# AP Calculus AB Course Study Guide
# Limits and Continuity

**From Simple Studies, [https://simplestudies.edublogs.org](https://simplestudies.edublogs.org) & @simplestudies4 on Instagram**

## What is a limit and how to find it:

Limit: If f(x) becomes **close to a unique number L as x approaches c from either side**, then the limit of f(x) as x approaches c is L.

- A limit refers to the y-value of a function

Lim f(x) = L

- The general limit exists when the right and left limits are the same/equal each other.
- ***DNE = does not exist.***

*Examples of estimating a limit numerically:*

| x | 1.9 | 1.99 | 1.999 | 2.0 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|---|
| f(x) | 3.700 | 3.970 | 3.997 | **4** | 4.003 | 4.030 | 4.4 |

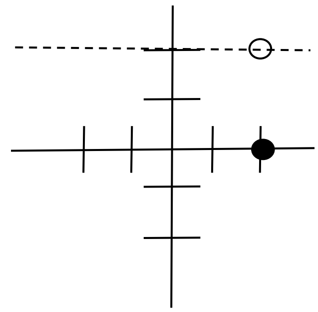
- Example 2: Given lim (3x-2), find what L would be when you plug in the constant of 2.

Lim f(3x-2) → Lim f(3(2)-2)

*Example of using a graph to find a limit:*

$$f(x) = \begin{cases} 2, & x \neq 3 \\ 0, & x = 3 \end{cases}$$

$$\lim_{x \to 3} f(x) = 2$$

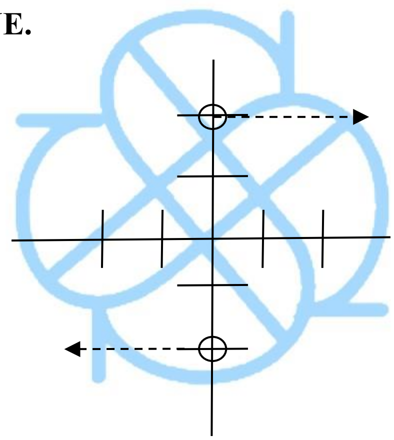


*2 is the limit.

## When limits don't exist:

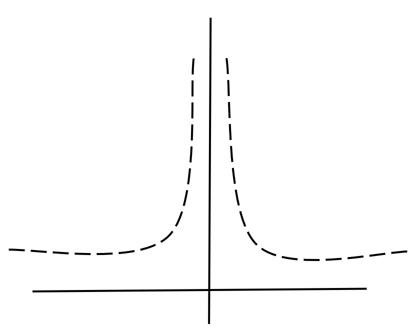
When the **Left limit ≠ Right limit,** then the limit is **said to not exist.**

- In the picture below, you can tell that the two limits don't equal each other, thus the answer to this limit is **DNE.**

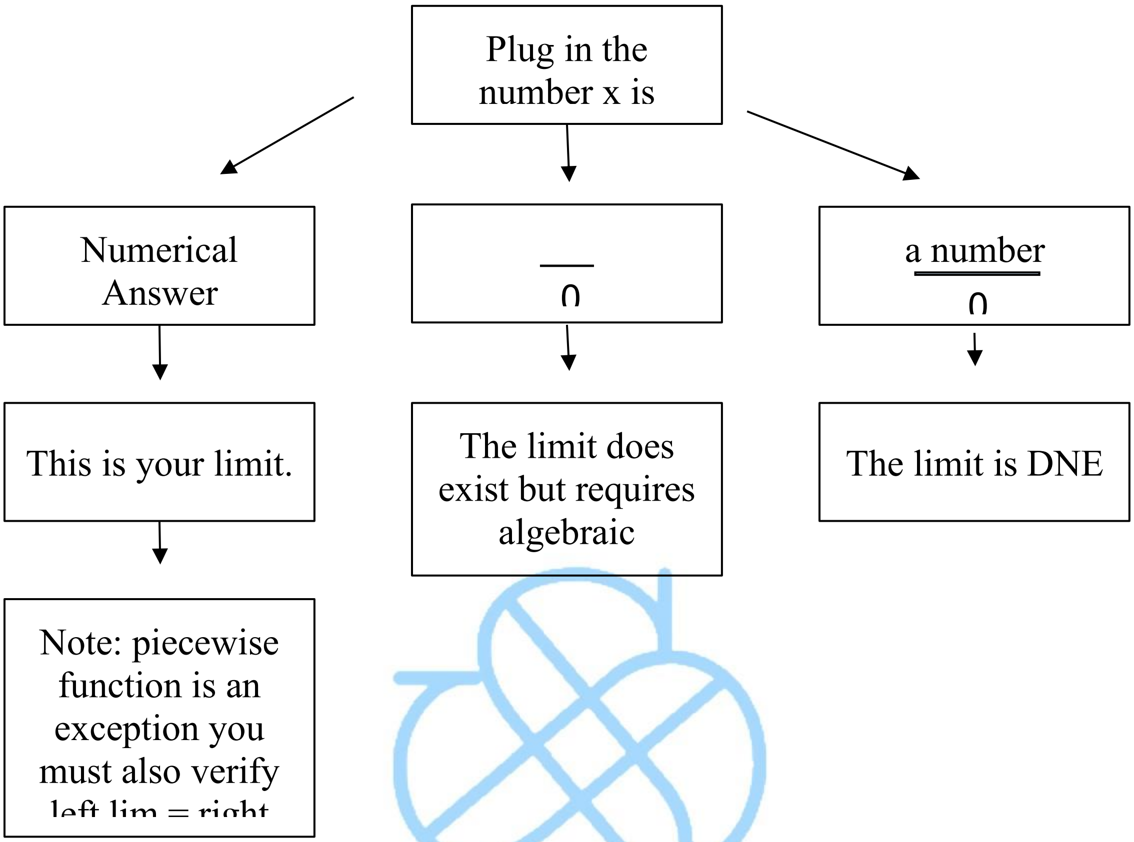


## Unbounded Behavior:

$$\lim_{x \to 0} \frac{1}{x^2}$$



## Evaluating Limits Analytically:



## Limits Theorem:

Given:

Lim and Lim

| | |
|---|---|
| **Scalar Multiple** | $\lim_{x\to c}[bf(x)] = bL$ |
| **Sum/Difference** | $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$ |
| **Product** | $\lim_{x\to c}[f(x)g(x)] = LK$ |
| **Quotient** | $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \left(\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\right) = \frac{L}{M}$ |
| **Power** | $\lim_{x\to a}\left(f(x)\right)^r = \left(\lim_{x\to a} f(x)\right)^r = L^r$ |

Picture Credits: need2knowaboutcalculus & khan academy

## Limits at Infinity

- If m<n, then the limit equals 0
- If m=n, then the limit equals a/b
- If m>n, then the limit DNE

$$\lim_{x\to \pm\infty} \frac{ax^m}{bx^n}$$

## Finding Vertical Asymptotes

The only step you have to do is ***set the denominator equal to zero and solve.***

- Example:

$$f(x) = \frac{x - 2}{x^2 - 4} = \frac{x - 2}{(x+2)(x-2)}$$

  - $(x+2)(x-2) = 0 \rightarrow x = 2, -2$
    - 2 is a removable hole while -2 is the non-removable vertical asymptote.

## Finding Horizontal Asymptotes

Use the ***two terms of the highest degree in the numerator and denominator***

- Example:

$$\lim_{x \to \infty} \frac{x-2}{x^2-4}$$

  - x and $x^2$ are the two terms of the highest degree in the numerator and denominator respectively. After finding it, use the limits at infinity rule to determine the limit.

## Intermediate Value Theorem

A continuous function on a **closed interval cannot skip values.**

- f(x) must be continuous on the given interval [a,b]
- f(a) and f(b) cannot equal each other.
- f(c) must be in between f(a) and f(b)

*Example #1*: Apply the IVT, if possible on [0,5] so that f(c)=1 for the function $f(x)=x^2+x+1$

1) f(x) is continuous because it is a polynomial function.
2) f(a)=f(0)=1
   f(b)=f(5)=29
3) By the IVT, there exists a value c where f(c)=1 since 1 is between -1 and 29.

*Example #2:*

| t(seconds) | 0 | 15 | 25 | 30 | 35 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| v(t) in ft/sec | -20 | -30 | -20 | -14 | -10 | 0 | 10 |

1) For $0<t<60$, must there be a time t when v(t) = -5?
2) f(a) = f(0) = -20

   f(b) = f(60) =10
3) By the IVT, there is a time t where v(t)=-5 on the interval [0,60] since $-20 < -5 < 10$

## The Squeeze Theorem

$$h(x) \le f(x) \le g(x)$$

$$\lim_{x\to a} h(x) = \lim_{x\to a} g(x) = L$$

therefore,

$$\lim_{X\to a} h(x) = L$$

that means f(x) equals h(x) and g(x)