fiveable

♾️AP Calculus AB – 2023 Cheat Sheet | 👉🏽 See all AP Calc study guides | @thinkfiveable

👑Unit 1
Limits & Continuity

- $\lim_{x \to c} f(x)$ is the value f(x) approaches when x → c from BOTH sides
- $\lim_{x \to c^{\pm}} f(x)$ is the value that f(x) approaches when x → c from ONLY the right (if +) or left (if -) side
- $\lim_{x \to c}(af(x)) = a \lim_{x \to c} f(x)$
- $\lim_{x \to c}(f(x) \pm g(x)) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$
- $\lim_{x \to c}(f(x)g(x)) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$
- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$
- $\lim_{x \to c} f(g(x)) = f(\lim_{x \to c} g(x))$
- Methods to algebraically simplify limits if you can't directly plug in: Completing the square, Rationalization, Factoring
- Order of growth rates from fastest to slowest: $x^x, x!, a^x, x^p, xln(x), ln(x)$
- For f(x)/g(x), if highest power of f > highest power of g: infinite limit DNE, if <, HA at y=0, if =, HA at y = ratio of first terms
- Continuity if $f(c) = \lim_{x \to c} f(x)$
- Removable: hole, Asymptote, and Jump: Piecewise where y-values different
- IVT PROBLEMS: Write "Since f(x) is continuous on [a,b] and f(c) is between f(a) and f(b), by the IVT there is a c in (a,b) such that f(c) = f(c)"

🤓Unit 2
Fundamentals of Differentiation

- AROC = $\frac{f(x+h) - f(x)}{x+h}$
- $f'(x) = \lim_{x \to h} \frac{f(x+h) - f(x)}{x+h} = \lim_{c \to x} \frac{f(x) - f(c)}{c}$
- When estimating f'(c) from a table, straddle both sides and use AROC, from a graph, slope of tangent line
- All differentiable functions are continuous, but not all continuous functions are differentiable
- Power Rule: $\frac{dy}{dx} x^n = nx^{n-1}$
- $\frac{dy}{dx} c = 0$
- $\frac{dy}{dx} f(x) \pm g(x) = f'(x) \pm g'(x)$
- $\frac{dy}{dx} kf(x) = kf'(x)$
- $\frac{dy}{dx} sin(x) = cos(x)$
- $\frac{dy}{dx} cos(x) = -sin(x)$
- $\frac{dy}{dx} e^x = e^x$
- $\frac{dy}{dx} ln(x) = \frac{1}{x}$
- Product Rule: $\frac{dy}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule: $\frac{dy}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
- $\frac{dy}{dx} tan(x) = sec^2(x)$
- $\frac{dy}{dx} cot(x) = -csc^2(x)$
- $\frac{dy}{dx} sec(x) = sec(x)tan(x)$
- $\frac{dy}{dx} csc(x) = -csc(x)cot(x)$

🤙🏽Unit 3
Composite, Implicit, & Inverse Functions

- $\frac{dy}{dx} a^x = a^x ln(a)$
- $\frac{dy}{dx} log_a(x) = \frac{1}{ln(a)x}$
- Chain Rule: $\frac{dy}{dx} f(g(x)) = g'(x) \cdot f'(g(x))$
- The chain rule is like unpeeling an onion, where you keep going from the outside in, you differentiate the outside function, plug in the inside function, and multiply by the derivative of the inside function.
- Implicit Differentiation: Differentiate each term with respect to the individual variables, and whenever you differentiate a y, multiply by $\frac{dy}{dx}$

  Ex. $\frac{d}{dx} xy = y + x\frac{dy}{dx}$
- Inverses:
- $\frac{dy}{dx} arcsin(x) = \frac{1}{\sqrt{1 - x^2}}$
- $\frac{dy}{dx} arctan(x) = \frac{1}{1 + x^2}$
- $\frac{dy}{dx} arcsec(x) = \frac{1}{|x|\sqrt{x^2 - 1}}$
- Derivatives of inverse trig cofunctions are the negative of the derivative of the other 3 inverse trig functions
- Higher order derivatives: Just repeat!
- Second derivatives of implicit functions are functions of $x, y, \frac{dy}{dx}$

fiveable

♾️AP Calculus AB – 2023 Cheat Sheet | 👉🏽 See all AP Calc study guides | @thinkfiveable

👀Unit 4
Contextual Applications of Differentiation

- The derivative of a function is the rate of change of that function
- If you are being asked about the rate of change of a rate of change, that's basically the derivative of f'(x), or f''(x)
- Particle motion: $\frac{d^2y}{dx^2}x(t) = \frac{dy}{dx}v(t) = a(t)$
- Steps for Related Rates:
  1. Draw picture
  2. List knowns and unknowns
  3. Write an equation to model the situation (DO NOT PLUG IN STUFF THAT CHANGES)
  4. $\frac{d}{dt}$
  5. Substitute for changing values
  6. Solve for desired value
- Linearization: $f(c+a) \approx f'(c) \cdot a + f(x)$
- L'Hopital's Rule (LHR): $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$ ONLY IF $\frac{f(x)}{g(x)}$ IS INDETERMINATE, that is $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ are both either 0 or ∞
- Sometimes you may need to use LHR multiple times
- Remember to plug in for the limit before doing LHR!!

✨Unit 5
Analytical Applications of Differentiation

- MVT PROBLEMS: Write "Since f(x) is continuous on [a,b] and differentiable on (a.b), there exists a c in (a,b) where $f'(c) = \frac{f(b) - f(a)}{b - a}$ by the MVT"
- Rolle's Theorem is MVT where f(b) = f(a)
- EVT PROBLEMS: Write "Since f(x) is continuous on (a,b), by the EVT, there exists at least one local maximum and one local minimum on (a,b).
- Critical Points: Where f'(x) = 0 or undefined
- Local Extrema: Point that is greater/less than surrounding points, always at critical points or endpoints
- Global Extrema: Greatest or Least value of function
- f'(x) > 0: inc, f'(x) < 0: dec
- If asking for whether the rate of change of f(x) is increasing or decreasing, this is asking for the sign of the second derivative!
- Where f'(x) = 0, if f(x): $- \to +$: local min, $+ \to -$: local max
- ENDPOINTS CAN BE EXTREMA TOO, REMEMBER THEM WHEN FINDING GLOBAL EXTREMA
- f''(x) > 0: ccu, f''(x) < 0: ccd, f''(x) = 0: inflection point
- Where f'(x) = 0, if f''(x) > 0: min, if f''(x) < 0: max, if f''(x) = 0: indeterminate
- Steps for Optimization:
  1. Draw picture
  2. Write primary equation
  3. Write constraint equation, solve for other variables, and plug into primary equation (if applicable)
  4. Find extrema of primary equation and solve for variables

🔥Unit 6
Integration of Accumulation of Change

- Accumulation/Integral is the area between a rate of change graph and the x-axis
- If below x-axis, then accumulating negative area
- Can use geometry to find integral from a graph (look for circles, triangles, squares, and places where positive and negative area cancel out)
- When a function is split into multiple subdivisions, y_1 is the left boundary, y_2 is the right boundary, y_3 is in the middle of an interval, and Δx is the interval width between the values that give y_1 and y_2

  LRS: $\sum y_1 \Delta x$

  RRS: $\sum y_2 \Delta x$

  MRS: $\sum y_3 \Delta x$

  Trap Rule: $\sum \frac{1}{2}(y_1 + y_2)\Delta x$
- Riemann Sum: $\int_a^b f(x)dx \approx \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\Delta x$ where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$
- FTOC Pt 1 (Definite Integrals): If F'(x) = f(x), then $\int_a^b f(x)dx = F(b) - F(a)$
- FTOC Pt 2: $\frac{d}{dx}\int_{a(x)}^{b(x)} f(x) = b'(x)f(b(x)) - a'(x)f(a(x))$
- Integrals can be used if there are jump or removable discontinuities
- $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$
- $\int cf(x)dx = c\int f(x)dx$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- If b in between a and c: $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$
- U-substitutions are your friend, use them!
- Good substitutions for u: inner functions, functions with higher powers, denominator/numerator, an antiderivative present in the function to integrate, may need long division or completing the square

💎Unit 7
Differential Equations

- Slope Fields show tangent line segments to the particular solution through that point
- If you can write the differential equation in the form $g(y)dy = f(x)dx$, it's a separation of variables problem, you can leave your answers implicitly (ex. $xy^2 = 3xln(y)$)
- When all constants from antidifferentiation are replaced with appropriate values, you get a particular solution when there is an initial value condition which the solution must go through
- $\frac{dy}{dx} = ky \to y = ce^{kx}$ where c = y(0) (exponential growth/decay)

🐶Unit 8
Applications of Integration

- Average Value of f(x) = $\frac{1}{b-a}\int_a^b f(x)dx$
- AVT PROBLEMS: Write "Since f(x) is continuous on (a,b), by the AVT, there must be a c in (a,b) where f(c) = $\frac{1}{b-a}\int_a^b f(x)dx$
- $\int(\int a(t)dt)dt = \int v(t)dt = x(t)$ where x(t) is displacement
- DISPLACEMENT $\neq$ DISTANCE TRAVELED, JUST LIKE HOW VELOCITY $\neq$ SPEED
- speed = |v(t)|
- Distance traveled = $\int |v(t)|dt$
- Area: If dx, then integral of top-bottom, if dy, then integral of right - left, same rules apply when finding radii for disk/washer method
- Washer Method Integrand: $\pi(R_{outer} - R_{inner})^2$ dx (or dy)
- Disk method is just washer method but with inner radius 0
- Sometimes you'll need to split your section - when the curves intersect in the middle
- Integrand for cross-sections is the area of the cross section

💯FRQ Tips

- Work on the parts you know you can do first before moving onto other parts!
- Be sure to show all your work still, even though it is a shorter test.
- Shorthand like IVT, MVT, FTOC for Intermediate Value Theorem, Mean Value Theorem, Fundamental Theorem of Calculus is fine!
- Don't simplify your answers if you don't need to! You don't want to unnecessarily lose points on steps you don't need to do!