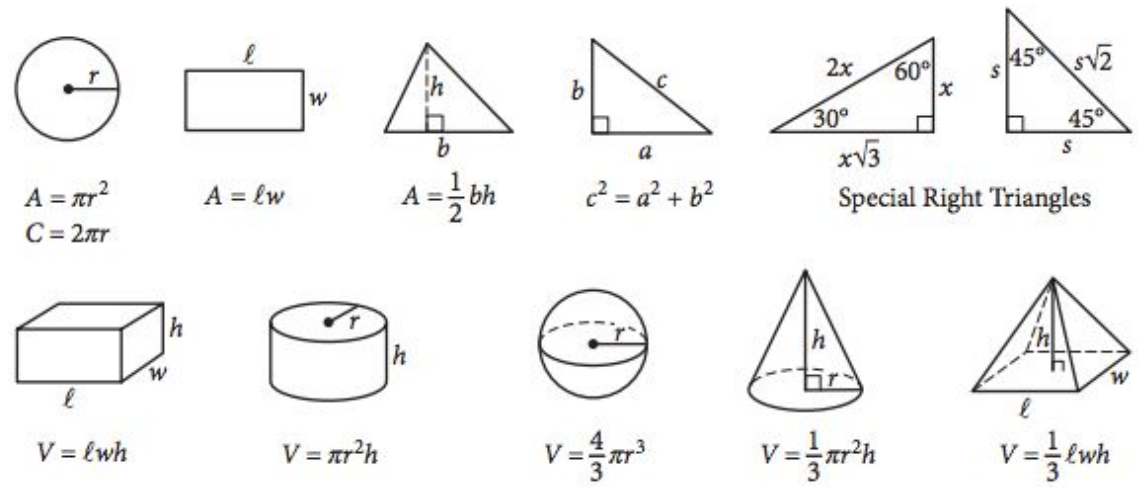
SAT Math Study Guide

From Simple Studies: https://simplestudies.edublogs.org & @simplestudiesinc on Instagram

SAT Test Information

Given Formulas

REFERENCE



The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

Math Test- No Calculator

- 25 Minutes for 20 Questions

Math Test- Calculator

- 55 Minutes for 38 Questions

Use of Calculator

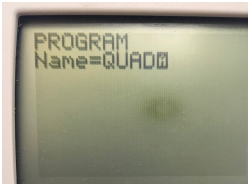
Programming

Programming a graphing calculator can be an **easy way to prepare for the test.** It allows you to more quickly solve easy problems. Here are a **few programs** you can create for your calculator:

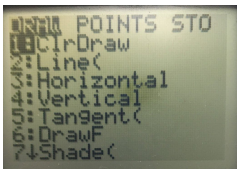
If you need any other programs you can download more off of the TI site.

1. Quadratic Formula

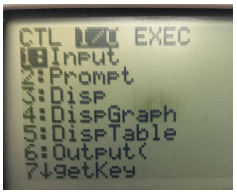
Step 1: Press the [PRGM] button and move over to the right twice to Create New.



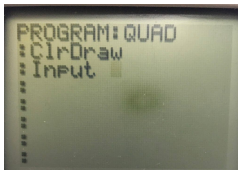
Step 2: Name the program Quad.



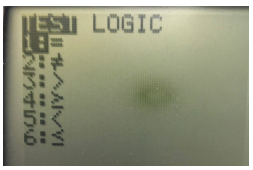
Step 3: Add ClrDraw to the program by going to [2ND] [PRGM].

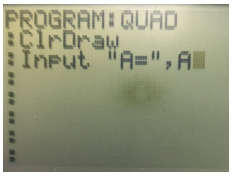


Step 4: Add Input by going to [PRGM] moving to the Right by one and pressing Input.

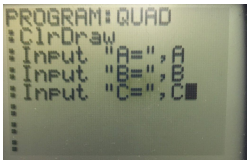


Step 5: Type in "A=". The equal sign can be found by going to [2ND] [MATH] and pressing =.

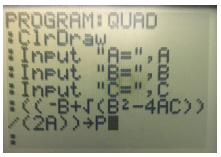


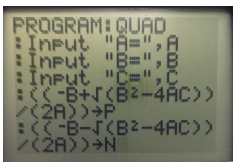


Step 6: After typing "A=" add a comma and an A.

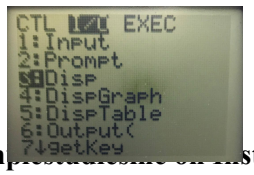


Step 7: Repeat Steps 4-6 twice more replacing A with B and C as shown.





Step 8: Write the Quadratic formula as follows: ((-B+√(B²-4AC))/(2A))



Step 9: Store that equation as P by pressing [STO➤].

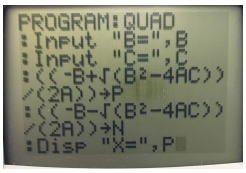
Then write P.

Step 10: Repeat the process in Steps 9-10 instead writing

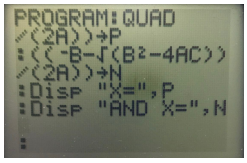
((-B-√(B²-4AC))/(2A)) and storing it as N.

Step 11: Add Disp by going to [PRGM] moving to the

Right by one and pressing Disp.



Step 12: Write "X=", P.

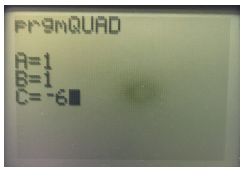


Step 13: Add Disp and write "X=", N.

<u>Example:</u>

$x^2 + x - 6$

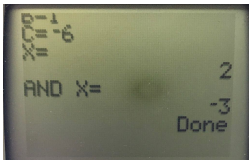
Plug in numbers for A, B, and C.



A=1

B=1

C=-6

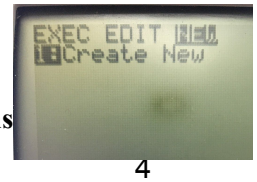


After pressing Enter, you should get the following answer:

X=2

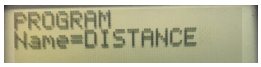
And X=-3

2. Distance Formula

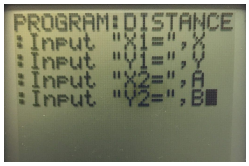


Step 1: Press the [PRGM] button and move over to the right twice to Create New.

Step 2: Name the Program Distance.

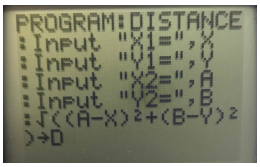


Step 3: Add 4 Inputs for X1, Y1, X2, and Y2 as shown in the picture.

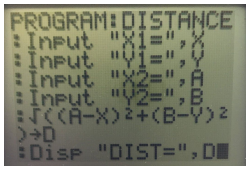


Step 4: Add the equation as follows:

$\sqrt{((A-X)^2+(B-Y)^2)}$



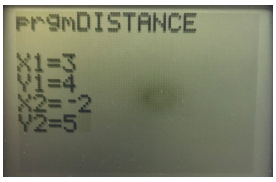
Step 5: Store the equation as D.



Step 6: Add the Disp and write "DIST=", D.

<u>Example:</u>

Find the distance between the points (3,4) and (-2, 5).



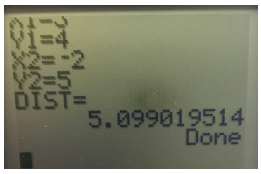
Plug in the numbers for X1, Y1, X2, and Y2,

X1= 3

Y1= 4

X2= -2

Y2= 5



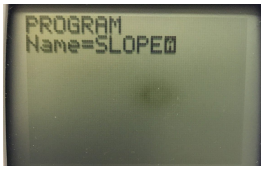
After pressing Enter, you should get the following answer:

DIST= 5.099019514

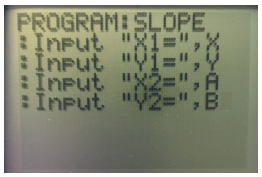
3. Slope Formula

Step 1: Press the [PRGM] button and move over to the right twice to Create New.

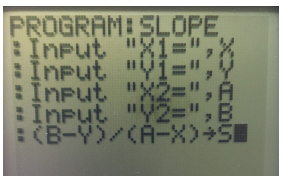
Step 2: Name the Program Slope.



Step 3: Add 4 Inputs for X1, Y1, X2, and Y2 as shown in the picture.

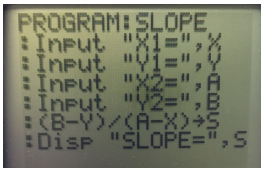


Step 4: Add the equation as follows:
(A-X)/(B-Y)



Step 5: Store the equation as S.

Step 6: Add the Disp and write "SLOPE=", S.



<u>Example:</u>

Find the slope of a line using the points (3,4) and (-2, 5).

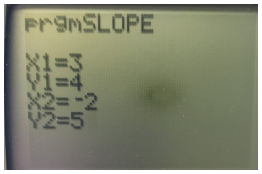
Plug in the numbers for X1, Y1, X2, and Y2.

X1= 3

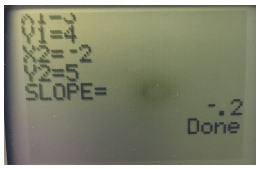
Y1= 4

X2= -2

Y2= 5



After pressing Enter, you should get the following answer:



SLOPE= -.2

Basic Use of Calculator

1. **How to Graph an Equation**

Step 1: Press [Y=].

Step 2: Enter Equation.

Step 3: Press [GRAPH] to see the graph or [2ND][GRAPH] to see the table.

2. **Analyzing a Graph**

   a. ***Calculating a value***

Step 1: Press [2ND][TRACE][1: value].

Step 2: Enter a value of X and press [ENTER].

   b. ***Finding zeros***

Step 1: Press [2ND][TRACE][2: zero].

Step 2: Move left of the zero and press [ENTER].

Step 3: Move right of the zero and press [ENTER].

   c. ***Finding minimum/maximum***

Step 1: Press [2ND][TRACE][3: minimum] or Press [2ND][TRACE][4: maximum].

Step 2: Move left of the minimum/maximum and press [ENTER].

Step 3: Move right of the minimum/maximum and

press [ENTER].

3. **Linear Regression/Line of Best Fit**

Step 1: Press [STAT][1: Edit…].

Step 2: Enter X and Y values.

Step 3: Press [STAT][CALC][4: LinReg(ax+b)].

Step 4: Add Lists to Xlist and Ylist and press [Calculate].

Linear Equations

- **Slope-intercept form**
  - $y = mx + b$
  - m is the slope and b is the y-intercept

- **Point slope form**
  - $y - y_1 = m(x - x_1)$
  - m is the slope and (x_1, y_1) is any point on the line

- **Standard form**
  - $ax + by = c$
  - *b in the Standard form is different from b in Slope-intercept form

- **Vertical Equation**
  - $x = a$
  - a is the x-coordinate throughout the whole line

- **Horizontal Equation**
  - $y = b$
  - b is the y-coordinate throughout the whole line

- **Examples:**
  - $y = 3x + 5$
    - Slope: 3, y-intercept: 5
  - $y - 5 = 4(x - 2)$
    - Slope: 4, Point on Line: (2,5)
  - $2x + 5y = 12$

        - Slope: -⅖, y-intercept: 12/5
    - $x = 2$
        - X-Coordinate: 2
    - $y = 6$
        - Y-Coordinate: 6

Solving Equations

Solving equations for a variable

- ***Step 1:*** Move all like terms to the same side of the equation
- ***Step 2:*** Combine like terms
- ***Step 3:*** Solve for variable

- Example:
  - Equation: $4x + 1 = x - 5$
  - ***Step 1:*** $4x - x = -5 - 1$
  - ***Step 2:*** 3x = -6
  - ***Step 3:*** $x = -2$

Solving Inequalities

Solving inequalities for a variable

- Similar to solving equations
  - ***Step 1:*** Move all like terms to the same side of the equation
  - ***Step 2:*** Combine like terms
  - ***Step 3:*** Solve for variable
- ***Don't forget to change inequality sign when multiplying/dividing by a negative***
- Example:
  - Equation: $-4x - 2 \geq -3x + 5$
  - ***Step 1:*** $-4x + 3x \geq 5 + 2$
  - ***Step 2:*** $-x \geq 7$
  - ***Step 3:*** $x \leq -7$
    - *Remember to flip the sign

Systems of Equations

Setting up Systems of Equations from a word problem:

- Example: Julie has \$11.50 and wants to buy a total of 15 apples and oranges. Each apple cost \$1.20 and each orange cost \$0.70. How many apples and oranges does she have?
  - ***Step 1: Create/Assign variables***
    - a = apples
    - o = oranges
  - ***Step 2: Set up 2 Equations***
    - $a + o = 15$
      - Total apples and oranges
    - $1.20a + 0.70o = 11.50$
      - Total cost of fruit
  - ***Step 3: Choose one of 3 Methods to Solve***
    - **Substitution:**
      - Plug in one equation into the other to solve for one variable.

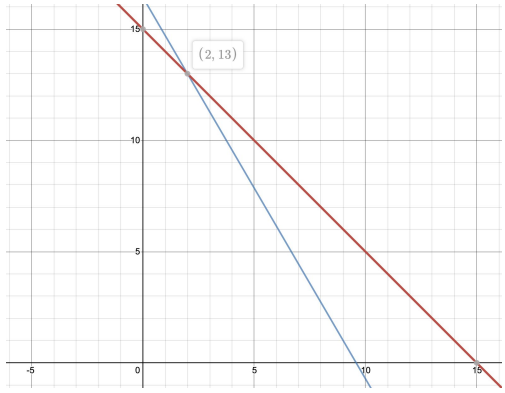
$a + o = 15$ and $1.2a + 0.7o = 11.5$
$a = 15 - o$ and $1.2a + 0.7o = 11.5$
$1.2(15 - o) + 0.7o = 11.5$
$18 - 1.2o + 0.7o = 11.5$
$18 - 0.5o = 11.5$
$-0.5o = -6.5$
$o = 13$

$a + 13 = 15$
$a = 2$
$o = 13$ and $a = 2$

- **Elimination:**
  - Manipulate equations so one variable can be canceled out.

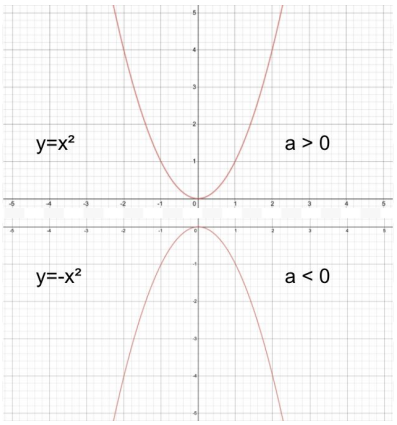
$a + o = 15$ $1.2a + 0.7o = 11.5$
$-1.2 * (a + o) = (15) *- 1.2$ $1.2a + 0.7o = 11.5$
$-1.2a - 1.2o = -18$ $1.2a + 0.7o = 11.5$
$-0.5o = -6.5$
$o = 13$
$a + 13 = 15$
$a = 2$
$o = 13$ and $a = 2$

- Graphing:
  - Graph both equations and see where they intersect.

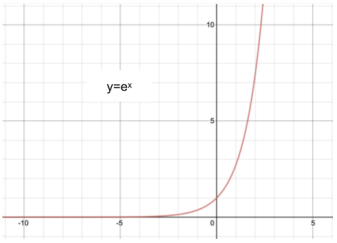


- All three methods come out to $a = 2$ and .

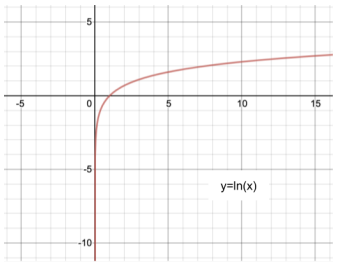
Nonlinear Equations



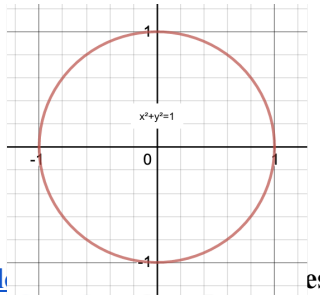
- **Quadratic Equation**
  - $ax^2 + bx + c = 0$
  - $y = a(x-h)^2 + k$
    - Vertex: (h,k)
    - If a > 0, the parabola opens up
    - If a < 0, the parabola opens down



- **Exponential Equation**
  - $y = e^x$

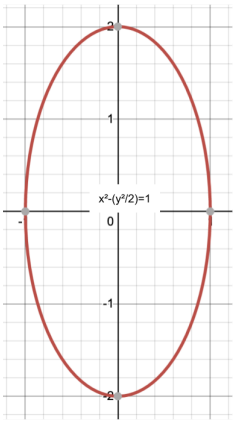


- **Logarithmic Equation**
  - $y = ln(x)$

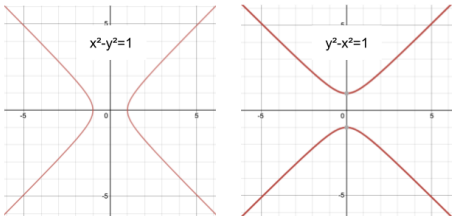


- **Equation of Circle**
  - $(x-h)^2 + (y-k)^2 = r^2$
  - r: radius
  - (h,k): center

- **Equation of Ellipse**
  - $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
  - (h,k): center
  - **a: horizontal distance from center to vertex**
  - **b: vertical distance from center to vertex**



- **Equation of Hyperbola**
  - $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
    - Hyperbolas open to the right/left
  - $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
    - Hyperbolas open up/down
  - (h,k): center
  - a: the distance from the center to the vertex



Solving Quadratic Equations

- Example: Solve for $x^2 + 4x - 12 = 0$.

Completing the Square

- ***Step 1: Move the constant to the other side***
  - $x^2 + 4x = 12$
- ***Step 2: Complete the square by using*** $(\frac{b}{2})^2$
  - $x^2 + 4x + 4 = 16$
- ***Step 3: Factor the expression***
  - $(x+2)^2 = 16$
- ***Step 4: Square root both sides***
  - $\sqrt{(x+2)^2} = \sqrt{16} = x + 2 = \pm 4$

- ***Step 5: Solve for x***
  - $x = -2 \pm 4 = -6 \text{ or } 2$

Factoring

- ***Step 1: Multiply the constants of the first and last term***
  - $1 * -12 = -12$
- ***Step 2: Find 2 terms that multiply to get the previous step and that add to get the middle term***
  - $6 * -2 = -12$ and $6 + (-2) = 4$
- ***Step 3: Factor***
  - $(x + 6)(x - 2)$
- ***Step 4: Solve for each factor***
  - $x = -6,\ 2$

Quadratic Formula

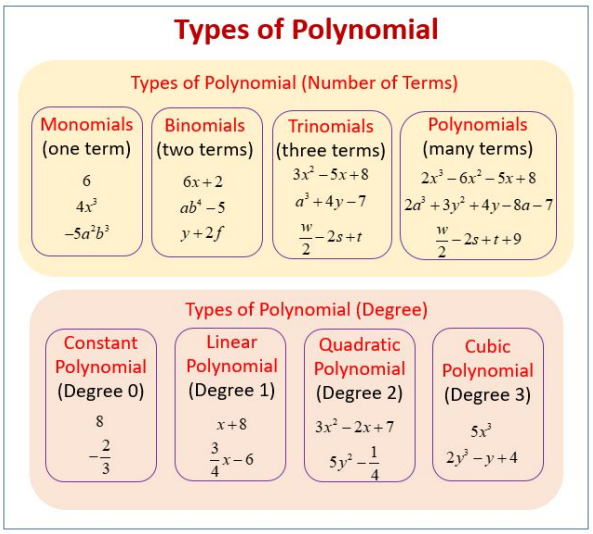
- $ax^2 + bx + c = 0$
- $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- $$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-12)}}{2(1)}$$
- $$x = \frac{-4 \pm \sqrt{16 + 48}}{2}$$
- $x = -2 \pm 4 = -6, 2$

Polynomials

- **Binomials** have two terms.
- **Trinomials** have three terms.
- **Polynomials** have four or more terms.

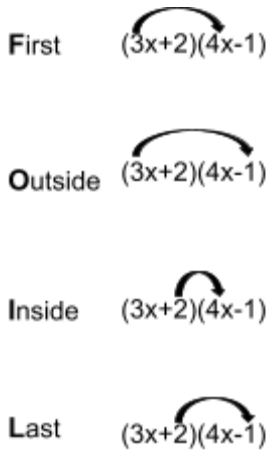
Adding/Subtracting Polynomials

- Add or subtract like terms.
- ***Example:***
  - $(3x^2 + 2x - 6) - (2x^2 - 6x + 3)$
  - $x^2 + 8x - 9$



Multiplying Polynomials

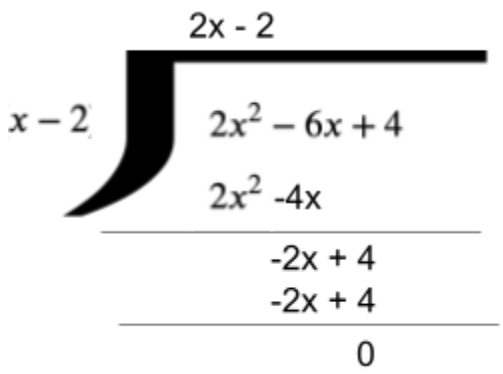
- **FOIL-** First, Outside, Inside, Last
- ***Example:***
- (3x+2)(4x-1)



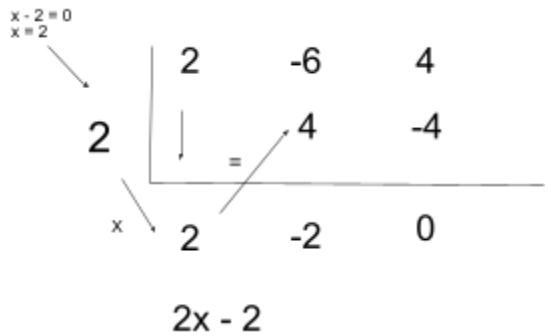
- $12x^2 - 3x + 6x - 2 = 12x^2 + 3x - 2$

Dividing Polynomials

- ***Example:***
- $\frac{2x^2-6x+4}{x-2}$
- **Polynomial Division**



- **Synthetic Division**



Rational Functions

Adding/Subtracting Rational Functions

- Add or subtract like terms.
- They have to have the same denominator.
- ***Example:***
  - $\frac{3x^2-2x+5}{3x} - \frac{2x^2+5x-6}{3x} = \frac{x^2-7x+11}{3x}$
  - $\frac{4x^2+2x-7}{x} + \frac{3x^2-6x+4}{2x} = \frac{2(4x^2+2x-7)}{2x} + \frac{3x^2-6x+4}{2x} = \frac{11x^2-2x-10}{2x}$

Multiplying Rational Functions

- Multiply denominator and numerator.
- ***Example:***
  - $\frac{2x+3}{4x^2} * \frac{x-4}{2x^3} = \frac{(2x+3)(x-4)}{8x^5} = \frac{2x^2-5x-12}{8x^5}$

Dividing Rational Functions

- Flip the second rational and change the division sign to a multiplication sign.
- ***Example:***
  - $\frac{4x-5}{2x^2} \div \frac{6x+1}{4x^3} = \frac{4x-5}{2x^2} * \frac{4x^3}{6x+1} = \frac{2x(4x-5)}{6x+1} = \frac{8x^2-10x}{6x+1}$

Finding Horizontal and Vertical Asymptotes

- **Vertical Asymptotes**
  - Set the denominator equal to 0.
- ***Example:***
  - $\frac{x^2-3x+1}{x+1}$
  - $x + 1 = 0$
  - $x = -1$
- **Horizontal asymptotes**
  - If the **highest degree of the numerator equals the degree of the denominator**, divide the highest degrees to get the horizontal asymptote.
  - If the **highest degree of the numerator is less than the degree of the denominator**, the horizontal asymptote is 0.
  - If the **highest degree of the numerator is greater than the degree of the denominator**, there are no horizontal asymptotes.

Radicals and Exponents

Exponent and Radical Rules

Product Rule	$x^m * x^n = x^{m+n}$
Quotient Rule	$\frac{x^m}{x^n} = x^{m-n}$
Power of Product Rule	$(x * y)^n = x^n * y^n$
Power Rule	$(x^m)^n = x^{m*n}$
Zero Exponent Rule	$(x^m)^0 = 1$
Negative Exponents	$x^{-m} = \frac{1}{x^m}$
Adding/Subtracting Exponents	$x^m \pm x^n = x^m \pm x^n$ $x^m + x^m = 2x^m$
Fractional Exponents/Radicals	$x^{\frac{m}{n}} = \sqrt[n]{x^m}$
Multiplying Radicals	$\sqrt{x} * \sqrt{y} = \sqrt{xy}$
Dividing Radicals	$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$

Sequences

Arithmetic Sequences

- Arithmetic sequences are when a **common number is added or subtracted from each number.**
- ***Example:*** 1, 3, 5, 7
  - +2
- **Formula:** $a_n = a_1 + (n-1)d$
  - a_n = nth term in sequence
  - a_1 = first term in sequence
  - n = number in sequence
  - d = common difference
- ***Example Problem:***

  The first five terms of an arithmetic sequence are given: -7, -2, 3, 8, 13,…

  What is the eighth term in the sequence?
  - Solution:

    $a_8 = -7 + (8-1) * 5$

    $a_8 = -7 + (7) * 5$

    $a_8 = -7 + 35$

    $a_8 = 28$

Geometric Sequences

- Geometric sequences are **always multiplied or divided by the same number** throughout the sequence.
- ***Example:*** 1, 4, 16, 64
  - *4
- **Formula:** $a_n = a_1 r^{n-1}$
  - a_n = nth term in sequence
  - a_1 = first term in sequence
  - r = common ratio

- n = number in sequence

- ***Example Problem:***

  The first term is 2 in the geometric sequence 2, -8, 32, -128…. What is the eleventh term of the geometric sequence?

  - Solution:

    $a_{11} = 2(-4)^{(11-1)}$

    $a_{11} = 2(1048576)$

    $a_{11} = 2097152$

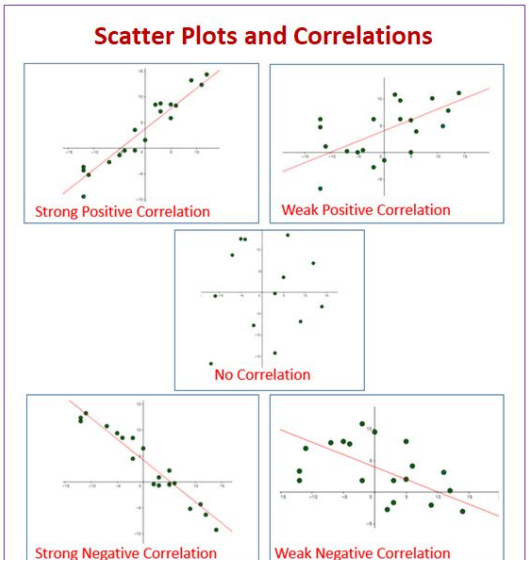
How to Tell the Difference

- ***Arithmetic Sequences will always go up by the same amount while geometric sequences will go up by multiples.***

Scientific Notation

- A way to write large or small numbers using powers of 10s.
- **Large Numbers**
  - Form: $x * 10^n$
  - Example: $284,000,000 = 2.84 * 10^8$
- **Small Numbers**
  - Form: $x * 10^{-n}$
  - Example: $0.0000000284 = 2.84 * 10^{-8}$

Scatterplots



Correlation

- **Strong Correlation**
  - The line of best fit is close to **most points.**
- **Weak Correlation**

From Simple Studies: https://simplestudies.edublogs.org &

        - The line of best fit is close to **some points.**
    - **No Correlation**
        - The line of best fit is close to a **few points.**
    - **Positive Correlation**
        - The slope of the line of best fit is **positive.**
    - **Negative Correlation**
        - The slope of the line of best fit is **negative.**

Linear vs Exponential Growth

Linear

- If the y changes by adding the same values, it represents **linear growth.**
    - Changes at a constant rate

Exponential

- If the y changes by multiplying the same values, it represents **exponential growth.**
    - Changes by a%
    - Halves/Doubles

Probability

Probability

- **AND Probability**
    - Probability of multiple things happening.
    - $P(A \cap B) = P(A) * P(B)$
    - Probability of A and B
- **OR Probability**
    - Probability of one thing OR another thing happening.
    - $P(A \cup B) = P(A) + P(B)$
    - Probability of A or B
- **Conditional Probability**
    - Probability of something happening GIVEN something else happens.

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- Probability of A given B.

Independent vs Dependent

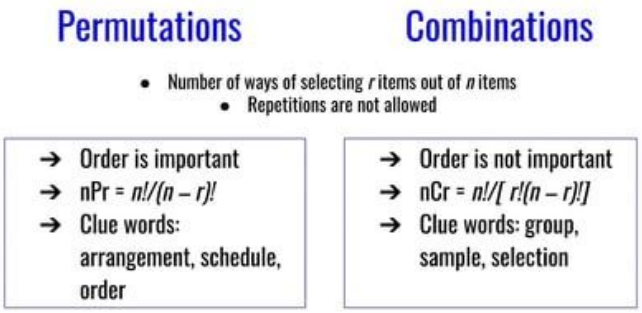
- **Independent Probability**
  - Probability is NOT affected by another event happening.
- **Dependent Probability**
  - Probability IS affected by another event happening.

Permutations

- **Order matters**
- $_nP_r = \frac{n!}{(n-r)!}$
  - n = total number of items
  - r = selected number of items

Combinations

- **Order does not matter**
- $_nC_r = \frac{n!}{r!(n-1)!}$
  - n=total number of items
  - r=selected number of items



Center and Distribution

Median

- **Number in the middle of a set of numbers**
- ***Example:***
  - 1, 3, 6, 3, 7, 4, 2, 8, 1, 5
  - ***Step 1:*** Reorder Numbers.
    - 1, 1, 2, 3, 3, 4, 5, 6, 7, 8
  - ***Step 2:*** Find the middle number (If there are 2 in the middle find the average of them).
    - Median= $\frac{3+4}{2}$ = 3.5

Mean

- **The average of all of the numbers**

- Add all of the numbers and divide by the amount of numbers there are.
- ***Example:***
  - 1, 3, 6, 3, 7, 4, 2, 8, 1, 5
  - ***Step 1:*** Add all numbers together.
    - $1 + 3 + 6 + 3 + 7 + 4 + 2 + 8 + 1 + 5 = 40$
  - ***Step 2:*** Find out how many numbers there are in the set.
    - There are 10 numbers.
  - ***Step 3:*** Divide the sum by the amount of numbers.
    - Mean= $\frac{40}{10} = 4$

Mode

- **The number that shows up the most in the set**
- ***Example:***
  - 1, 3, 6, 3, 7, 4, 2, 8, 1, 5
  - Mode= 3 and 1

Standard Deviation

- **Standard deviation shows the spread of the data**
- The more spread out the data is, the **greater** the standard deviation is.
- The less spread out the data is, the smaller the standard deviation is.

Range

- **The difference between the largest and smallest number**
- ***Example:***
  - 1, 3, 6, 3, 7, 4, 2, 8, 1, 5
  - ***Step 1:*** Find the largest and smallest number.
    - Largest= 8 and Smallest=1
  - ***Step 2:*** Subtract the smallest number from the largest number.
    - Range= 8-1=7

Writing Formulas in Terms of Variable

Solving to get a variable alone on one side

- ***Example:***
  - $V = \pi r^2 h$

    The formula gives the volume V with a radius r and height h. Write r in terms of V and h.
  - ***Step 1:*** Get r alone on one side.

- $\frac{V}{\pi h} = r^2$
- ***Step 2:*** Solve for r.
  - $r = \sqrt{\frac{V}{\pi h}}$

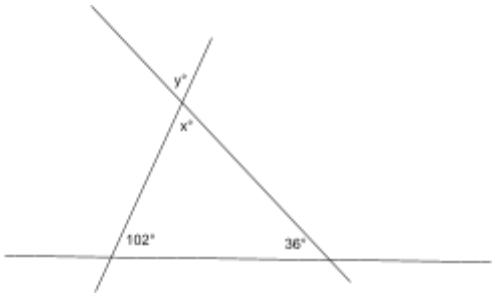
Surface Area and Volume Word Problems

- Most formulas for solids are given to you in the formula area of the math section.
- Always draw a Picture.
- **Rectangular Prism**
  - Volume= lwh
  - Surface Area= $2lw + 2lh + 2wh$
  - Diagonal= $\sqrt{(l^2 + w^2 + h^2)}$

Angles

- **Parallel Lines**
  - Lines that point the same direction and will never meet.
- **Perpendicular Lines**
  - Lines that meet at a 90° angle.
- **Supplementary Angles**
  - Angles that add up to 180°.
- **Complementary Angles**
  - Angles that add up to 90°.
- **Opposite Angles**
  - Angles opposite of each other when two lines intersect are equal.
- Remember that the interior angles of a triangle equal 180°.
- ***Example:***

What is the value of y



- ***Step 1:*** Find the value of x
  - X is the third value in the triangle, so you can use the equation $180 = 102 + 36 + x$
  - $180 = 138 + x$
  - $x = 42$
- ***Step 2:*** Find the value of y using x
  - The angle y is opposite the angle x, so the value of y will be the same as x
  - y=42

Trigonometry

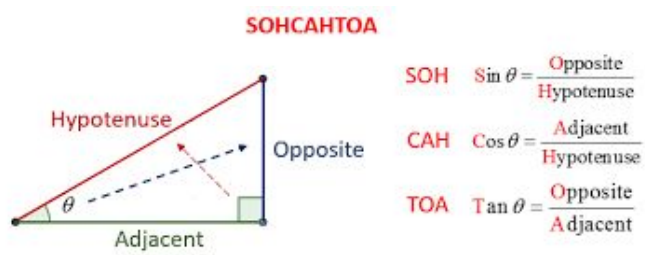
Right Triangle

- Remember Soh Cah Toa to remember how to get sine, cosine, and tangent from a triangle.
  - **Soh:**

    $Sine = \frac{opposite}{hypotenuse}$
  - **Cah:**

    $Cosine = \frac{adjacent}{hypotenuse}$
  - **Toa:**

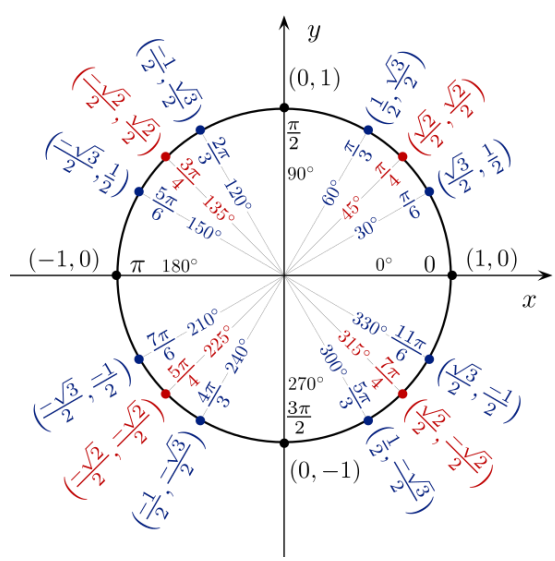
    $Tangent = \frac{opposite}{adjacent}$



- $sin(x°) = cos(90° - x°)$

Radians

- **Unit Circle**
- **Angles to Radians**
  - Multiply by $\frac{\pi}{180°}$
  - Example:
    - $270 \circ * \frac{\pi}{180°} = \frac{3\pi}{2}$
- **Radians to Angles**
  - Multiply by $\frac{180°}{\pi}$
  - Example:



- $\frac{3\pi}{2} * \frac{180°}{\pi} = 270$

Complex Numbers

- i is a complex number that equals $\sqrt{-1}$.
- $i^2 = -1$
- $i^3 = -\sqrt{-1}$ or $-i$
- $i^4 = 1$
- **Multiplying by the conjugate**
  - Example:
  - $\frac{3+2i}{4-i}$
  - $\frac{3+2i}{4-i} * \frac{4+i}{4+i}$
  - $\frac{(3+2i)(4+i)}{16-i^2}$
  - $\frac{12+11i+2i^2}{16+1}$
  - $\frac{10+11i}{17}$

Sources

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