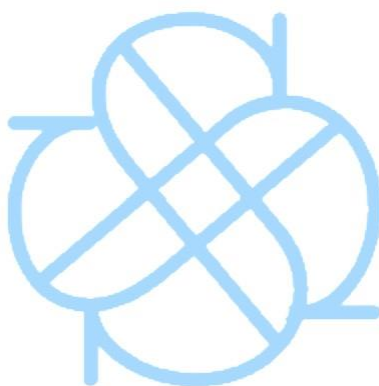


**AP Physics C: Electricity and Magnetism**  
**UNIT 3: ELECTRIC CIRCUITS**



## Before You Begin

This study guide will guide you through each of the topics covered on the AP Physics C: Electricity and Magnetism exam, and cover core concepts, formulas, and other important info. However, it's important to note that there's a lot of stuff you should know before starting on E&M.

Firstly, you're going to need a decent grasp of basic calculus. Both Physics C exams are **calculus-based**, rather than algebra-based, like Physics 1 and 2 are. You will need to know how to do basic integration and differentiation, as well as solve separable differential equations. It's not like Calculus BC: there won't be any integration by parts or anything, so don't fret about not being able to comprehend any super-advanced calculus concepts, because you won't need them here.

You should also have a decent grasp of regular high school curriculum-level physics concepts. If you've taken Physics 1 or 2, that works as well. The course and exam expand on some of the electricity and magnetism concepts covered there in greater detail, so if you already have a grasp of the basics, it'll really help you when trying to wrap your head around E&M concepts.

Also, something else of note: I will **BOLD** any variables in formulas that are vector quantities. That's how I'll be notating vectors in formulas in this study guide. Keep that in mind as you use this guide: some people bold variables instead, to mark them as vectors, but I'll be using the arrows.

With all that said, let's get started!

# Current

Conventional current is the rate at which positive charges flow through a wire.

$$I = \frac{dQ}{dt}$$

Current can also be thought of as the flux of “current density” that flows past a surface.

$$I = \int J \cdot dA$$

$dA$  is defined like it is with Gauss’s law, with magnitude being the infinitesimally small surface area  $dA$  and the direction being outward normal from the surface.

## Things To Note About Current

Above, we describe **conventional current**. Conventional current views current as positive charges flowing in one direction in a wire: electron flow (how current actually works) is electrons (which are negative charges) flowing in the opposite direction of conventional current. Either one works: if you keep it consistent when solving for stuff, you’ll get the right answer.

Also: here, current is treated as a scalar quantity. However, we’re going to have to treat it as a vector quantity in the future.

## Resistivity

When charges move through a conductor, they will lose potential energy. The amount of energy lost depends on the **resistance** of the material. The **resistivity** of a material is proportional to the electric field and current density.

$$E = \rho J \text{ and in scalar: } \rho = \left| \frac{E}{J} \right|$$

$\rho$  is resistivity, in ohm meters, while  $E$  is electric field in newtons per coulomb and  $J$  is current density in amperes per square meter (A/m<sup>2</sup>).

Generally, in a conductor, electrons are free to move, and therefore the electric field is usually very small, and resistivity is low. In dielectric (non-conducting materials), electrons don't move easily, and can only polarize themselves. The electric fields are pretty strong, and resistivity is higher.

## Resistance Of A Conductor

The resistance of a conductor is proportional to resistivity and length, and inversely proportional to cross-sectional area.

$$R = \rho \frac{L}{A}$$

Here,  $R$  is the resistance in ohms ( $\Omega$ ),  $L$  is the length of the conductor in meters, and  $A$  is the cross-sectional area of the conductor, in square meters.

## Ohm's Law

The electric potential difference  $V$  across an ohmic load (resistor) is the product of the current  $I$  through the load and the resistance  $R$  of the load.

$$V = IR$$

$V$  is potential difference in volts,  $I$  is current in amps, and  $R$  is resistance in ohms.

## Power Dissipated By A Resistor

Power is the rate of work  $W$  being done. Remember from unit 1 that change in potential energy  $\Delta U_q$  is proportional to charge  $q$  and voltage  $V$ . This means that we can get an easy expression for power through a resistor.

$$P = \frac{dW}{dt} = \frac{d(qV)}{dt} = \left(\frac{dq}{dt}\right)V = IV$$

$\downarrow P = IV$

$P$  is the power through a resistor (load) in watts, while  $I$  is the current through the load in amps, and  $V$  is the voltage across the resistor in volts.

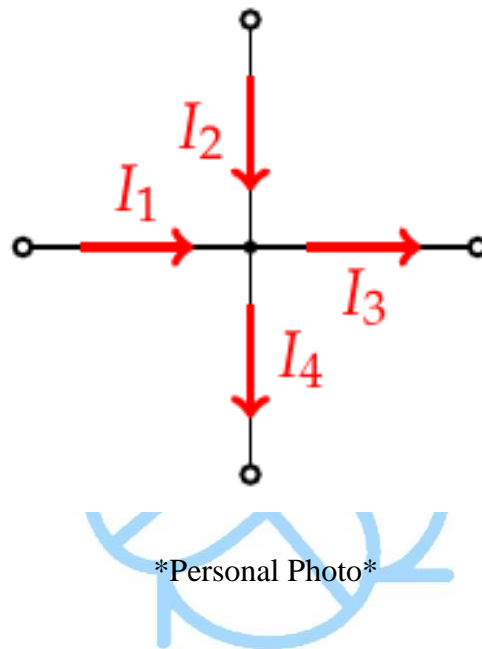
Because of Ohm's Law, there are also a couple other equations you can use for power, should the need arise.

$$P = \frac{V^2}{R} \text{ or } P = I^2 R$$

## Kirchhoff's Current Law

Kirchhoff's current law states that the electric current flowing into any junction in a circuit must be equal to the current that flows out of the junction.

Essentially, charges can't accumulate anywhere in the circuit. It's a consequence of the conservation of energy.



For example: take a look at this junction. We can see 4 paths to the center junction, with  $I_1$  and  $I_2$  going into the junction and  $I_3$  and  $I_4$  coming out.

Kirchhoff's current law means that

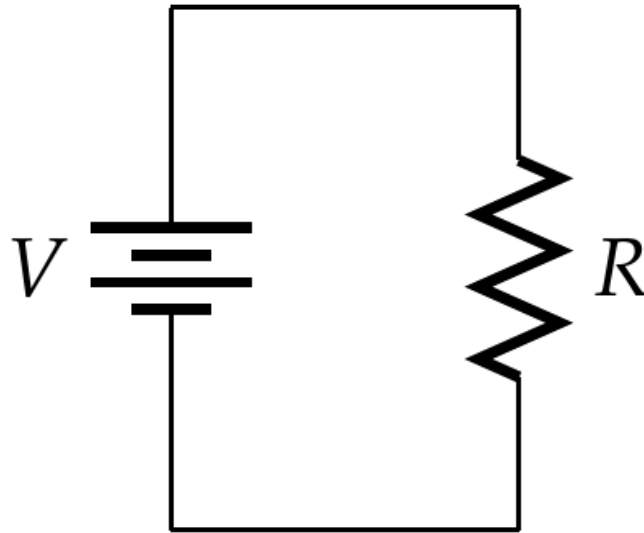
$$I_1 + I_2 = I_3 + I_4$$

You can also see it this way:

$$I_1 + I_2 - I_3 - I_4 = 0$$

## Kirchhoff's Voltage Law

Kirchhoff's voltage law states that no matter what path you take through an electric circuit, the voltage changes must sum to zero.



\*Personal Photo\*

Let's examine this very simple circuit, as an example. If we assume that the current flows clockwise, and we draw a clockwise loop through the circuit, we get

$$V - V_R = 0 \rightarrow V - IR = 0$$

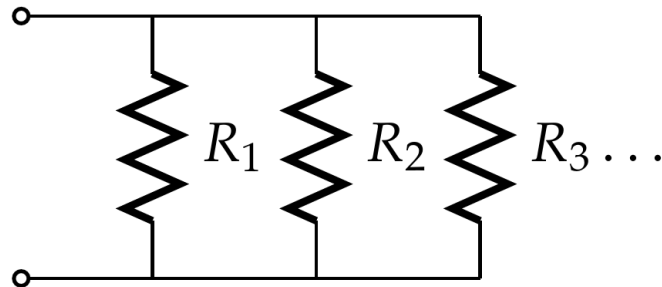
$V_R$  is the voltage drop across the resistor. If we instead guess that current  $I$  is flowing counterclockwise in the circuit, we will have a similar expression, but with the voltage change from the battery being negative.

$$-V_R - V = 0 \rightarrow -V - IR = 0$$

However, when we solve for  $I$  through the circuit using the latter equation,  $I$  turns out to be negative. We can't have this, so the current can't be flowing counterclockwise.

## Equivalent Resistance In Parallel

If we have some resistors in parallel, like shown, how do we figure out the equivalent resistance of all of them?



\*Personal Photo\*

It's not as hard as you think! Firstly, we know that the total current is the sum of the currents through each resistor, from the current law.

$$I = I_1 + I_2 + I_3 \dots = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \dots$$

However, we also know that the voltage through each resistor must be the same, because of the voltage law. Therefore...

$$I = \frac{V}{R_{eq}} = V \left( \frac{I}{R_1} + \frac{I}{R_2} + \frac{I}{R_3} \dots \right)$$

Simplifying a little, we get the equation for equivalent resistance of resistors in parallel.

$$\frac{I}{R_{eq}} = \frac{I}{R_1} + \frac{I}{R_2} + \frac{I}{R_3} + \dots + \frac{I}{R_N}$$



## Equivalent Resistance In Series



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Resistors in series are easier. We know that the current through each resistor must be the same, again from the current law.

$$I = I_1 = I_2 = I_3 = \dots$$

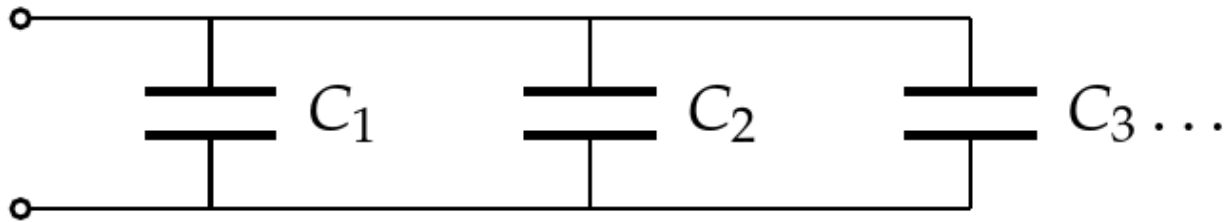
The total voltage drop across all the resistors is as follows...

$$V = V_1 + V_2 + V_3 + \dots = I(R_1 + R_2 + R_3 \dots)$$

In the end, the equivalent resistance for a bunch of resistors in series is

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N$$

## Capacitors In Parallel



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From Kirchhoff's voltage law, we know that the voltage across each of the capacitors is the same.

We can express the total charge in terms of capacitance and the common voltage  $V$ .

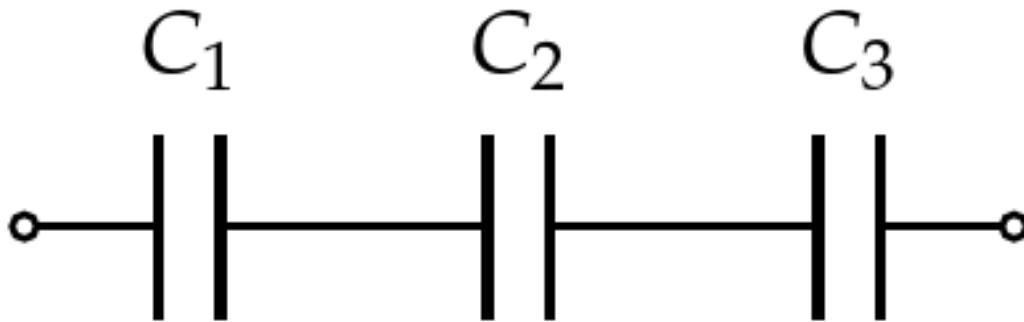
$$Q_{tot} = Q_1 + Q_2 + Q_3 + \dots = C_1V + C_2V + C_3V + \dots$$

We can then factor out  $V$ .

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$

This is the equivalent capacitance of capacitors in parallel.

## Capacitors In Series



\*Personal Photo\*

Let's now look at some capacitors in series.

The total voltage across the capacitors is the sum of the voltages across each of them.

Also, each capacitor here must be carrying the same amount of charge  $Q$ . Why? Well, the capacitor plates and the wire between the capacitor is really one great big conductor. The charges are all staying inside the conductor, and so when 1 capacitor accumulates charge, the other capacitors must collect the same charge, because of conservation of charge.

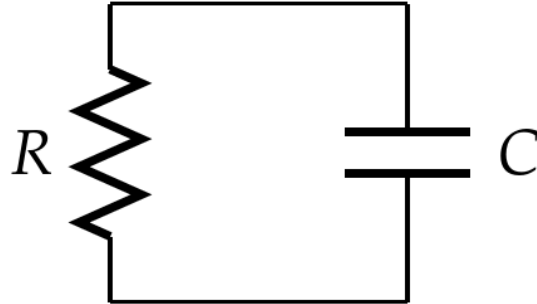
$$V_{tot} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots$$

We can factor once again to get the equivalent capacitance.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

## Discharging Capacitors In A Circuit

I won't go into the derivations for these equations in these unit-by-unit notes because it's too long.



\*Personal Photo\*

The charge and current in this circuit aren't necessarily easy to find: as current flows out of the capacitor, it decreases in charge, and the current decreases. Eventually the capacitor is fully discharged.

Let's say that the capacitor is charged to  $V_c = Q_{tot} / C$ . The charge across the resistor is given by the time-dependent equation

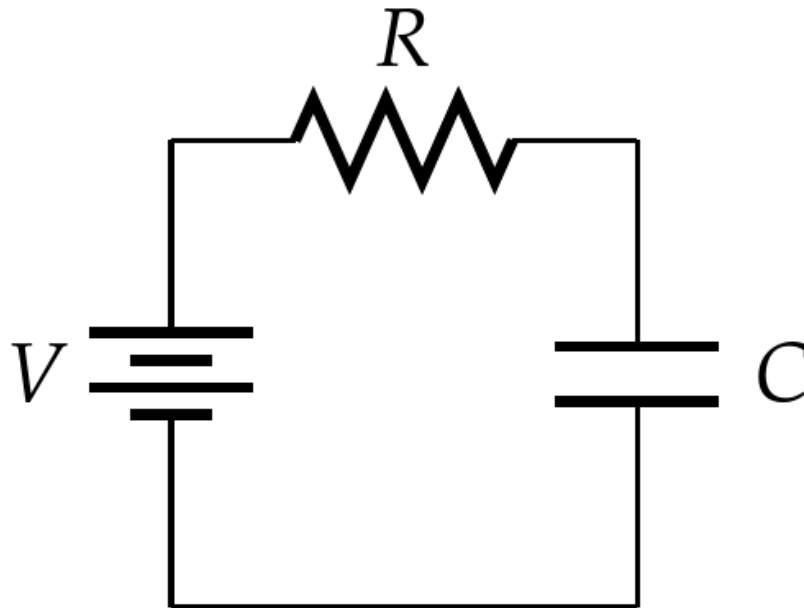
$$Q(t) = Q_0 e^{-t/\tau}$$

$\tau = RC$  is called the **time constant**. The current is the change in charge, and is also time-dependent:

$$I(t) = \frac{dQ}{dt} = I_0 e^{-t/\tau}$$

The initial current  $I_0$  at  $t = 0$  is given by  $I_0 = Q_{tot} / \tau = Q_{tot} / RC$ .

## Charging Capacitors In A Circuit



\*Personal Photo\*

The time-dependent equation for charge in this case is

$$Q(t) = Q_{tot}(1 - e^{-t/\tau})$$

The time constant  $\tau = RC$  is the same here as during discharge.

Differentiating to find the current yield us this:

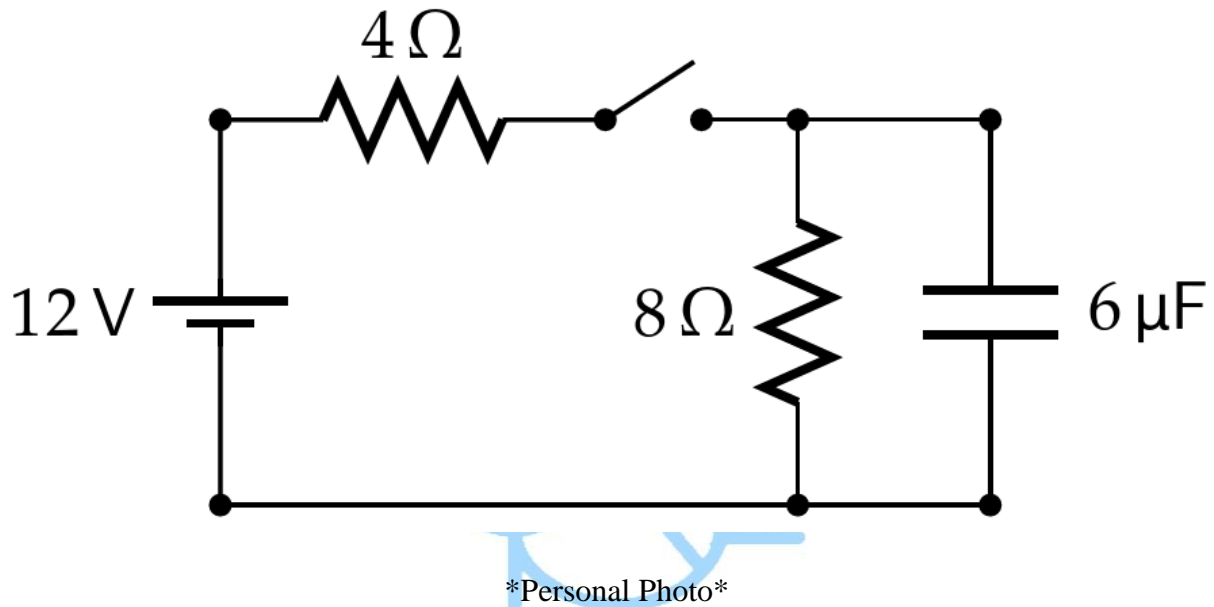
$$I(t) = \frac{dQ}{dt} = I_0 e^{-t/\tau}$$

The initial current is given by  $I_0 = Q_{tot} / \tau = V / R$ .

## Some Final Notes

- When a capacitor is fully uncharged, there's no voltage across the plates, and it acts like a short circuit.
- When a capacitor is charged, there is voltage across it, but no current flows *through* it. It acts like an open circuit.

With that in mind, try solving this.



The capacitor is initially uncharged.

Find the current that flows through the battery:

- Immediately after the switch is closed (capacitor uncharged)
- A long time after the switch is closed (capacitor fully charged)