of an Inverse Trig Function:

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{ x \sqrt{x^2-1}}$
$\csc^{-1} x$	$-\frac{1}{ x \sqrt{x^2-1}}$

Unit 4

Contextual Applications of Differentiation

- Particle Motion:
 - Position= $s(t)$
 - Velocity= $v(t)=s'(t)$
 - Acceleration= $a(t)=v'(t)=s''(t)$
- If velocity is negative, the particle is moving to the left.
- If velocity is positive, the particle is moving to the right.
- If velocity and acceleration have the same sign, the particle is speeding up.
- If velocity and acceleration have different signs, the particle is slowing down.
- Steps for Related Rates:
 - Draw a picture and label the picture (assign variables)
 - List your knowns and unknown values
 - Write an equation to model the situation.
 - Take the derivative of both sides. Remember: d/dt
 - Plug in known values and solve for desired values. DON'T FORGET UNITS!
- Linearization:** $f(c+a) \approx f'(c) \cdot a + f(c)$
- If $f(x)/g(x)$ are indeterminate that $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ and $\lim_{x \rightarrow c} g(x)$ are both equal to 0 or ∞ . Use **L'Hopital's Rule:**

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

Unit 5

Analytical Applications of Differentiation

- Mean Value Theorem:** Write "Since $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) , there exists a c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$ by the MVT.
- Extreme Value Theorem:** Write "Since $f(x)$ is continuous on (a,b) , by the EVT, there exists at least one local maximum and one local minimum on (a,b) ."
- Critical Points:** where $f'(x)=0$ or does not exist
- $f'(x) > 0$: increasing; $f'(x) < 0$: decreasing
- First Derivative Test:** where $f'(x) = 0$, if $f(x): - \rightarrow +$: local min; if $f(x) + \rightarrow -$: local max
- Determining **Concavity:** $f''(x) > 0$: concave up; $f''(x) < 0$: concave down; $f''(x) = 0$: inflection point
- Second Derivative Test:** $f'(x) = 0$, if $f''(x) > 0$: min; if $f''(x) < 0$: max; if $f''(x) = 0$: indeterminate
- Steps for **Optimization:**
 - Draw picture
 - Label your picture and assign variables
 - Write an equation and use given information to find relationships among variables
 - Find extrema (min/max) and evaluate the function

Unit 6

Integration of Accumulation of Change

- The **integral** is the area between the graph and the x-axis
- Riemann Sum** can be used to approximate area (includes left, right, midpoint, and trapezoidal sum)

$$\int_a^b f(x) dx \approx \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x \quad \text{where } \Delta x = \frac{b-a}{n} \text{ and } x_k = a + k\Delta x$$
- Fundamental Theorem of Calculus (FTC)-** Definite Integrals

$$\int_a^b f(x) dx = F(b) - F(a)$$
- Integration Formulas (NEVER FORGET + C):
 - $\int \sin(x) dx = -\cos(x) + C$
 - $\int \cos(x) dx = \sin(x) + C$
 - $\int \sec^2(x) dx = \tan(x) + C$
 - $\int \csc^2(x) dx = -\cot(x) + C$
 - $\int \sec(x)\tan(x) dx = \sec(x) + C$
 - $\int \csc(x)\cot(x) dx = -\csc(x) + C$
 - $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$
 - $\int e^x dx = e^x + C$
 - $\int a^x dx = \frac{a^x}{\ln(a)} + C$
- Integration by Parts (IBP):** $\int u dv = uv - \int v du$
- Learn the Tabular Method to make IBP easier
- Use **Partial Fraction Decomposition** to integrate rational functions
- Improper Integrals:** $\int_0^{\infty} f(x) dx = \lim_{n \rightarrow \infty} \int_0^n f(x) dx$



Unit 7

Differential Equations

- A **slope field** is a graphical representation of a differential equation in the form $dy/dx = f(x,y)$
- Logistic Differential Equation:** $\frac{dP}{dt} = kP(1 - \frac{P}{L})$ where P is the population, L is the carrying capacity, and k is a constant
- Euler's method** can be used to find the numerical values of functions based on a given differential equation and an initial condition



Unit 8

Applications of Integration

- Average Value** = $\frac{1}{b-a} \int_a^b f(x)dx$
- Average Value Theorem:** Write "Since $f(x)$ is continuous on (a,b) by the AVT, there must be a c in (a,b) where $f(c) = \frac{1}{b-a} \int_a^b f(x)dx$ "
- Acceleration = $a(t)$
- Velocity = $v(t) = \int a(t) dt$
- Position = $s(t) = \int v(t) dt$
- Speed = $|v(t)|$
- Distance traveled = $\int |v(t)| dt$
- Volume using the **Washer Method:** $\pi \int (Radius_{outer} - Radius_{inner})^2$
- Volume using the **Disc Method:** $\pi \int (Radius_{outer})^2$
- Arc Length:** $\int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx$



Unit 9

Parametric Equations, Polar Coordinates, & Vector Valued Functions

- Second Derivative of Parametric Equation: $\frac{(\frac{d}{dt})(\frac{dy}{dx})}{\frac{dx}{dt}}$
- Arc Length for Parametric Functions: $S = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$
- Slope of a Tangent Line for Polar Equations: $\frac{dy}{dx} = \frac{r \cos \theta + \frac{dr}{d\theta} \sin \theta}{-r \sin \theta + \cos \theta \frac{dr}{d\theta}}$
- Polar Conversions: $x = r \cos \theta$ $y = r \sin \theta$ $r = \sqrt{x^2 + y^2}$
- Area under Polar Curves: $A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$
- Area under two Polar Curves: $A = \int_{\alpha}^{\beta} \frac{1}{2} [(r_1)^2 - (r_2)^2] d\theta$
- Arc Length for Polar Functions: $L = \int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$



Unit 10

Infinite Sequences & Series

- Sequences: **arithmetic** and **geometric**
- Series:
 - Harmonic series** $\frac{1}{n}$ diverges, but $\frac{(-1)^n}{n}$ converges
 - Power Series** with terms $\frac{1}{n^p}$ converges when $p > 1$, else it diverges
 - Alternating Series:** For an alternating series (terms change sign), converges if $\lim_{n \rightarrow \infty} a_n = 0$, else it diverge
 - Taylor Series:** $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(x-c)}{k!} (x-c)^k$
 - MacLaurin Series** (make sure you memorize them!)
- Various tests are used to determine **convergence** and **divergence**
 - nth Term Test**
 - Limit Comparison Test**
 - Direct Comparison Test**
 - Integral Test**
 - Alternating Series Test**
 - Ratio Test**



100 FRQ Tips

- Be sure to show ALL of your work (even when using a calculator).
- Work on all of the parts that you know first before moving onto other parts. (Get those easy points!)
- Do not round any values as you complete the problem! Wait all the way until the end to round your answer to 3 or 4 decimal places.
- You can use abbreviations like IVT (Intermediate Value Theorem), MVT (Mean Value Theorem), and FTC (Fundamental Theorem of Calculus)
- Do not simplify your answers unless specified. You don't want to lose points on steps you don't need to do!
- Memorize your important theorems and convergence tests! You'll need to know the conditions where the theorems and tests are met.
- Keep an eye on the time and pace yourself.



Formulas

- Squeeze Theorem:** If $f(x) \leq h(x) \leq g(x)$ and $\lim_{x \rightarrow c} f(x) = L = \lim_{x \rightarrow c} g(x)$ then $\lim_{x \rightarrow c} h(x) = L$
- Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$
- Double Angle Formulas: $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$
 $\cos 2x = 2 \cos^2 x - 1$ $\cos 2x = 1 - 2 \sin^2 x$