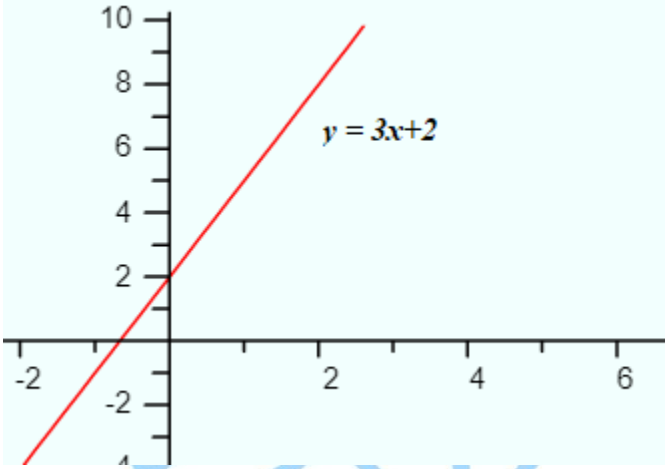
SAT Math Study Guide

Linear Equation

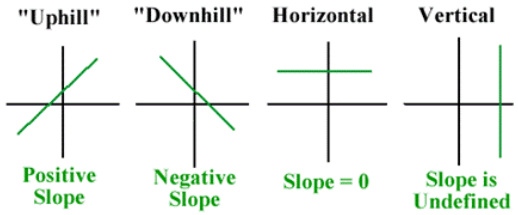
- An equation that lies on the graph and goes in a continuous straight line
- Slope-intercept form: y = mx +b
    - m is the slope
    - b is the y-intercept



Source:https://www.calculushowto.com/types-of-functions/linear-function/

4 Types of Slopes

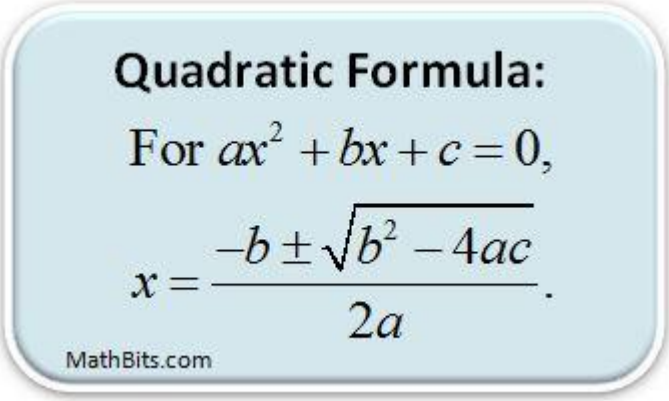
- Positive slope: If m is positive, then the line is uphill/upward-sloping
- Negative slope: If m is negative, then the line is downhill/downward-sloping
- Slope = 0: If m is 0, then the line is horizontal
- Undefined slope: If m is undefined, then the line is vertical

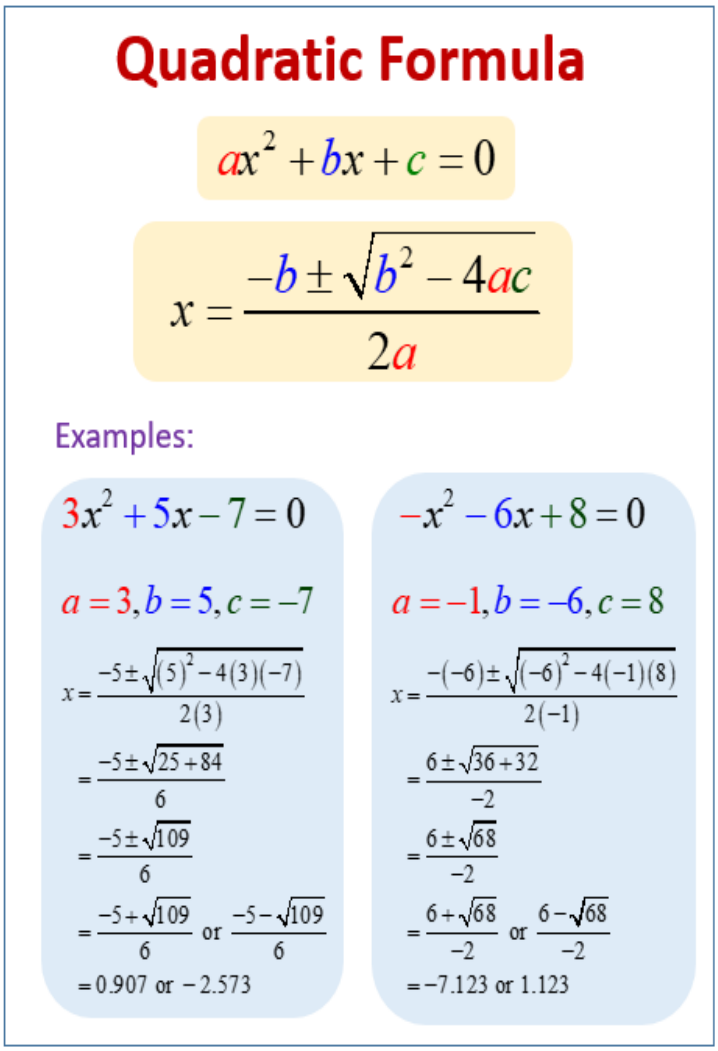


Source: https://courses.lumenlearning.com/beginalgebra/chapter/read-or-watch-find-slope-from-a-graph/

- To memorize the slope - use VUX HOY
    - V: vertical line
    - U: undefined
    - X: x = #
    - H: Horizontal
    - O: slope = 0
    - Y: y = #
- How to find the slope (m) of an equation: $m = (X_2 - X_1) / (Y_2 - Y_1)$

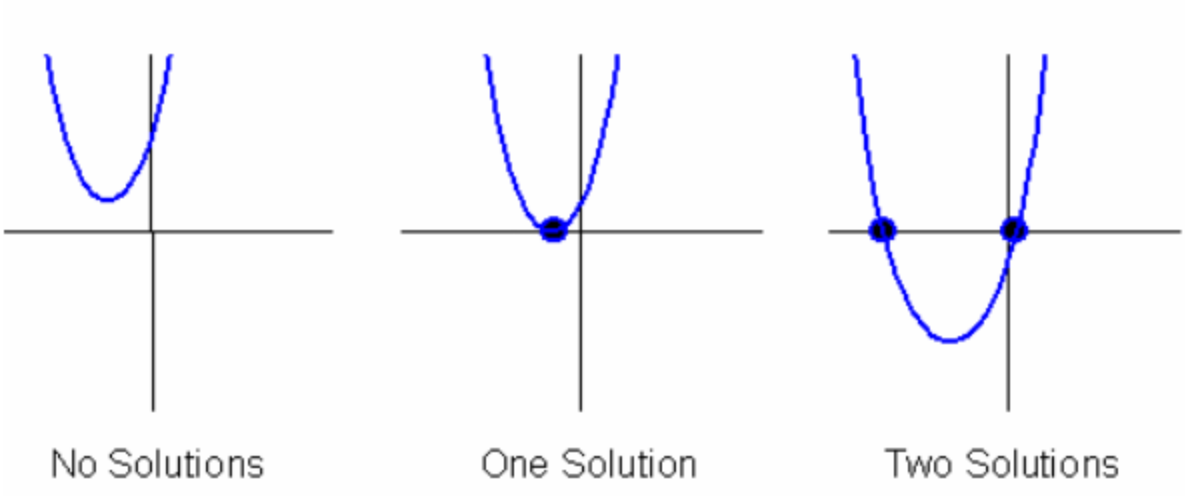
Quadratic Equations





Source: https://www.onlinemathlearning.com/quadratic-formula.html

- Discriminant
  - Definition: the discriminant tells us the type of roots the equation has
  - Formula: $D = b^2 - 4ac$
  - If $D < 0$, there are 2 imaginary roots (conjugates)
  - If $D > 0$, there are 2 real roots
  - If $D = 0$, there's 1 real root (twice)



Source:http://herodquadratics.weebly.com/the-discriminant-q18---q27.html

- Vertex Form: $y = a(x - h)^2 + k$
    - (h,k) is the point of the vertex
    - How to find "a"
        i. Find the point of the vertex
        ii. Find the point that passes through the line
        iii. Substitute the "h" and "k" into the equation
        iv. Substitute the "x" and "y" into the equation
        v. Solve the equation in terms of "a"

Graphing Terms

- Maximum - the highest point of "y" in the graph
- Minimum - the lowest point of "y" in the graph
- Domain - all the values "x" for which f(x) is defined
- Range - the set of all "y" values for which f(x) is defined
- Intercepts
    - x - intercepts: values that makes f(x) = 0 and lies on the x -axis
    - y - intercepts: values that makes f(0) = # and lies on the y - axis

Looking at the Graph to make an Equation

- Find the points on the graph that intersect the x - axis, and then make it into factored form
    - Example: The graph g(x) crosses the x-axis at x = 1, -2
        - $g(x) = (x - 1)(x + 2)$
    - Look at how many turns does the graph has (turn = when the slope changes)
        - To find the number of turns, you look at how many times it curves then subtract by 1.

Factor Formula to know

- Difference of Squares
    - $a^2 - b^2 = (a + b)(a - b)$
        - Example: $9x^2 - 121 = (3x + 11)(3x - 11)$
- Factor by Grouping
    - Example: $2x^3 - 10x^2 + 3x - 15$
        - $2x^2(x - 5) + 3(x - 5)$
            - Answer: $(x - 5)(2x^2 + 3)$
- Perfect Binomial Squared
    - $(x + y)^2 = x^2 + 2xy + y^2$
        - Example: $(3x + 2y)^2 = 9x^2 + 12xy + 4y^2$
    - $(x - y)^2 = x^2 - 2xy + y^2$
        - Example: $(6x - 3y)^2 = 36x^2 - 36xy + 9y^2$

- Complex Numbers
    - Definition: numbers that contain a real number and an imaginary number
    - Formula: a + bi
        - "a" is a real number
        - "b" is an imaginary number
    - $i = \sqrt{-1}$
    - $i^2 = -1$
    - $i^3 = -i$
    - $i^4 = 1$
    - $i^5 = i$
    - Conjugate Complex numbers

Given a complex number	Its conjugate
$a + bi$	$a - bi$
$a - bi$	$a + bi$

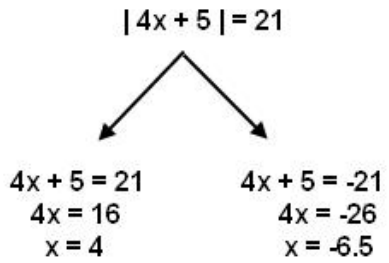
Source: https://www.chilimath.com/lessons/advanced-algebra/dividing-complex-numbers/

$$
\begin{aligned}
(3 + 2i)(1 - 4i) &= 3 - 12i + 2i - 8i^2 \\
&= 3 - 10i - 8(\sqrt{-1})^2 \\
&= 3 - 10i - 8(-1) \\
&= 3 - 10i + 8 \\
&= 11 - 10i
\end{aligned}
$$

Source:https://medium.com/i-math/complex-numbers-explained-fad9a6793019

Absolute Value

- Solving Equations with absolute value
    - 1. Isolate the absolute value
    - 2. Separate the equation into two, making one solution positive and one negative
    - 3. Solve for it in terms of the variable
    - 4. Check for extraneous solutions



Source: https://www.veritasprep.com/blog/2013/03/gmat-gurus-speak-out-absolute-value-in-data-sufficiency/

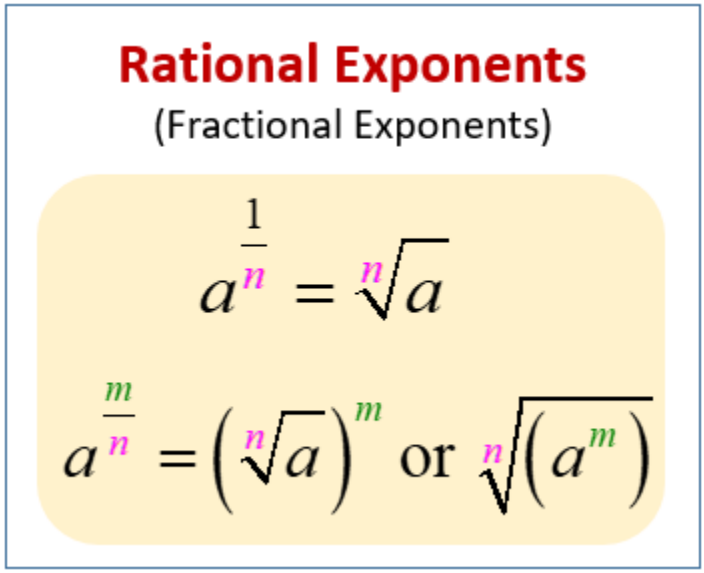
- Graphing
    - Formula: y =a |x - h| + k

- "a": stretch or compress the graph by a factor
- (h,k) is the vertex
- Axis of symmetry: vertical line through the vertex, x = h
- "h": denotes how much the graph shifts to the left or right
- "k": denotes how much the graph shifts up or down

Exponent

- Product Rule
  - Formula: $a^m \times a^n = a^{m+n}$
  - Example: $4^2 \times 4^6 = 4^8$
- Quotient Rule
  - Formula: $a^m/a^n = a^{m-n}$
  - Example: $4^{12}/4^8 = 4^4$
- Power Rule
  - Formula: $(a^m)^n = a^{mn}$
  - Example: $(6^4)^2 = 6^8$
- Zero Rule
  - $a^0 = 1$, provided that $a \neq 0$
  - Example: $5^0 = 1$
- Negative Exponent
  - $a^{-4} = 1/a^4$
  - Example: $9^{-2} = 1 / 9^2$
- Power of a Product Rule
  - Formula: $(ab)^n = (a^n)(b^n)$
  - Example: $(3(2))^3 = (3^3)(2^3)$
- Power of a Fraction Rule
  - Formula: $(a/b)^n = a^n/b^n$
  - Example: $(4/5)^5 = 4^5/5^5$

- Rational Exponents
    - A rational exponent has a base that is raised to the power with a fraction
    - Converting rational exponents to radicals
        - The denominator goes out of the square root to the index
        - The numerator goes inside the square root, becoming the exponent



Source: https://www.onlinemathlearning.com/fractional-exponent.html

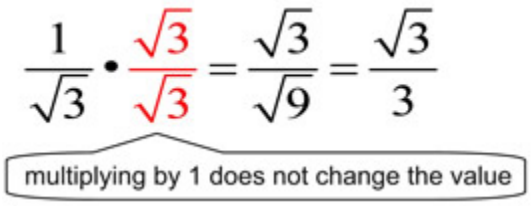
Radicals

- Definition - An expression that uses a root
- To get rid of the radical in an equation, square both sides

$$\left(\sqrt{x-7}\right)^2 = (3)^2$$
$$x-7=9$$
$$x-7+7=9+7$$
$$x=16$$

Source: https://www.chilimath.com/lessons/intermediate-algebra/solving-radical-equations/

- Radical in the denominator
    - Whenever you see a radical in the denominator, you have to rationalize it
        - To get rid of it, multiply the top and bottom with the same term as that in the denominator



Source: https://mathbitsnotebook.com/Algebra2/Radicals/RDSimplifyingRadicals.html

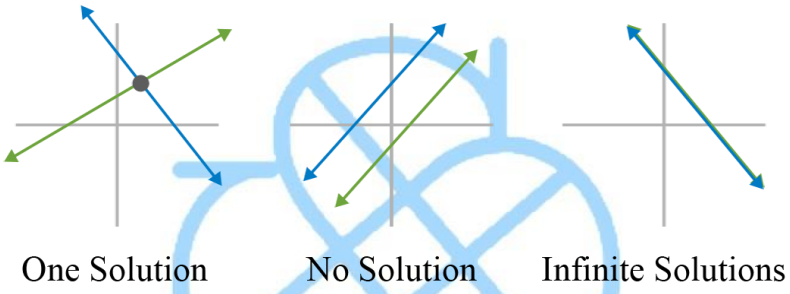
- Whenever you take the square root of both sides, it ends up being +/-

System of Equations

- Definition - solving two different equations for two or more variables
- Substitution with steps
    - 1. Solve for one of the variables by isolating one variable in an equation
    - 2. Plug your answer to step 1 into the other equation, and then solve for that equation in terms of the variable (there should only be 1 variable in the equation)
    - 3. Plug in the value you got in step 2 into either equation, and then solve for the other variable
    - 4. Check your work by plugging in the given values in either equation
- Elimination with steps
    - 1. Get one of the variables to have the same coefficient in both equations using multiplication or division
    - 2. Add the two equations together (this should leave you with one equation that has only one variable) and then solve the equation in terms of the variable
    - 3. Plug your answer to step 2 into either equation and then solve for it in terms of the variable
    - 4. Check your work by plugging in the given values in either equation

NOTE: SUBSTITUTION WOULD BE THE BEST METHOD TO USE FOR SAT

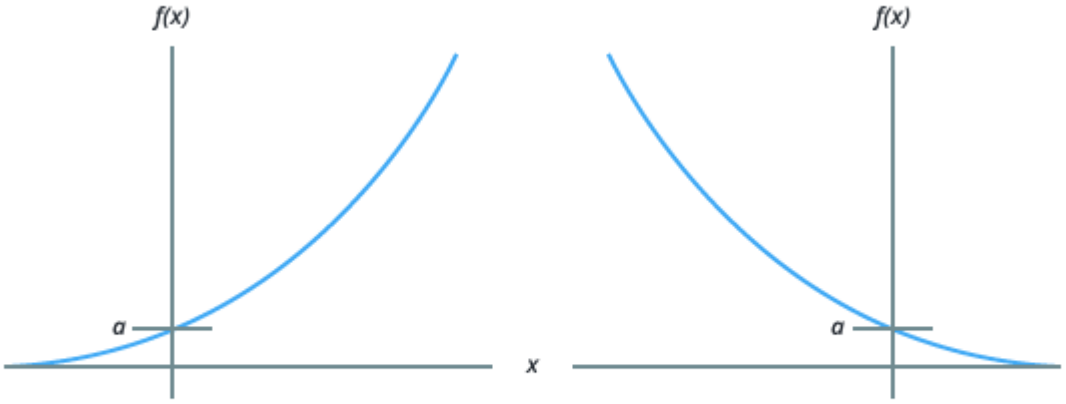
- Solutions
    - One Solution
        - The functions intersect at only one point, meaning that they have perpendicular slopes and opposite reciprocals
    - No Solutions
        - Two equations do not intersect, meaning that they have parallel and equal slopes parallel but different y-intercepts.
    - Infinite Solutions
        - Two equations overlap, meaning that they have the same slopes and y-intercepts (the two equations are essentially the same)



Source: https://calcworkshop.com/systems-equations/graphing-method/

Exponential Equations

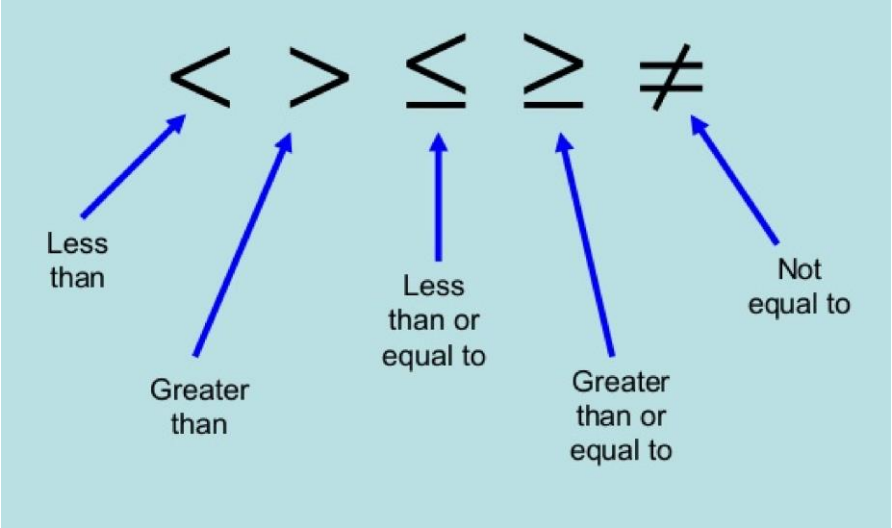
- Formula: $y = ab^x$
    - a is not equal to 0
    - b and b > 0 is not equal to 1



Source:https://www.visionlearning.com/en/library/Math-in-Science/62/Exponential-Equations-in-Science-I/206

- Word Problems
  - Formula: $P = a(1 + r)^t$
  - a = initial amount
  - r = rate of increase
  - t = time

Inequalities



Source:https://www.crackverbal.com/gre-inequalities/

- Inequalities in Word Problem
  - Greater than symbol
    - Above
    - Greater than
    - Exceeding
    - More than
  - Greater than or equal to symbol
    - At minimum
    - No less than
    - At least
  - Less than symbol
    - Less than
    - Below
    - Fewer than
  - Less than or equal to symbol
    - At most
    - No more than
    - Does not exceed

- Graphing
    - Greater than symbol
        - The line is dashed; the section above the line is shaded
    - Greater than or equal to symbol
        - The line is solid; the section above the line is shaded
    - Less than symbol
        - The line is dashed; the section below the line is shaded
    - Less than or equal to symbol
        - The line is solid; the section below the line is shaded
- Multiplying and dividing by a negative number
    - Flip the inequality symbol

$$-7x > 98 \qquad \text{(Divide both sides by } -7\text{)}$$

$$\frac{-7x}{-7} < \frac{98}{-7} \qquad \text{(Change } > \text{ into } <\text{)}$$

$$x < -14$$

Source:https://www.mathsteacher.com.au/year10/ch02_linear_equations/12_dividing_negative/neg.htm

Function Notation

- How to solve function notation
    - Plug the x value into the equation and solve the equation for f(x)
- Examples
    - $f(x) = x^2 + 2x + 4$
        - $f(-4) = x^2 + 2x + 4$
        - $f(-4) = (-4)^2 + 2(-4) + 4$
        - $f(-4) = 16 - 8 + 4$
        - $f(-4) = 12$

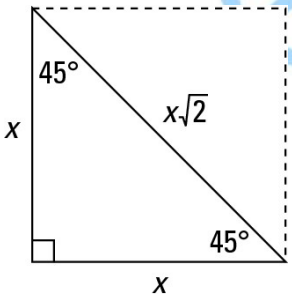
- f(x) = 3x + 2
  - f(2) = 3(2) + 2
  - f(2) = 6 + 2
  - f(2) = 8

Combining Like Terms

- Definition - Terms that have the same variable and are raised to the same power
- Example
  - $xy + 2xy^2 + 3xy$
    - $2xy^2 + 4xy$
  - $(x^2y - 3y^2 + 5xy^2) - (-x^2y + 3xy^2 - 3y^2)$
    - $x^2y - 3y^2 + 5xy^2 + x^2y - 3xy^2 + 3y^2$
    - $2x^2y + 2xy^2$

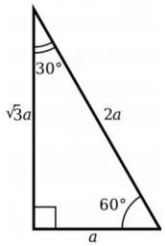
Geometry

- Special right triangle
  - $45^o - 45^o - 90^o$



Source: https://www.dummies.com/education/math/geometry/identifying-the-45-45-90-degree-triangle/

- 30^o - 60^o - 90^o

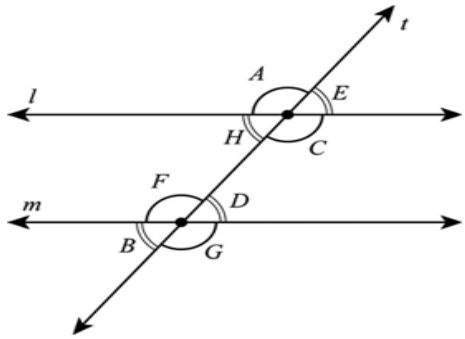


Source:https://www.voovers.com/geometry/30-60-90-triangle/

- Pythagorean Theorem
  - Definition - Pythagorean Theorem is used to find the length of the third side of a right triangle, given the values of two sides already
  - Formula: $a^2 + b^2 = c^2$
  - Pythagorean Theorem triples to know
    - 3, 4, 5
    - 5, 12, 13
    - 7, 24, 25
    - 8, 15, 17
    - 9, 40, 41

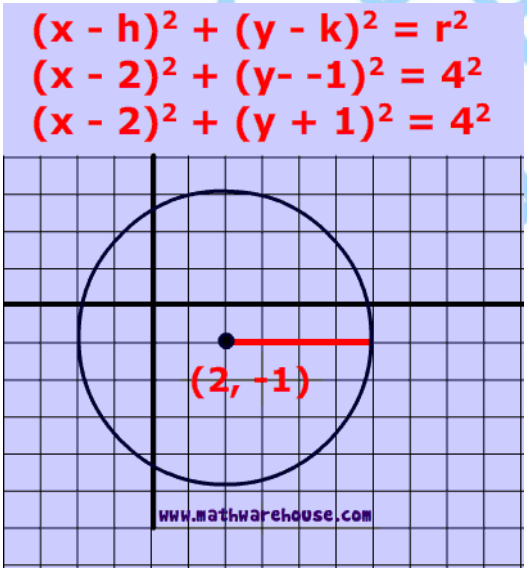
- Types of Angles
  - **Complementary Angles**: 2 angles whose sum is 90^o
  - **Supplementary Angles**: 2 angles whose sum is 180^o
  - **Vertical Angles**: 2 angles opposite of each other and are made by intersecting lines
  - **Alternate Interior Angles**: 2 angles that are equal in measure if two parallel lines are intersected by a transversal; lie within the two parallel lines
  - **Alternate Exterior Angles**: 2 angles that are equal in measure if two parallel lines are intersected by a transversal; lie outside the two parallel lines
  - **Same-side Interior Angles**: 2 angles that are supplementary if two parallel lines are intersected by a transversal; lie within the two parallel lines

- **Same-side Exterior Angles**: 2 angles that are supplementary if two parallel lines are intersected by a transversal; lie outside the two parallel lines



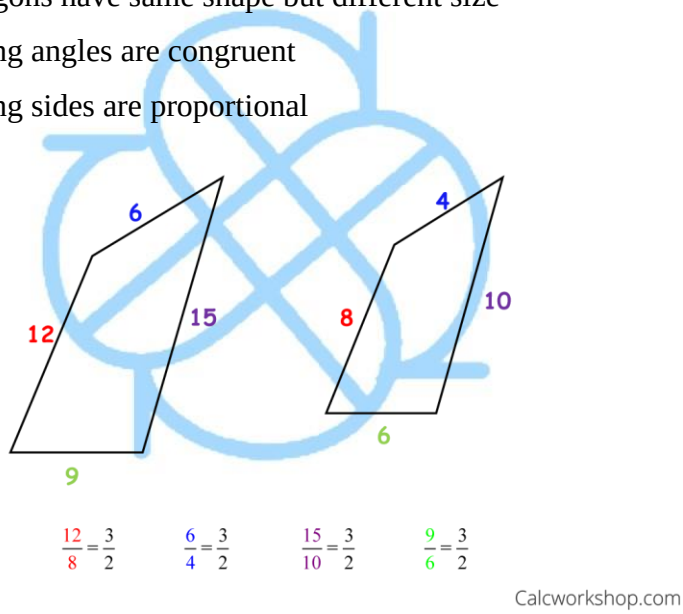
Source:https://www.chegg.com/homework-help/definitions/angle-theorems-63

- Equation of a Circle
  - Formula: $(x-h)^2 + (y-k)^2 = r^2$
  - (h, k) is the center point of the circle
  - r is the radius



Sources:https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.mathwarehouse.com%2Fgeometry%2Fcircle%2Fequation-of-a-circle.php&psig=AOvVaw3tNjMBujenjVGF8yVp_ZWD&ust=1606671916208000&source=images&cd=vfe&ved=0CAMQjB1qFwoTCJj6yfvlpe0CFQAAAAAdAAAAABAL

- Converting into Standard Form of the Circle Equation
    - Standard Form: $(x - h)^2 + (y - k)^2 = r^2$
    - Steps
        - 1. Make sure the terms match up with the same variable
        - 2. Complete the square
        - 3. Add the constant to the right side of the equation
        - 4. Factor the variables by making the binomial squared
        - 5. To determine the center, use “h” and “k”
        - 6. To find the radius square root “r”
- Similar Polygon
    - Similar polygons have same shape but different size
    - Corresponding angles are congruent
    - Corresponding sides are proportional



Sources: https://calcworkshop.com/similarity/similar-polygons/

- Arc of a Circle
    - Formula: $(2\pi r)$ central angle/360^o = arc length
    - Steps
        i. Find the circumference
        ii. Find the central angle value
        iii. Solve the equation in terms of arc length

- Distance Formula
  - Definition: the distance between two points is a straight line
  - Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

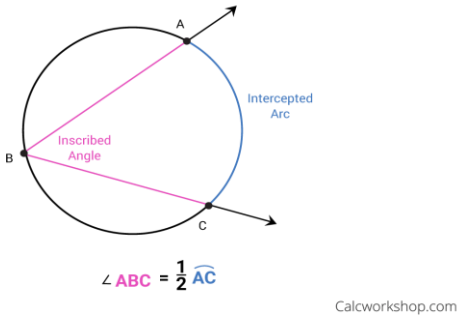
- Midpoint Formula
  - Definition: points that lies halfway between two ordered pairs
  - Formula: midpoint = $(x_1 + x_2/2, y_1 + y_2/2)$

- Circumference of the Circle
  - Definition - The distance around the circle
  - Formula: $C = 2\pi r$
    - "C" is the circumference
    - "r" is the radius

- Area of a Circle
  - Definition: region inside the circle
  - Formula: area = πr^2

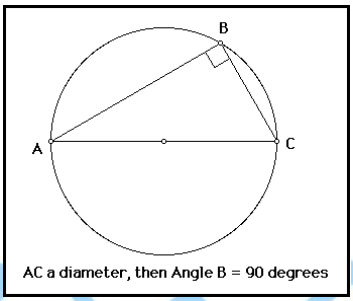
- Area of a Sector
  - Definition - Portion of the area of a circle
  - Formula: A = (πr^2) measure of the central angle / 360^o

- Inscribed Angles
  - Definition - An angle with its vertex on the circle and two sides that are chords
  - Formula: $\angle ABC = \frac{1}{2} * \text{measure of intercepted arc}$



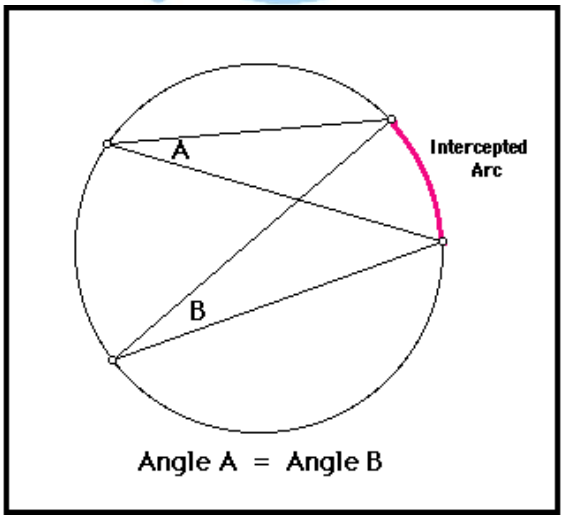
Source: https://calcworkshop.com/circle/inscribed-angle/

- If the inscribed angle is intercepted with a diameter, then it is a right triangle



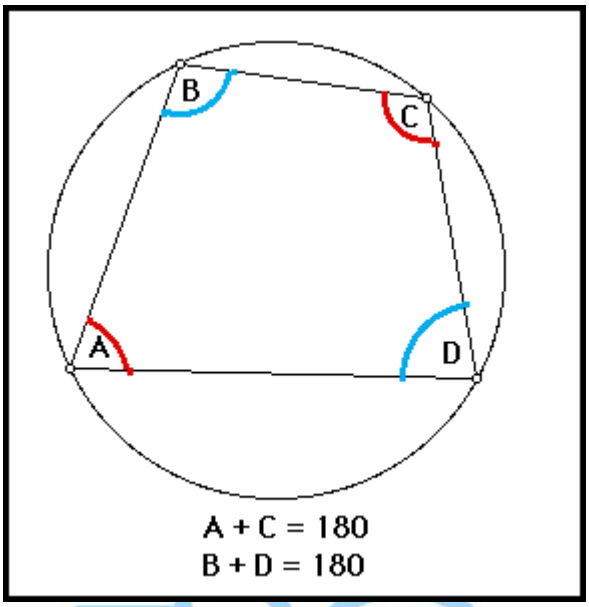
Source: http://www.geom.uiuc.edu/~dwiggins/conj44.html

- Finding the Measure of Overlapping Arcs
  - When two or more inscribed angles intercept the same arc, an overlapping arc occurs, which means the angles are congruent



Source: http://www.geom.uiuc.edu/~dwiggins/conj44.html

- When you inscribe a quadrilateral into a circle
  - A quadrilateral is inscribed in a circle if all four vertices are on the circle; this means the opposite angles in the quadrilateral are supplementary

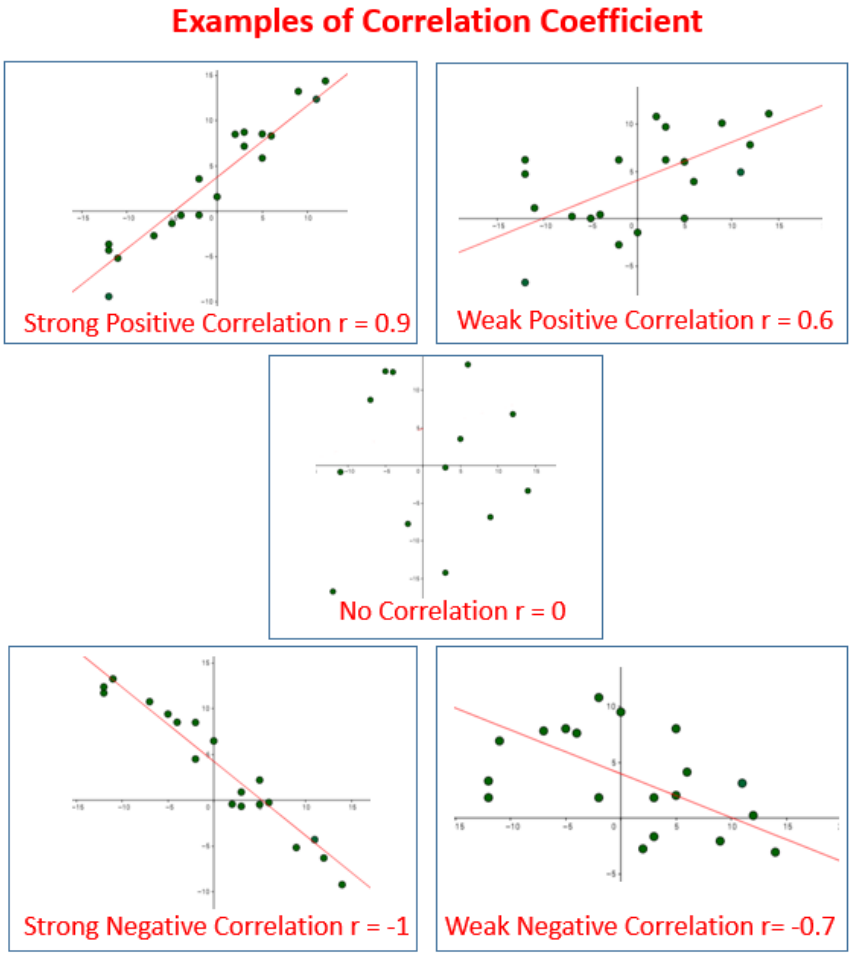


Source: http://www.geom.uiuc.edu/~dwiggins/conj47.html

Statistics

- Mean
  - Definition: the average of the numbers
  - Formula: sum of all data/number of data
- Median
  - Definition: the middle number in a data set
  - How to find the median
    i. Order the numbers in ascending order
    ii. Find the halfway point of the data set, using the formula: (n +1)/2
    iii. Starting from the smallest number, find the number that is in the middle of the data set, using the value you got in step ii

- Mode
    - Definition: the number that shows up the most in a data set
    - There can be more than one mode
    - How to find the mode
        i. Order the numbers in ascending order
        ii. Find the number(s) that repeat(s) the most
- Range
    - Definition: the difference between the highest and lowest values in a data set
    - How to find the range
        i. Order the numbers in ascending order
        ii. Find the lowest and highest number in the data set and subtract them
- Standard Deviation
    - Definition: the measure of variation of the values in a data set from the mean
    - How to interpret standard deviation
        - Low standard deviation means that the values in the data set are closer to the mean and that the data is not spread out
        - High standard deviation means that the values in the data set are far from the mean, or vary a lot from the mean, and the data is spread out
- Correlation
    - Definition: the relationship between two variables
    - Correlation is known as “r”
    - The correlation is positive if $r > 0$
    - The correlation is negative if $r < 0$
    - If $r = 0$, there is no correlation



Sources: https://www.onlinemathlearning.com/correlation-coefficient-id8.html

- **Problem Solving**
  - Ratio
    - Definition - The number of times the number contains another
    - Form: “a : b”
    - Order matters based on what the question is ask for
    - Ratio needs to be in the simplest form
    - Example: In the classroom, 26 of the students are boys and 24 of the students are girls. What is the ratio of girls to boys?

        - Ratio of girls to boys is **12:13** not 13:12.
            - 13:12 would be incorrect because this would mean that there are more girls than boys, which isn't true
            - 24:26 would be incorrect because it is not simplified
    - Proportion
        - Definition - Two fractions that are equal to each other
        - Form: a / b = c / d
        - You can check your work by making sure that the fractions are equal
        - Solving for "x"
            - 1. Set the proportion up and make sure that the units match up horizontally
            - 2. Cross multiply
            - 3. Solve the equation in term of "x"
            - 4. Check your work by making sure the fractions are equal
    - Percentage
        - Formula:

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100} \text{ or } \frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

Percentage formula

Source: https://www.basic-mathematics.com/formula-for-percentage.html

        - Sales tax
            - 1 + %
        - Discount
            - 1 - %
        - % increases or decrease

Percent Change

$$\text{Percent Change} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100\%$$

If the result is positive, it is an increase.
If the result is negative, it is a decrease.

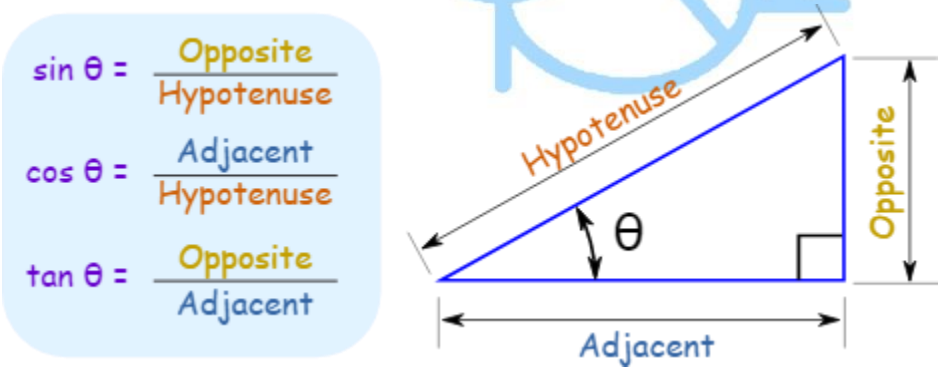
Source: https://www.onlinemathlearning.com/percent-change-algebra.html

Probability

- Definition - The "odds" of any particular event happening
- Formula: Probability = desired outcome / all possible outcomes

Precalculus

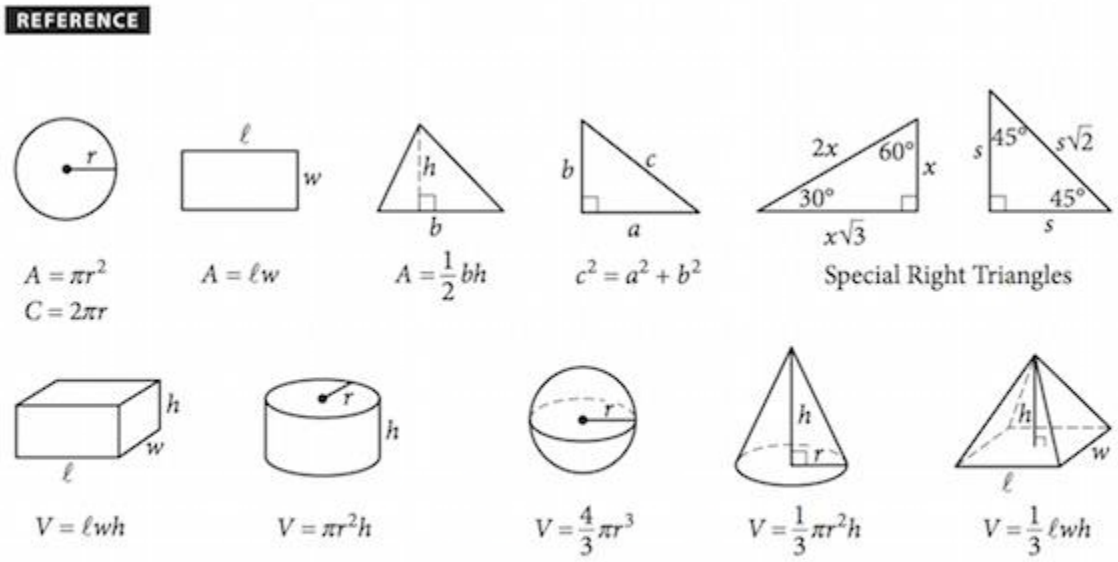
- Trigonometry
  - Sin = opposite/hypotenuse
  - Cos = adjacent/hypotenuse
  - Tan = opposite/adjacent
  - Trick to remembering it
    - SOH CAH TOA



Source: https://www.mathsisfun.com/sine-cosine-tangent.html

- Converting Radians to Degree and Degree to Radians
    - Radians = (π/180^o) x degrees
    - Degree = (180^o/π) x radians

SAT Math Formula Sheet



Source: https://blog.prepscholar.com/critical-sat-math-formulas-you-must-know

Calculator Tricks

- Use the “Math” button
    - Convert a decimal into a fraction
    - Find the maximum and minimum
    - Take the cube root or another type of root
- Factoring
    - Only if the equation is $ax^2 + bx + c$
    - Find the simplest form

- Graphing Functions
    - Graph the equation to give a visualize
    - Can help you find the min and max of an equation
    - Graph inequalities and system of inequalities
- System of Equation
    - Use the graph to find the solution
    - Use the tool to find it

Free Online Practice SAT Math Questions

- Khan Academy
    - https://www.khanacademy.org/sat
- College Board
    - https://collegereadiness.collegeboard.org/sat/practice/full-length-practice-tests
- Top 5 online SAT Math practice
    - https://blog.prepscholar.com/complete-list-of-free-sat-math-practice
- Free SAT Math Problems
    - https://www.freesatmath.com/

Best SAT Books

- College Board
    - https://www.amazon.com/Official-SAT-Study-Guide-2020/dp/1457312190/ref=sr_1_3?crid=3BYIRDQBWXSJ2&dchild=1&keywords=sat+book&qid=1607034919&sprefix=sat+boo%2Caps%2C209&sr=8-3
- The Princeton Review
    - https://www.amazon.com/Princeton-Review-Premium-Prep-2021/dp/0525569340/ref=sr_1_4?crid=3BYIRDQBWXSJ2&dchild=1&keywords=sat+book&qid=1607035042&sprefix=sat+boo%2Caps%2C209&sr=8-4
- The College Panda
    - https://www.amazon.com/College-Pandas-SAT-Math-Advanced/dp/1733192727/ref=sr_1_8?crid=3BYIRDQBWXSJ2&dchild=1&keywords=sat+book&qid=1607035042&sprefix=sat+boo%2Caps%2C209&sr=8-8

- Barron’s
    - https://www.amazon.com/Premium-Study-Guide-Practice-Tests/dp/1438012225/ref=sr_1_3?crid=3D4ZKXJ51BQYM&dchild=1&keywords=sat+book+barrons&qid=1607035164&sprefix=sat+book+ba%2Caps%2C175&sr=8-3