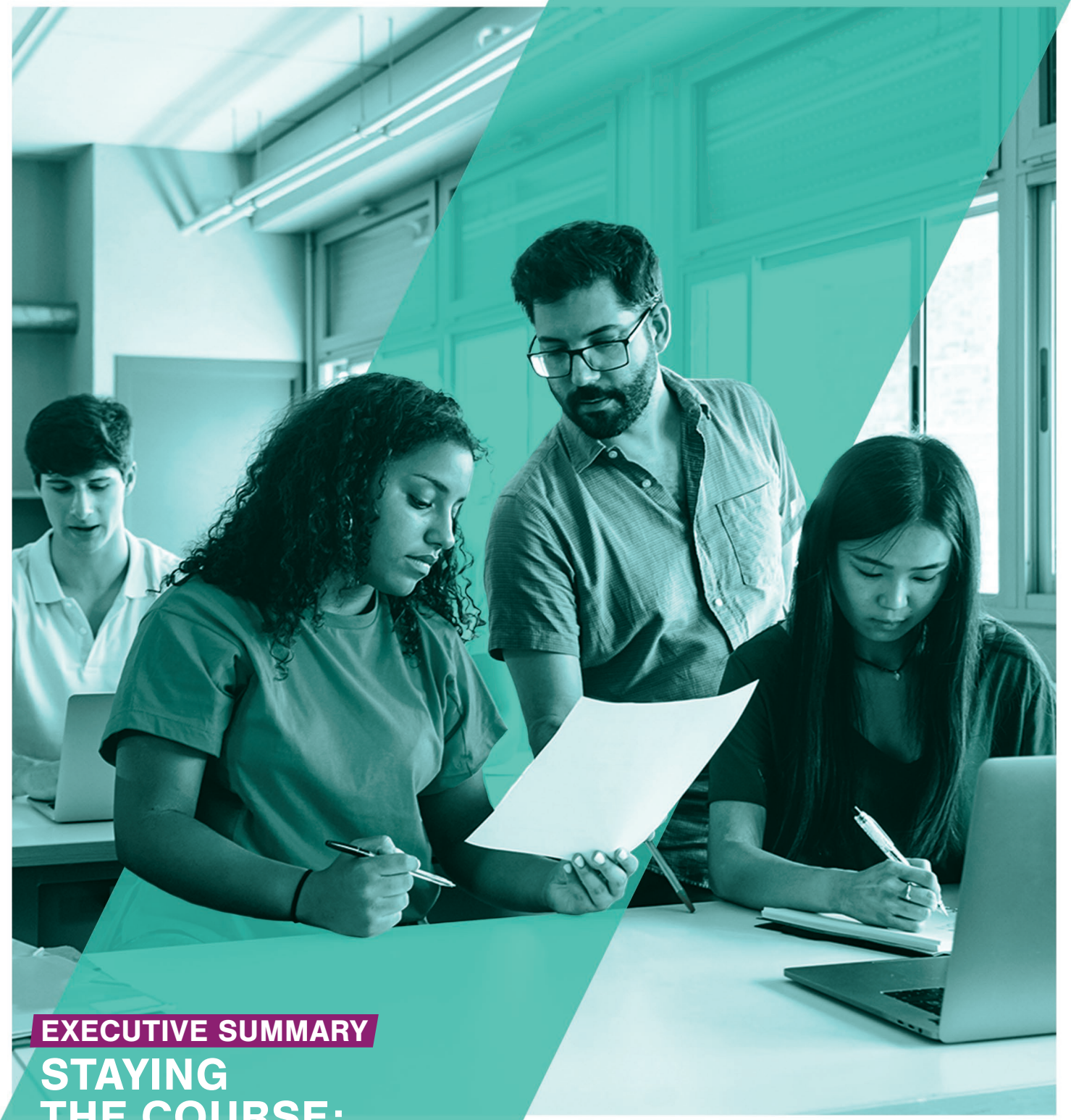


EXECUTIVE SUMMARY

STAYING THE COURSE:

Examining College Students' Paths to Calculus

By Marcelo Almora Rios and Pamela Burdman

January 2023

JUST EQUATIONS

Countless college students abandon their STEM ambitions after an encounter with calculus—or to avoid taking the intimidating course altogether.

While traditional approaches have focused on prerequisite courses—which often repeat high school coursework in algebra, precalculus and trigonometry—research has increasingly called that strategy into question.

ACKNOWLEDGMENTS

The authors are grateful to the California State University faculty and administrators who took the time to participate in interviews and respond to inquiries about Calculus foundation courses in the CSU system. The report could not have been written without their willingness to discuss current practices and their commitment to supporting students. Math faculty Geoffrey Buhl, Anne Cawley, Kimberly R. Elce, Rick Ford, Berit Givens, Julie Glass, Eric Hsu, Michael O'Sullivan, and Kate Stevenson participated in interviews. They and several others responded to email inquiries. Matthew Jones and Chenoa S. Woods also provided input on early findings. The final version of the report is stronger thanks to the experts who reviewed an initial draft: Lauren Asher, Kathy Reeves Bracco, Francesca Henderson, David T. Kung, Ricardo Moena, Lark Park, Laura M. Schueller, Rob Shorette, and Myra Snell. We also thank Just Equations staff members Katrina Goto for indispensable research assistance and Jenn BeVard for consummate project management. Lastly, we once again appreciate the expert editing of Jane Steinberg, the polished proofreading of Yael Katzwer, and the last in a series of dazzling designs by Christopher Artalejo-Price (we'll miss you, Christopher!).

ABOUT JUST EQUATIONS

Just Equations reconceptualizes the role of mathematics in ensuring education equity for students. An independent resource on the equity dimensions of math education in the transition from high school to college, Just Equations advances evidence-based strategies to ensure that math policies give all students the quantitative foundation they need to succeed in college and beyond. Just Equations' work is supported by College Futures Foundation, the Bill & Melinda Gates Foundation, and Valhalla Foundation.

ABOUT THE AUTHORS

Marcelo Almora Rios, a research fellow at Just Equations, recently completed his master's degree in mathematics at the University of Montana, having previously earned his bachelor's at Harvey Mudd College, also in mathematics. He has supported students' mathematics learning in many roles. His research interests lie at the intersection of mathematics education, psychology, and philosophy, with a particular interest in the concept of mathematical well-being. A first-generation, Peruvian-born college student, he was the recipient of the 2020 Hispanic Foundation of Silicon Valley Latinos in Technology Scholarship.

Pamela Burdman, founder of Just Equations, is a policy analyst and strategist on college access, readiness, and success, with a focus on equity in mathematics. She works at the intersection of education research, policy, and practice to synthesize knowledge from the field to define problems and advance strategies to support student success. She began her career as a reporter for the San Francisco Chronicle more than 20 years ago and first focused on math opportunity as a program officer at the William and Flora Hewlett Foundation.

Suggested citation: Almora Rios, M., & Burdman, P. (2023). *Staying the course: Examining college students' paths to Calculus*. Just Equations.

EXECUTIVE SUMMARY

Calculus is a stepping stone to STEM degrees and careers. But it is also known to be a stumbling block to those very objectives. Each year, nearly 1 million students attempt a college calculus or precalculus course without success (Gobstein, 2021, p. ix), and many abandon their STEM ambitions as a result. Women and minoritized students are most likely to be affected by this weed-out function (Seymour & Hunter, 2019; Hatfield et al., 2022).

While numerous strategies have been identified to improve success in college calculus courses (Bressoud & Rasmussen, 2015; Burdman et al., 2021), one challenge is that struggles in calculus are often attributed to varying levels of preparation among students interested in pursuing a STEM major. Traditional approaches have focused on prerequisite courses—which often repeat high school coursework in algebra, precalculus and trigonometry—but research reveals that strategies aimed at "fixing" students have not effectively improved STEM success for historically excluded groups (Hatfield et al., 2022).

Addressing this issue is especially relevant in the wake of the COVID-19 disruptions that compromised students' exposure to advanced math in high school.

About this report. This report focuses on the role of calculus prerequisites, courses that are considered college level but do not meet the requirements for most STEM majors. For STEM-interested students considered unprepared for college Calculus, do prerequisite sequences enhance their chances of succeeding in Calculus as intended? Or do they, like remedial courses, present barriers rooted in deficit perspectives of students?

We examine the apparent tension between the calculus prerequisite practices of many math departments and the implications of existing research by considering several questions, including:

- How have math departments structured their calculus prerequisite sequences?
- How do colleges determine students' eligibility for Calculus and, for those not ready, assign them to prerequisite courses or sequences?

We focused on the 23-campus California State University system. Besides being the nation's largest public university system, the CSU is also among the most diverse. The CSU has a long history of working to address college preparation issues, dating back to the 1990s. The system has also been a leader among postsecondary systems in adopting evidence-based reforms to remedial mathematics programs.

Our research included an analysis during summer 2022 of online information from all 23 CSU campuses, a review of relevant research literature, and interviews with math leaders from eight CSU campuses.

RESEARCH CONTEXT: STEM PERSISTENCE, COURSE SEQUENCES, AND PLACEMENT

Cultivating a talented and diverse STEM workforce requires solving the dilemma associated with attrition in students' pathways to STEM—attrition that disproportionately affects women and minoritized students (Eagan et al., 2014; Gates et al., 2012; Riegle-Crumb et al., 2019; Seymour & Hunter, 2019). Since attrition begins even before students reach Calculus, work to address Calculus' gatekeeper role needs also to consider students' pathways to Calculus.

Remedial prerequisites and exponential attrition. Research has consistently linked remedial sequences, especially long remedial math sequences, with reduced success (Bailey et al., 2010; Brathwaite et al., 2020; Cuellar Mejia et al., 2021; Edgecombe, 2011; Hern et al., 2020; Hodara, 2013). However, attrition has also been observed at four-year institutions (Complete College America, 2012).

Sequence length places a ceiling on ultimate success in Calculus and STEM. As such, an intended remedy for perceived underpreparedness can instead be a structural barrier to improving Calculus completion.

However, this issue will go unaddressed unless colleges begin to monitor sequence success rates in addition to course success rates.

Precalculus and other calculus prerequisites. No known studies have validated particular prerequisite courses or sequences for students at given preparation levels. However, some research has found significant attrition for students who enroll in college Precalculus, even those who have been successful in the course (Hsu & Bressoud, 2015; Thompson et al., 2007) and little benefit in Calculus for those who took Precalculus in college (Sonnert & Sadler, 2015). Such findings appear to implicate the length of the prerequisite sequences.

Course placement. Math department chairs rate placement as the most important feature of calculus paths but report low satisfaction with their placement solutions (Rasmussen et al., 2019). Placement practices need to consider sequence success, sometimes called throughput. Doing so generally counsels against placing students into lengthy sequences, which often lower a student's chances of eventually completing Calculus.

Similar to research on college admission tests (Geiser, 2017; Geiser & Santelices, 2007; Kurlaender & Cohen, 2019), studies on remedial placement tests repeatedly find that high school transcripts outperform placement tests in predicting student achievement, with less racially disparate impact (Hayward, 2017; Scott-Clayton, 2012; Scott-Clayton et al., 2014).

Calculus placement tests have received far less research attention, but it is common for math departments to use them (Burdman, 2017; Hsu & Bressoud, 2015). Approaches to improve the effectiveness of course placement include self-placement with appropriate guidance (Kosiewicz & Ngo, 2019; Cuellar Mejia et al., 2020; Brathwaite et al., 2022) and practice tests (Lewis, 2019).

Tension between research and practice. Practices such as multicourse prerequisite sequences and standardized placement tests have been criticized as reflecting a "deficit orientation to student performance," yet these practices are the norm at many public institutions. Further research is needed to understand the range of Calculus

prerequisite practices, the rationale behind them, and the associated outcomes for students at different preparation levels in the current environment.

CALCULUS PREREQUISITES: VARIATIONS IN FOUR THEMES

New policies adopted by the CSU system in 2017 to eliminate traditional remedial courses have enhanced students' chances of completing a college-level math class. However, it is less clear how those policies have affected the success of students pursuing STEM majors. A recent analysis found that only a fraction of entering students pursuing STEM majors enrolled in a Calculus I course in fall 2019 and identified racial disparities in access to calculus courses (Burdman et al., 2021).

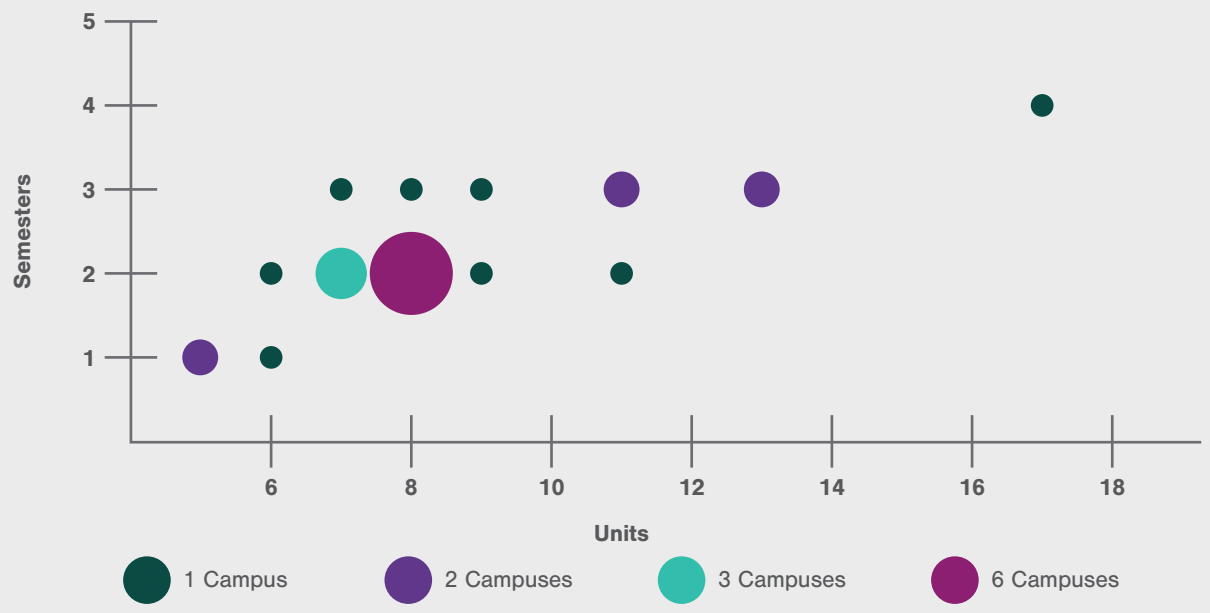
While uniform system policies preclude the use of remedial placement tests and remedial courses, campus-level policies determine the length and composition of prerequisite sequences, as well as individual students' entry points into those sequences. The intersection of these system and campus policies ultimately influences how, when, and whether such students enroll in and succeed in Calculus. Yet, there is little consensus in California or nationally on how to design pathways to Calculus.

The majority of CSU campuses have multiple routes to Calculus, depending on a student's placement level and other factors. Four themes stood out in our analysis: long on-ramps, course confusion, reliance on testing, and range of supports.

Long on-ramps. Prerequisite pathway lengths vary across CSU campuses, with some three- and four-course sequences that seem to mimic the remedial course sequences that were eliminated via executive order. At some campuses, students are disproportionately enrolled in longer routes to Calculus.

Maximal Pathways by Semesters and Units

With Counts



***SOURCE:** Just Equations' analysis of campus course offerings. See appendix for campus-specific data.*

The length of prerequisite pathways—whether in semesters or units—is important, because it reflects the opportunity cost (in terms of time, money, and forgone classes) of getting to Calculus. The risk of lengthy sequences is that the opportunity cost can be too great for students to bear, even when they may be passing individual courses along the way.

Prerequisite pathway length, as measured by semesters and units, varies across the CSU system. The most common pattern appears to be a two-semester sequence. However, seven campuses offer pathways of three or more semesters, meaning that students with the lowest placements taking the maximal sequence could take Calculus, at the earliest, in the middle of their sophomore year. The longest pathway we identified is Chico State's four-semester, 17-unit pathway. Three campuses offer only a one-semester prerequisite, though these can have high unit counts: six at Monterey Bay and five each at Sonoma State and Cal Maritime.

Interestingly, the length of maximal sequences did not appear to be tied to the preparation levels of students at the campus, based on historical data on campuses' math proficiency levels published by the CSU.

Many campuses defaulted to a two-course pathway, regardless of students' incoming proficiency levels. Of the seven campuses with proficiency rates over 80 percent, four had three-semester maximal pathways.

Stretching the content. One way of offering longer sequences is by spreading out a single course over a longer period. Many campuses offer both a one-semester version of College Algebra or Precalculus and a two-semester "stretch" version, for example.

Adding lower-level courses. Several campuses have added a college-level prerequisite since eliminating remedial courses under Executive Order 1110, which took effect in 2018. Examples include San Diego, Chico, and San José State.

Predominance of longer routes. At many campuses, students are more likely to be taking longer routes than shorter routes to Calculus. Lower-level prerequisites frequently enroll proportionately more students, as indicated by the numbers of course sections offered for each course. This trend appears at some campuses that enroll fewer students considered proficient, as well as at campuses with higher preparation levels.

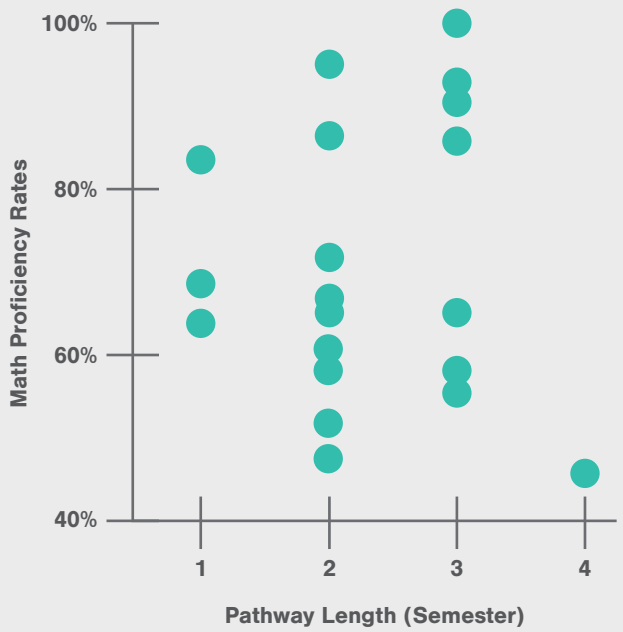
Course confusion. Some campuses offer multiple paths to one or more calculus courses, with limited transparency about which are required or optimal, and a variety of course titles, which may complicate students' choices (when they have choices).

Most campuses have two or more pathways to one or more calculus courses. The number of paths does not appear to be correlated with campus size. Campuses with more routes tend to have longer pathways and more entry points. The sheer array of paths and starting points can complicate students' coursetaking decisions and counselors' advice. The varying designs across campuses also create challenges for researching the effectiveness of different pathway designs and placement practices.

Variation of course content, units, and titles. How campuses translate units and semesters into math content and course titles varies widely. Some campuses confer fewer credits than others for a given course or sequence length. Taking longer routes might entail trudging through more math content than is covered in shorter pathways. It also could mean covering the same content at a slower pace—or in greater depth. The variety of course titles adds to the lack of clarity.

Math Proficiency and Pathway Length

Not Correlated



Graph relates campuses' 2020 math proficiency rates (defined as proportion of students ready for a college-level math course or who have completed a college-level math course) to fall 2022 maximal pathway length in semesters (or quarters, in one case). Source: California State University (July 2021). See appendix for campus-specific data.

Transparency and accuracy. Information about math sequences in course catalogs and on websites was not always accurate or up to date. Counselors, advisors, and first-year students often rely on the accuracy of online information; this is especially true for students who have less access to guidance during the enrollment process.

Most campuses lacked clear visual guides to their math sequences. A few had guides that were out of date or inconsistent with the information in their course catalogs. Seven sites still reference required scores on an exam that was retired four years earlier, in 2018.

RELIANCE ON CALCULUS PLACEMENT TESTS. Although remedial placement tests have been eliminated, most campuses continue to emphasize standardized placement tests for students seeking to enroll in Calculus, and the use of these tests plays a role in the longer routes to Calculus.

The fact that students deemed unprepared for Calculus appear disproportionately to enter longer routes to Calculus raises the question of how students arrive there. A continued reliance on placement exams is part of the explanation. Since the CSU system eliminated placement testing and traditional remedial courses in 2018, all incoming students are eligible

to be placed directly into college-level math courses, with support as needed.

But, because campuses have one to four college-level prerequisites for Calculus, eligibility for a college-level math course does not ensure that a student seeking to pursue a STEM field will have immediate access to Calculus. At most campuses, such students are subjected to two placement processes—the systemwide process for placement into a general education math course and a campus process for access to Calculus.

Examining math placement tests. Most CSU campuses use placement tests in addition to high school transcripts, but not in consistent ways. Most use an exam called Assessment and Learning in Knowledge Spaces for Placement, Preparation, and Learning, or ALEKS PPL, though with a variety of cutoff scores. Our research did not uncover an explanation for the wide variation.

Range of supports. There is large variation in the kinds of support strategies and structures campuses across the CSU landscape use. Among the most common structures are Early Start Program summer bridge courses, corequisites, and stretch courses.

CSU math leaders described a range of strategies to support math learning, including evidence-based approaches for promoting students' positive attitudes about mathematics and addressing the marginalization that some students experience in math class. There has been less examination, however, of how those **strategies** relate to specific support **structures.** These include:

Corequisite support models. The aim of the corequisite model is to provide support to students while they are enrolled in a college-level class, to allow them to progress toward a degree instead of taking remedial courses. In all, 21 out of 23 CSU campuses offer some form of corequisite support, with campuses offering up to six courses classified as support courses, workshops, or recitations for students taking Calculus I or prerequisite courses. Campuses can require only one unit of noncredit support, and only for students at the lowest preparation level. To improve effectiveness, some campuses offer two hours per week of corequisite support, despite being funded for only one.

Stretch models. One feature of the CSU's policies to eliminate remedial courses is the use of stretch courses that spread one semester of content over two semesters. This feature contributes to the wide variation in pathways across CSU campuses, with Precalculus offerings, for example, ranging from three units in some places to 10 units in others. Little research has assessed the effectiveness of this model as compared to single-semester corequisites.

Summer bridge courses. Under system policies, incoming students with the lowest preparation levels are expected to take summer bridge courses through the CSU campuses' Early Start Program.

More research is still needed to understand how various support structures can remain sustainable and effective resources for large, public universities seeking to advance students' progress in the math courses required for STEM majors.

PREREQUISITES THROUGH A MATH DEPARTMENT LENS

Foremost among faculty concerns informing these practices was an unease about the ability of students with weaker math backgrounds to succeed in Calculus.

Fear of failure. In discussing their sequences and placement policies, some faculty cited specific data on or experience with course failure rates of less-prepared students. Absent rigorous research, it is hard to know whether these faculty concerns, while well-intended, accurately reflect what is needed to prepare CSU students. None of the faculty we spoke with provided data to demonstrate that lengthier sequences enhanced student success in completing Calculus and entering STEM fields.

Student choice. A few campuses prioritize student choice through self-placement approaches. When San Francisco adopted this approach, many students who likely could have succeeded in a single semester of Precalculus were choosing the two-semester stretch course. Students' success rates in completing the sequence have improved, but more research is needed to show whether self-placement is disadvantaging any groups of students by decreasing their likelihood of completing Calculus.

FURTHER RESEARCH AND COURSE CORRECTIONS

The interviews for this report as well as existing research point to numerous opportunities for pursuing the goal of STEM preparation in ways that reduce racial and gender inequities:

Further research. Absent evidence, campuses run the risk of unintentionally relying on biases regarding student capabilities, reinforcing and replicating societal inequities. The most important recommendation emerging from this report, therefore, is longitudinal research to ensure that campuses' prerequisite approaches are effective in promoting success in Calculus and in STEM fields. Attention should be paid to:

- Comparing sequence success for students placed directly into Calculus with concurrent support to that for students at similar placement levels required to take prerequisite courses.

- Analyzing the range of placement test cutoffs and placement policies to validate placement practices, including self-placement, by determining which of them maximize completion in equitable ways.

What campuses can do. Steps that universities can take to support students' success in the pathway to and through Calculus include:

- Implementing the shortest possible sequences that support students' success and persistence into STEM majors, replacing lengthy sequences with just-in-time support to the extent possible.

- Promoting positive math classroom experiences by adopting course content relevant to students' contexts and interests and addressing faculty implicit bias.

What higher education systems or state-level agencies can do. College systems, university systems, and higher education boards can contribute by:

- Investing in data collection and research to address the questions highlighted above and support campus-level data collection and research.

- Make access to success in STEM fields a component of efforts to improve completion of general education mathematics.

- Support efforts to increase alignment of practices, including prerequisite sequences and placement policies, to increase transparency, support transfer and other transitions, and enhance research into the effectiveness of practices.

JUST EQUATIONS

justequations.org