

AP Physics Study Guide

Rotation and Angular Momentum

From Simple Studies, <https://simplestudies.edublogs.org> & @simplestudiesinc on Instagram

All images are from the Openstax college physics textbook

Angular Acceleration is the rate of change of angular velocity

- $\alpha = \Delta\omega/\Delta t$
 - If ω increases, alpha is positive
 - If ω decreases, alpha is negative
- Its units are (rad/s)/s

Tangential Acceleration is the acceleration in a direction tangent to the circle at the point of interest in circular motion

- It refers to changes in the magnitude of velocity but not its direction
- It is perpendicular and independent to centripetal acceleration
 - It is instead directly related to angular acceleration
 - $\alpha = a_t/r$
 - It is also linked to an increase or decrease in the velocity, but not its direction



The **kinematics of rotational motion** describe the relationships among rotation angle, angular velocity, angular acceleration, and time

- $\theta = \omega t$
- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + .5\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha\theta$

These are the steps to take for rotational kinematics problems:

- Examine the situation to determine that rotational kinematics are involved
- Identify exactly what needs to be determined in the problem
 - Draw a sketch of the situation
- Make a list of what is given or can be inferred from the problem as stated
- Solve the appropriate equation or equations for the quantity to be determined
- Substitute the known values along with their units into the appropriate equation
 - Obtain numerical solutions complete with units
- Check your answer to see if it is reasonable

The **moment of inertia** is the mass times the square of perpendicular distance from the rotation axis

- $I = mr^2$
- I is analogous to m in translational motion
 - So, net $\tau = I\alpha$
 - The larger the torque, the larger the angular acceleration
 - Distribution of mass relative to the axis around which it rotates also plays a part

These are the steps to take for rotational dynamics problems:

- Examine the situation to determine that torque and mass are involved in the rotation
 - Draw a sketch of the situation
- Determine the system of interest
- Draw a free body diagram
 - Draw and label all external forces acting on the system of interest
- Apply net $\tau = I\alpha$ to solve the problem
- Check the solution to see if it is reasonable

Rotational kinetic energy is the kinetic energy due to the rotation of an object

- $\text{Net}W = (\text{net } \tau)\theta$
- $KE_{\text{rot}} = .5I\omega^2$

These are the steps to take for rotational energy problems:

- Determine that energy or work is involved in the rotation
- Determine the system of interest
 - Draw a sketch of the situation
- Analyze the situation to determine the types of work and energy involved
- For closed systems, mechanical energy is conserved
 - Kinetic energy may include translational and rotational contributions
- For open systems, mechanical energy may not be conserved
 - Other forms of energy may enter or leave the system (heat)
 - Determine what they are, and calculate them as necessary
- Eliminate terms wherever possible to simplify the algebra
- Check the answer to see if it is reasonable

Angular momentum is the product of moment of inertia and angular velocity

- $L = I\omega$
 - An object that has a large moment of inertia has a large angular momentum
 - An object that has a large angular velocity has a large angular momentum
- Net $\tau = \Delta L / \Delta t$
 - If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases

The **law of conservation of angular momentum** states that the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system.

- $L = L'$
- $I\omega = I'\omega'$

