Engaging Math Circles **Syllabus 2022-2023**Algebra B (Intermediate)



Algebra B

Course Summary

This course is meant for students ages 13 to 16 who have completed, at a minimum, the Common Core (or equivalent) curriculum for Grade 7, or Algebra 1 or an Introductory Algebra coursework.

Expected Outcomes

Students will expand on applications of linear equations and systems of linear equations and build their understanding of functions with respect to describing quantitative relationships. Use of variables will be extensive in this course, with a focus on quadratic and other non-linear relationships. Algebra B will focus more on higher-order polynomials: factoring, graphing, irrational roots, non-real roots, multivariable polynomials. Students will learn arithmetic and geometric sequences and series. They will further analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence. By the end of this course students should be able to:

- Write and manipulate expressions and equations in equivalent forms to solve problems
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Understand and solve equations involving exponents and logarithms
- Solve systems of linear and nonlinear equations
- Represent and solve equations and inequalities graphically
- Define, evaluate, and compare functions
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres

Focus Areas

Concepts, skills, and learning tools students see in this course include, but are not limited to:

- Polynomials
- Exponents & Logarithms
- Special Functions
- Sequences & Series
- Manipulations of Functions
- Complex Numbers
- Quadratic Functions
- Conics
- Polynomial Division
- Polynomial Roots
- Factoring Multivariable Polynomials
- Inequalities
- Radicals
- Piecewise Functions

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Complementary Coursework

- Logic puzzles incorporated in every class
- Math competition practice in a low-stress environment
- Fun games challenging students' observational skills, logical reasoning, spatial reasoning, and pattern recognition
- Dynamic discussions and debate on problem solving strategies

Pre Requisites

Students registering for this course should be comfortable with the following Math Topics:

- Algebra 1, Introductory Algebra or equivalent
- Arithmetic with positive and negative numbers, fractions, and decimals
- Arithmetic operations with variables
- Solving linear equations and systems of equations
- Factoring quadratic equations
- Manipulations of exponents and radicals
- Arithmetic by hand with at least 6-digit numbers
- Drawing and graphing points on a Cartesian plane

Students should also be willing and/or able to:

- Communicate in English either verbally or in writing
- Be respectful of other students in their classes
- Write down answers to any non-trivial problems
- Share their thoughts with the instructors to help them discover solutions to their problems
- Take constructive criticism when it comes to their learning habits

Course Materials (Required)

- All classes will be taught online, via **Zoom**. Your student will need a device with a microphone and camera.
- Homework will be assigned via <u>The Art of Problem Solving: Intermediate Algebra</u> textbook. (https://artofproblemsolving.com/store)

Course Materials (Optional)

- Additional *Optional* homework will be assigned in the <u>The Art of Problem Solving: Introduction to Algebra</u> textbook.
- We use the ebook of both texts in class. You may choose the physical textbook or ebook; both versions are identical in content.

Students should also have access to:

- Ruler, Protractor, Compass (to make circles)
- Calculator (to check your work only)
- Paper, Pencils and Erasers
- Colored pencils or markers
- Reliable internet connection and digital device



Course Itinerary

Below is a list of the topics that will be covered in this class in order:

Week (Dates)	Chapter	Title	Summary (Required Homework Reading)	Exercises
1 (Aug.22-28)	2	Functions Review	2.1 Functions Basics 2.2 Graphing Functions 2.3 Composition	Required 2.2.1 to 2.2.3 2.3 - all Exercises Optional Read and review all of Chapter 1: Basic Techniques for Solving Eqns
2 (Aug.29-Sept.4)			2.4 Inverse Functions Ch.2 Review & Challenge	Required 2.4 - all Exercises Review & Challenge - all problems not done in class Optional Intro to Algebra Chapter 16: Functions - Review Problems
3 (Sept.5-11)			3.1 Arithmetic of Complex No. 3.2 Complex Plane	3.1 - all Exercises 3.2.1 to 3.2.6
4 (Sept.12-18)	3	Complex Numbers	3.3 Real & Imaginary Parts 3.4 Graphing in Complex Plane Ch.3 Review & Challenge	3.3.1 to 3.3.6 3.4.1 to 3.4.3 Review & Challenge - all problems not done in class
5 (Sept.19-25)	4	Quadratics	4.1 Factoring Quadratics 4.2 Relating Roots & Coeff. 4.3 Completing the Square	4.1 - all Exercises 4.2.1 to 4.2.4 4.3 - all Exercises Check-In #1: Functions and Complex Numbers Covering Weeks 1-4
6 (Sept.26-Oct.2)			4.4 The Discriminant 4.5 Quadratic Inequalities Ch.4 Review & Challenge	4.4 - all Exercises 4.5 - all Exercises Review & Challenge - all problems not done in class
7 (Oct.3-9)			5.1 Parabolas 5.2 Problem Solving 5.3 Maxima & Minima	5.1.1 to 5.1.4 5.2 - all Exercises 5.3.1 to 5.3.4
8 (Oct.10-16)	5	Conics	5.4 Circles 5.5 Ellipses	5.4 - all Exercises 5.5 - all Exercises



9 (Oct.17-23)			5.6 Hyperbolas Ch.5 Review & Challenge	5.6.1 to 5.6.4 Review & Challenge - all problems not done in class
10 (Oct.24-30)	6	Polynomials	*Intro to Algebra Chapter 18: Polynomials 18.1 Addition & Subtraction 18.2 Multiplication	Required 6.1 - all Exercises Check-In #2: Quadratics and Conics Covering Weeks 5-9
			Intermediate Algebra 6.1 Polynomial Review	Optional Intro to Algebra Chapter 18: Polynomials - Review Problems
11 (Oct.31-Nov.6)			6.2 Intro to Polynom. Division 6.3 Synthetic Division	6.2 - all Exercises 6.3 - all Exercises
12 (Nov.7-13)			6.4 Remainder Theorem Ch.6 Review & Challenge	6.4.1 to 6.4.3 Review & Challenge - all problems not done in class
13 (Nov.14-20)	7	Polynomial Roots Pt.1	7.1 The Factor Theorem 7.2 Integer Roots 7.3 Rational Roots	7.1 - all Exercises 7.2.1 & 7.2.2 7.3.1 & 7.3.2 Check-In #3: Polynomials I Covering Weeks 10-12
14 (Nov.21-23)		THANKSGIVING WEEK (½ week)	FUN WEEK! (Games & Math Comp. problems)	NONE Happy Thanksgiving!
Thanksgiving (Nov.24-27)				
15 (Nov.28-Dec.4)	7	Polynomial	7.4 Bounds 7.5 Graphing & The Fundamental Thm of Algebra	7.4.1 & 7.4.2 7.5 - all Exercises
16 (Dec.5-11)		Roots Pt.1	7.6 Applications of Fundamental Thm of Algebra Ch.7 Review & Challenge	Review & Challenge - all problems not done in class
17 (Dec.12-18)	8	Polynomial Roots Pt.2	8.1 Irrational Roots 8.2 Nonreal Roots 8.3 Vieta's Formulas	8.1.1 to 8.1.4 8.2.1 to 8.2.5 8.3.1 to 8.3.6 Check-In #4: Polynomials II Covering Weeks 13-16
18 (Dec.19-22) (Jan.6-8)		Midterm Review (½ week)	Review Ch.1-7	NONE Happy Holidays!



19 (Jan.9-15)	8	Polynomial Roots Pt.2	8.4 Using Roots to Make Eqs Ch.8 Review & Challenge	8.4.1 & 8.4.2 Review & Challenge - all problems not done in class	
20 (Jan.16-22)	9	Factoring Multivariable Polynomials	9.1 Grouping 9.2 Sums & Diff. of Powers	9.1.1 to 9.1.7 9.2.1 to 9.2.7	
21 (Jan.23-29)			9.3 The Factor Thm for Multivariable Polynomials Ch.9 Review & Challenge	9.3.1 & 9.3.4 Review & Challenge - all problems not done in class	
22 (Jan.30-Feb.5)	10		Sequences &	*Intro to Algebra Chapter 21: Polynomials 21.1 Arithmetic Sequences 21.2 Arithmetic Series Intermediate Algebra 10.1 Arithmetic Sequences 10.2 Arithmetic Series	(Intermediate Algebra) 10.1.1 to 10.1.5 10.2.1 to 10.2.7 Check-In #5: Polynomials III Covering Weeks 17-21
23 (Feb.6-12)		Series	*Intro to Algebra Chapter 21: Polynomials 21.3 Geometric Sequences 21.4 Geometric Series Intermediate Algebra 10.3 Geometric Sequences 10.4 Geometric Series	(Intermediate Algebra) 10.3.1 to 10.3.5 10.4.1 to 10.4.5	
Ski Week (Feb.20-26)					
24 (Feb.13-19)	10	Sequences & Series	Intermediate Algebra 10.5 Sequences, Summation, Product Notation 10.6 Nested Sums *Skip Ch.10 Review & Summary	10.5.1 to 10.5.6 10.6 - all Exercises	
25 (Feb.27-Mar.5)	11	11 Manipulations & Induction	Manipulations	11.1 Brute Force 11.2 Ratios 11.3 Induction	11.2.1 to 11.2.4 11.3.1 to 11.3.4 Check-In #6: Sequences and Series Covering Weeks 22-24
26 (Mar.6-12)			11.4 Binomial Theorem *Skip Ch.11 Review & Summary	11.4 - all Exercises	
27 (Mar.13-19)	12	Inequalities	12.1 Manipulating Inequalities 12.2 The Trivial Inequality 12.3 AM-GM Inequality with Two Variables	12.1.1 to 12.1.4 12.2.1 & 12.2.2 12.3.1 to 12.3.5	



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28 (Mar.20-26)			12.4 AM-Gm w/ More Var. 12.5 Cauchy-Schwarz Ineq.	12.4.1 to 12.4.4 12.5 - all Exercises
29 (Mar.27-Apr.2)			12.6 Maxima & Minima Ch.12 Review & Challenge	12.6 - all Exercises Review & Challenge - all problems not done in class
30 (Apr.3-9)	13	Exponents & Logarithms	*Intro to Algebra Chapter 19: Exponents & Logarithms 19.1 Exponential Functions 19.2 Show Me the Money 19.3 Interest Problems Intermediate Algebra 13.1 Expon. Functions Basics	Required (Intermediate Algebra) 13.1.1 - all Exercises 10.4.1 to 10.4.5 Check-In #7: Logic and Inequalities Covering Weeks 25-29 Optional Intro to Algebra Chapter 19: 19.1 - all Exercises 19.2.1 to 19.2.4
Spring Break (Apr.10-16)				
31 (Apr.17-23)	13	Exponents & Logarithms	*Intro to Algebra Chapter 19: Exponents & Logarithms 19.4 What is a Logarithm? Intermediate Algebra 13.2 Intro to Logarithms 13.3 Logarithm Identities 13.4 Using Log Identities	(Intermediate Algebra) 13.2.1 to 13.2.5 13.3.1 to 13.3.5 13.4 - all Exercises
32 (Apr.24-30)			Intermediate Algebra 13.5 Switching btw Log-Exp. 13.6 Natural Log & Exp .Decay *Skip Ch.13 Review & Summary	13.5 - all Exercises 13.6 - all Exercises
33 (May 1-7)	14	Radicals	14.1 Raising Radicals to Powers 14.2 Eval. Express. w/ Radicals	14.1.1 to 14.1.6 14.2 - all Exercises Check-In #8: Powers and Logs Covering Weeks 30-32
34 (May 8-14)		4)	14.3 Radical Conjugates Ch.14 Review & Challenge	14.3 - all Exercises Review & Challenge - all problems not done in class
35 (May 15-21	15	Special Classes of Functions	15.1 Rational Functions & Graphs *Skip 15.2 15.3 Even & Odd Functions 15.4 Monotonic Functions	15.1 - all Exercises 15.3 - all Exercises

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36 (May 22-28)	16	Piecewise Functions	16.1 Intro to Piecewise Funct. 16.2 Absolute Value 16.3 Graphing Ab.Value	16.1 - all Exercises 16.2.1 to 16.2.6
37 (May 29-Jun.4)			16.4 Floor & Ceiling 16.5 Problem Solving	16.4.1 to 16.4.6 16.5.1 to 16.5.4 Check-In #9: Other Functions Covering Weeks 33-37
38 (June 5-11)	N/A	REVIEW	Review all of Algebra B	NONE - Have a great summer