AP Calculus AB Course Study Guide Differentiation: Composite, Implicit, and Inverse Functions

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The Chain Rule

The chain rule helps us **find the derivative of a composite function.** For the formula, g'(x) would be the chain.

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Picture Credits: calcworkshop

Example:

We're given this function:	$f(x) = (3x^2 + 8)^2$
 Derive it by using the chain rule. g(x) = 3x²+8 When you chain 3x²+8, 3x² turns to 6x and 8 becomes 0. 	$f'(x)=2(3x^2+8)\cdot (6x)$
This is your final answer. You bring 6x to the front to make the equation look neater and multiply it by 2.	$f'(x)=12x(3x^2+8)$

General Rule Power

$$\frac{d}{dx}\Big[\Big(f(x)\Big)^n\Big] = n\Big(f(x)\Big)^{n-1} \cdot f'(x)$$

We use the general rule power when finding the **derivative of a function that is raised to the nth power.** In the formula given, f'(x) is the chain.

Implicit Differentiation

Step 1: Differentiate both sides of the	$y^3+y^2-5y+x^2=-4$
equation with respect to x. Whenever you	$3y^2(dy/dx)+2y(dy/dx)-5(dy/dx)-2x=0$
derive y, write dy/dx.	
Step 2: Move all dy/dx terms on one side of	$3y^2(dy/dx)+2y(dy/dx)-5(dy/dx)=2x$
the equation; move all other non dy/dx to the	
other side.	
Step 3: Factor out dy/dx	$dy/dx(3y^2+2y-5)=2x$
Step 4: Solve for dy/dx by dividing (3y ² +2y-	$dy/dx=2x/(3y^2+2y-5)$
5) on both sides.	

Inverse Trig Functions: Differentiation

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\csc^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$$
Picture Credits:

Example:

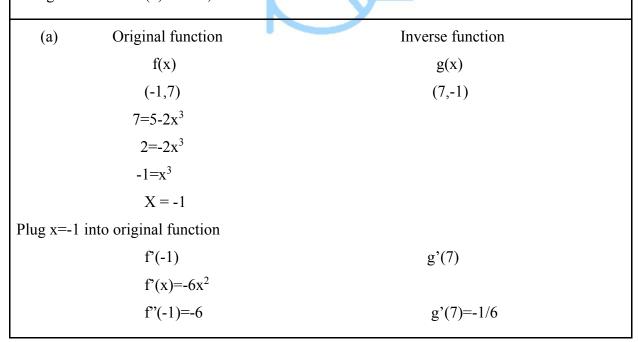
Function	h(x)=arcsinx-2arctanx

First, derive the inverse trig function arcsinx which is the same thing as sin ⁻¹ x	$h(x)=\arcsin x$ $h'(x)=\frac{1}{\sqrt{1-x^2}}$
Next, derive the inverse trig function 2arctanx.	$h(x)=2\arctan x$ $h'(x)=\frac{2}{x^2+1}$
Last, combine them and you have your answer.	$-\frac{1}{\sqrt{1-x^2}}-\frac{2}{x^2+1}$

Derivatives of Inverse Functions

Find:

- (a) The derivative of the inverse function at x=a
- (b) The equation of the tangent line to the inverse function at the point x=a Using the function $f(x)=5-2x^3$, a=7



(b)
$$y-y_1=m(x-x_1)$$

 $y-(-1)=-1/6(x-7)$
 $y+1=-1/6(x-7)$

Higher-Order Derivatives

Function	$f(x)=10x^3-3x^2+9x-7$
Finding the first derivative	$f'(x)=30x^2-6x+9$
Finding the second derivative	f''(x)=60x-6
Finding the third derivative	f'''(x)=60

