AP Physics C: Electricity and Magnetism

UNIT 1: ELECTROSTATICS

Before You Begin

This study guide will guide you through each of the topics covered on the AP Physics C: Electricity and Magnetism exam, and cover core concepts, formulas, and other important info. However, it's important to note that there's a lot of stuff you should know before starting on E&M.

Firstly, you're going to need a decent grasp of basic calculus. Both Physics C exams are **calculus-based**, rather than algebra-based, like Physics 1 and 2 are. You will need to know how to do basic integration and differentiation, as well as solve separable differential equations. It's not like Calculus BC: there won't be any integration by parts or anything, so don't fret about not being able to comprehend any super-advanced calculus concepts, because you won't need them here.

You should also have a decent grasp of regular high school curriculum-level physics concepts. If you've taken Physics 1 or 2, that works as well. The course and exam expand on some of the electricity and magnetism concepts covered there in greater detail, so if you already have a grasp of the basics, it'll really help you when trying to wrap your head around E&M concepts.

Also, something else of note: I will **BOLD** any variables in formulas that are vector quantities. That's how I'll be notating vectors in formulas in this study guide. Keep that in mind as you use this guide: some people bold variables instead, to mark them as vectors, but I'll be using the arrows.

With all that said, let's get started!

What Is Electrostatics?

Electrostatics is the study of charges *at rest*. It is weighted at about 26-34% of the exam. We study charges at rest (that aren't moving relative to each other) in this unit only because it simplifies the learning a bit. When charges move, they generate a *magnetic field* as well. This complicates stuff beyond what we're concerned with for the moment, so for this unit you only consider charges that are at rest or moving **very slowly**.

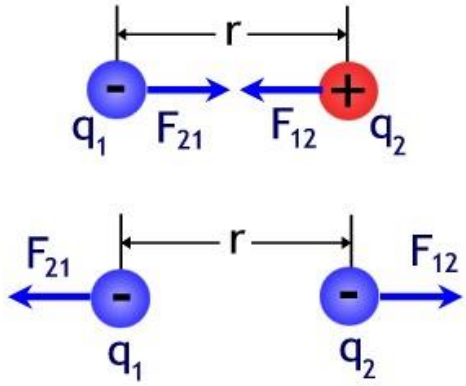
Charged Particles Review

Here’s a quick review on charged particles:

- Protons carry a positive charge.
- Electrons carry a negative charge.
- An object with a *net charge* has an excess of protons or electrons.
- Similar charges repel, opposite charges attract.

Charge is measured in the unit of **coulombs**. A single proton has a charge of +e, while an electron has a charge of -e. e is the *elementary charge*, defined as the electric charge a proton carries, and is about 1.602×10^{-19} coulombs. Charged particles exert force on other charged particles, with the direction determined by their charge (same or opposite to each other).

The Electrostatic Force



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The **electrostatic force** that a point charge of q_1 would exert on another charge q_2 is given by **Coulomb's Law for Electrostatic Force:**

$$F_{12} = \frac{kq_1q_2}{|r_{12}|^2}\hat{r}_{12}$$

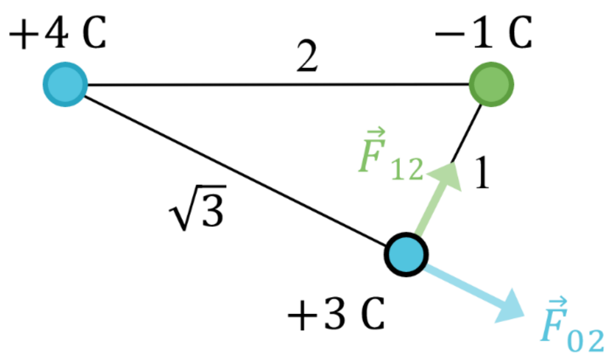
Here's what the symbols mean:

- $\boldsymbol{F}_{12}$ is the force that charge q_1 exerts on charge q_2, in Newtons.
- k is **Coulomb's constant**, about 8.99×10^9 N m^2/C^2.
- q_1 and q_2 are charges 1 and 2 being considered, in coulombs.
- $\boldsymbol{r}_{12}$ is the distance vector from q_1 to q_2.
    - $|\boldsymbol{r}_{12}|^2$ is the square of the scalar distance between charges 1 and 2.
    - $\hat{r}_{12}$ is the unit vector pointing from charge 1 to 2. This partially determines the direction of the electrostatic force that charge 1 will exert.

If the vector notation scares you, here's the scalar form that often also works:

$$F_{12} = \frac{kq_1q_2}{r^2}$$

Electrostatic Force With Multiple Charges



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When you have multiple point charges that are close to each other, each of them is exerting an electrostatic force on each other. To find the total electrostatic force ***F*** exerted on charge Q by N charges q_i, we simply do a vector sum of all the forces.

$$F = \sum_{i} F_i = kQ\left(\sum_{i=1}^{N} \frac{q_1}{r_i^2}\hat{r}_i\right)$$

If the summation notation looks scary, don't fret. This is just a fancy way of writing "sum up each of the force vectors from each charge to find the net force."

Electrostatic Force With Objects That Take Up Space

The electrostatic force equations that we just went over only cover interactions between **point charges**. A point charge is a charge that doesn't take up any space: it's a single point with some charge. When you have a charged object with **spatial extent** (that takes up space), however, you have to approach it a little differently. Let's go back to our sum of forces from before.

$$F = \sum_{i} F_i = kQ\left(\sum_{i=1}^{N} \frac{q_1}{r_i^2}\hat{r}_i\right)$$

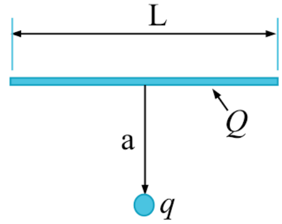
We can think of charged objects that take up space as a massive collection of an infinite number of small charges. Now, what would happen if we had an infinite amount of tiny charges, e.g. $N \to \infty$? Well, we get:

$$F = kQ\sum_{i=1}^{\infty} \frac{q_1}{r_i^2}\hat{r}_i$$

If you've done some calculus, you'll know that this looks suspiciously like it can be represented with an integral. If you thought that, you'd be right!

$$F = \int dF = kQ\int \frac{dq}{r^2}\hat{r}$$

So, to solve a problem like the one below, all you need to do is set up the right equations and integral and solve. Try finding the force exerted by the charged rod Q on q.



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Electric Field

What Is An Electric Field?

The electric field ***E*** created by a charge q_1 is a vector function called a *vector field*, that shows how the charge affects other charges around it. It is very similar to the concept of the gravitational field generated by objects with mass.

Electric Field Near A Point Charge

The electric field a distance *r* away from point charge *q* is given by:

$$E(q,r) = \frac{kq}{|r|^2}\hat{r}$$

The direction of ***E*** is radially outward from a positive point charge and radially inward towards a negative charge.

Once again, if you don't like or need the vector notation, you can use the scalar form of the equation.

$$E(q,r) = \frac{kq}{r^2}$$

To find the electrostatic force exerted on a charge due to the electric field it's in, multiple *E* and the charge *q*, just like you would with gravity.

$$F_q = Eq \longrightarrow F_g = mg$$

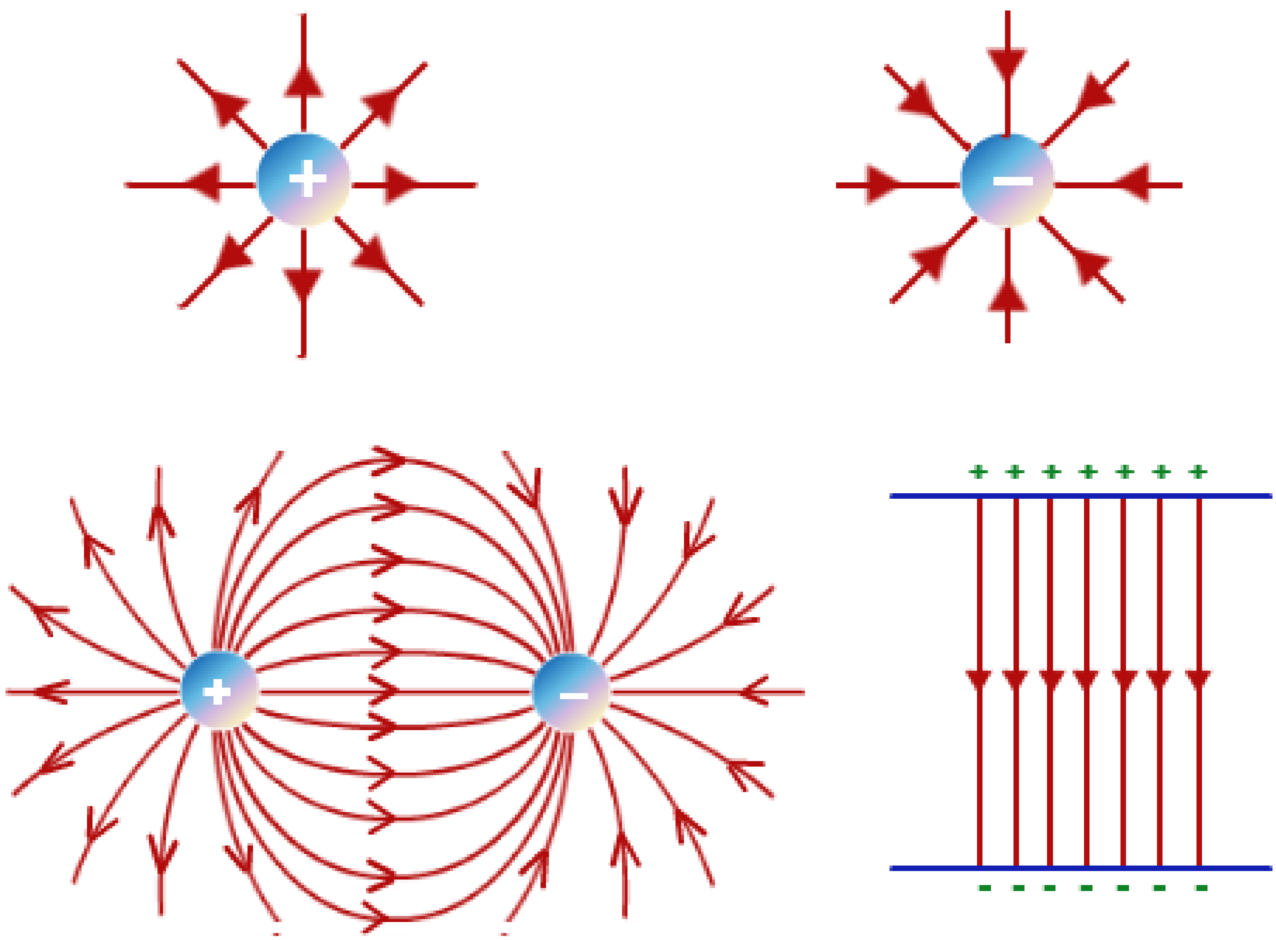
See the similarity?

Electric Field Lines

Electric field lines visually describe an electric field. The lines in an electric field line diagram describe the direction a positive test charge would accelerate if placed where the line was.

They begin from a charge and end at infinity, and never intersect. They also don’t have any ends: they extend out to infinity.

They point outwards from positive charges and inwards from negatives charges.



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Electric Field With More Than One Charge

The notation and thinking for electric fields with multiple charges is the same as with electrostatic force: simply sum up the fields created by each charge. This is the equation for the electric field at any position ***r.***

$$E = \sum_{i} E_i = k\left\{\sum_{i} \frac{q_i}{r_i^2}\hat{r}_i\right\}$$

Electric Field Of Charges With Spatial Extend

Like with electrostatic force, we can conceptualize charged objects that take up space as an infinite amount of small point charges. Therefore, we can apply the same method to turn the infinite sum into an integral. We get the following equation.

$$E = \int dE = k\int \frac{dq}{r^2}\hat{r}$$

This integral isn't always trivial. However, they're often ways you can simplify your equations and your integral to make your life easier, like looking for symmetry. Check out this HyperPhysics page to see an example of this integral in use.

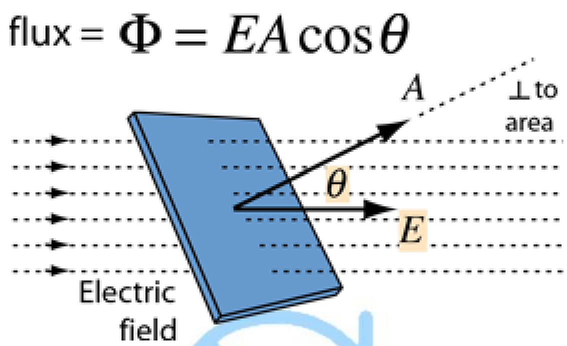
Something To Remember

Electric fields, like gravitational fields, don't do anything *until a charge enters the field.* The field is a description of how a charge will influence other charges, so if there aren't any other charges, then the electric field isn't actually doing anything at all.

Gauss's Law And Electric Flux

What Is Flux?

Flux is a concept that is important to many areas of physics. The flux of a vector quantity **X** is the amount of the quantity flowing through a surface.



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The direction of infinitesimal area *d***A** is **outward normal** to the surface.

Electric Flux

Flux can be of something physical, like water, or of something abstract, like an electric field, which is what we are looking at right now. You can compute a flux with a surface and a vector field **X** = **X**(*x*,*y*,*z*). With electric flux, our vector field **X** is just referring to the electric field **E**.

$$\Phi_q = \int \quad E \cdot dA$$

Here, we are taking the dot product of the field with the "area vector", to get the amount of the vector pointing in the direction perpendicular to the surface. (This is equivalent to the $EAcos(\theta)$ in the diagram above: that one just uses the angle instead of the dot product.)

Gauss's Law

Gauss's Law tells us the electric flux if we have a closed surface, like a sphere or cube. The formula is as follows:

$$\Phi_q = \oint E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

The scary integral symbol with a circle in it is a *surface integral*: it means you are adding up the infinitesimal bits over the surface you are considering. Q_{encl} is the charge enclosed by the closed surface, while ϵ_0 is the permittivity of free space, about $8.85 \times 10^{-12} C^2/Nm^2$. This is basically how easily an electric field can permeate in a vacuum.

You remember that constant mentioned in the previous 2 chapters, *k*? Well, it's quite closely related to the permittivity of free space:

$$k = \frac{1}{4\pi\epsilon_0}$$

Now let's figure out **why**. Imagine a charge q that is surrounded by a sphere of radius r. Let's try calculating the electric flux.

$$\Phi_q = \oint E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

$$EA = \frac{q}{\epsilon_0} \longrightarrow E = \frac{q}{A\epsilon_0}$$

$$A = 4\pi r^2 \longrightarrow E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2} = \frac{kq}{r^2}$$

Pretty interesting, huh? Flux is already showing us new things. Let's go on.

Electric Potential Energy

The work done by any force is $W = \int F \cdot dr$. Let's try taking the integral of the electrostatic force!

$$W = \int_{r_1}^{r_2} F_q \cdot dr = kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{kq_1q_2}{r}\Big|_{r_1}^{r_2} = -\Delta U_q$$

So, electric potential energy is given by

$$U_q = \frac{kq_1q_2}{r}$$

Now, let's consider what happens when we have positive/negative charges.

- If both charges are positive or negative, U_q will be **positive**. This means that it takes positive work to bring to 2 charges together from $r = \infty$ to r. Makes sense, right? When the 2 charges are both positive or negative, they will repel, which means you will have to do work to bring them together.
- If 1 charge is positive and the other negative, U_q will be **negative**. This means that it will take **negative work** to bring the 2 charges together. This also makes sense, intuitively: the 2 charges will be attracting each other!

Electric Potential (NOT energy!)

Electric potential is electrical potential energy per unit charge and is measured in units of joules per coulomb. For a charge q in a field created by q_{source}:

$$V = \frac{U_q}{q} = \frac{kq_s}{r}$$

Electric potential is measured in volts (joules per coulomb: 1V = 1 J/C). When a charge of q and a charge of $2q$ are displaced in the same way from one point to another in an electric field, in both cases, the ratio of the change in potential energy to the charge being displaced is equal is to the change in electric potential (the electric potential difference) from the first point to the endpoint.

Electric Potential Difference

The change in electric potential is called the **electric potential difference**, or **voltage**. It's also measured in volts (joules per coulomb).

$$V = \frac{U_q}{q}$$

↓

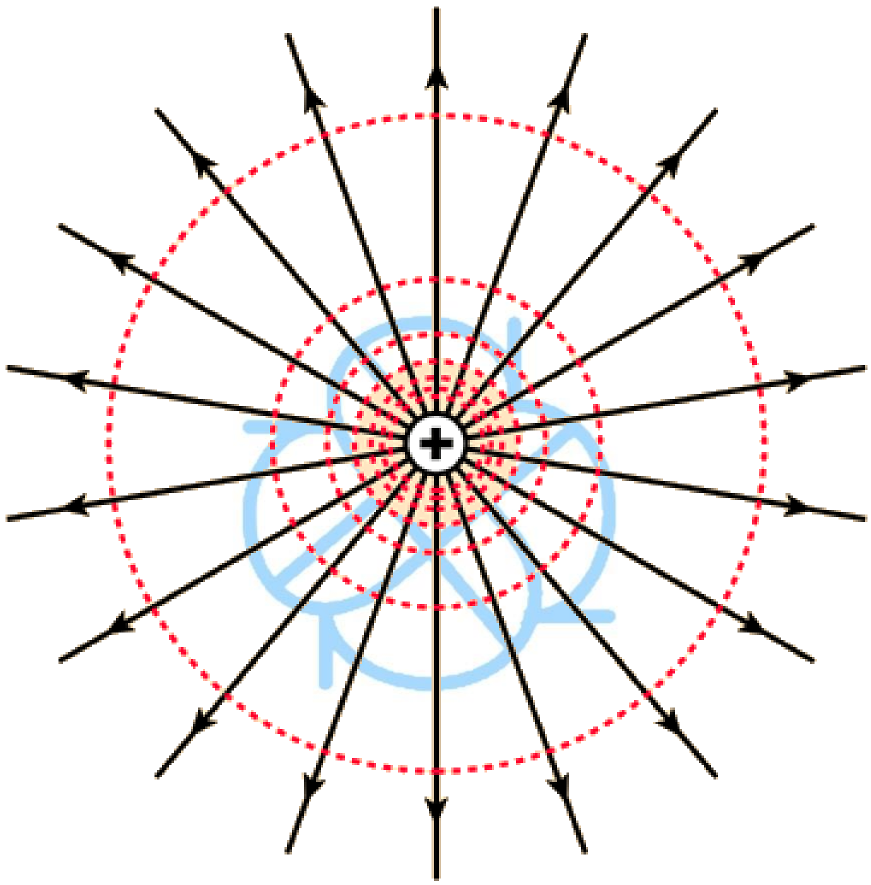
$$\Delta V = \frac{\Delta U_q}{q} \text{ and } dV = \frac{dU_q}{q}$$

You can also represent electric potential difference as an integral.

$$\Delta U_q = -\int F \cdot dr \longrightarrow \frac{\Delta U_q}{q} = \Delta V = -\int \frac{F}{q} \cdot dr$$

$$\Delta V = -\int E \cdot dr$$

Equipotential Lines



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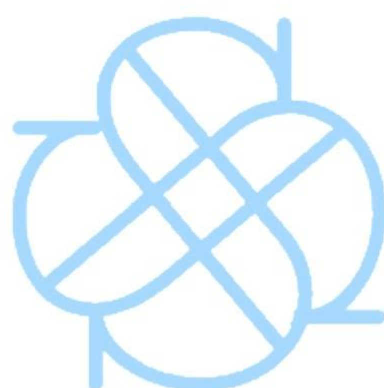
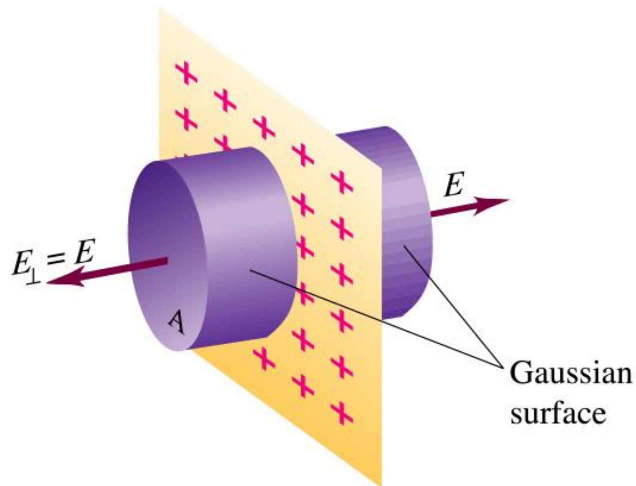
Equipotential lines are lines perpendicular to the electric field lines, where the electric potential is the same anywhere on the line.

Because the electric potential anywhere on an equipotential line is the same, no work is done when moving between points on an equipotential line.

Weird questions sometimes come up involving this: remember it.



Electric Field Near An Infinite Plane Of Charge



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Let's talk about the field near an infinitely large plane of charge. It's got a charge density of σ. By symmetry, the field **E** must be perpendicular to the plane. We have a Gaussian surface (in this case, a cylinder) with base area *A*. The height doesn't really matter.

Let's try and calculate flux: nothing is flowing out of the side, only out the ends of the cylinder. The flux is therefore $\Phi_q = E(2A)$.

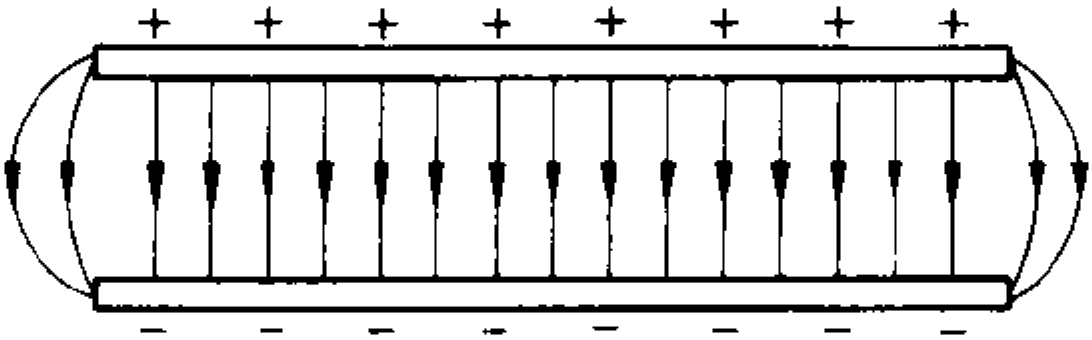
We can then use Gauss's Law.

$$\oint E \cdot dA = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\frac{\sigma A}{\epsilon_0} = E(2A) \longrightarrow E = \frac{\sigma}{2\epsilon_0}$$

We can see that *E* is a constant, independent of distance from the plane. Also, both sides of the plane are the same.

Electric Field Between 2 Parallel Charged Plates



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Let's imagine 2 parallel plates of uniform charge density, with electric field flowing in the same direction between them, separated by distance d with a potential difference of ΔV. What is the electric field inside the 3 plates?

Because it's like 2 charged planes, the electric field between the 2 plates will be uniform.

Imagine the work needed to move a charge q from the positive plate (call it plate A) to the bottom plate (call it plate B).

$$W = \Delta U_q = q\Delta V$$

The potential difference V_{AB} between A and B is

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}.$$

Work is *also* $W = Fd$ because of the constant field. $F = Eq$, so $W = Eqd$.

$$Eqd = qV_{AB} \longrightarrow E = \frac{V_{AB}}{d}$$

Again, we have a field that is the same, this time throughout the area between the plates.