Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

Exam style practice papers

Exercise MM1B, Question 1

Question:

A car, of mass 1200 kg, accelerates from rest to a speed of 10 $m s^{-1}$, as it travels a distance of 40 m. Assume that the acceleration of the car is constant.

(a) Calculate the acceleration of the car and the magnitude of the resultant force on the car.
(4 marks)

(b) If the car continues with the same acceleration, how long would it take to reach a speed of 20 $m s^{-1}$ and how far would it have travelled since setting off. (4 marks)

(c) Explain why it is unlikely that the car would continue to move with the same acceleration.
(2 marks)

Solution:

(a)

Using $v^2 = u^2 + 2as$,

$10^2 = 0 + 2 \times a \times 40$

$a = \frac{100}{80}$

$a = 1.25$

$\therefore$ Acceleration is 1.25 $m s^{-2}$

Using $F = ma$

$F = 1200 \times 1.25$

$= 1500$

$\therefore$ Resultant force is 1500 N

(b)

Using $v = u + at$,

$20 = 0 + 1.25t$

$t = 16$

$\therefore$ Time is 16 s

To find the distance, use $v^2 = u^2 + 2as$

$20^2 = 0 + 2 \times 1.25 \times s$

$s = 160$

$\therefore$ Distance is 160 m

(c) Air resistance would increase as speed increases, and decrease the resultant force and the acceleration.

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Exercise MM1B, Question 2

Question:

Two particles, *A* and *B*, have masses of 3 kg and 7 kg, respectively. They are moving on a horizontal surface when they collide. Just before the collision the velocity of *A* is $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ m s^{-1} and the velocity of *B* is $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$ m s^{-1}.

(a) Find the velocity of the particles after the collision if they coalesce. (3 marks)

(b) Find the velocity of *A* after the collision, if the velocity of *B* is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ m s^{-1}. (3 marks)

Solution:

(a) Using conservation of momentum

$$3\begin{bmatrix} 2 \\ -6 \end{bmatrix} + 7\begin{bmatrix} -3 \\ 4 \end{bmatrix} = 10v$$

$$\begin{bmatrix} 6 \\ -18 \end{bmatrix} + \begin{bmatrix} -21 \\ 28 \end{bmatrix} = 10v$$

$$\begin{bmatrix} -15 \\ 10 \end{bmatrix} = 10v$$

$$v = \begin{bmatrix} -1.5 \\ 1 \end{bmatrix}$$

Velocity is $\begin{bmatrix} -1.5 \\ 1 \end{bmatrix}$ m s^{-1}

(b) Using conservation of momentum

$$3\begin{bmatrix} 2 \\ -6 \end{bmatrix} + 7\begin{bmatrix} -3 \\ 4 \end{bmatrix} = 3v + 7\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -15 \\ 10 \end{bmatrix} = 3v + \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

$$3v = \begin{bmatrix} -15 \\ 3 \end{bmatrix}$$

$$v = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

$\therefore$ Velocity is $\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ m s^{-1}.

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Exercise MM1B, Question 3

Question:

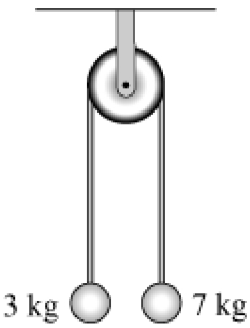
Two particles of mass 3 kg and 7 kg are joined by a light, inextensible string that passes over a light, smooth pulley. Assume that there is no air resistance.
The particles are initially released from rest at the same level.

(a) By forming an equation of motion for each particle, find the acceleration of the particles. (4 marks)

(b) Find the tension in the string. (2 marks)

(c) Find the speed of the particles when they are 20 cm apart. (2 marks)

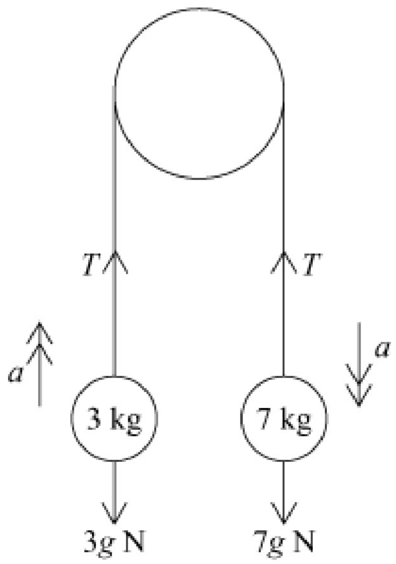
(d) If the string was not light, what would happen to the acceleration of the particles? (2 marks)



Solution:

Using $F = ma$

for 7 kg particle; $7g - T = 7a$
3 kg particle; $T - 3g = 3a$ [1]
Adding $4g = 10a$
$a = \frac{4}{10}g$
$\therefore$ Acceleration is 3.92 m s^{-2}



(b) From [1] $T = 3g + 3a$

$= 41.16$

Tension is 41.16 N

(c) When the particles are 20 cm apart, each particle has moved 10 cm or 0.1 m

To find the speed, use $v^2 = u^2 + 2as$

$v^2 = 2 \times 3.92 \times 0.1$

$v = 0.8854$

Speed is 0.885 m s^{-1}

(d) If the string was not light, as time went by more of the string, and hence more weight, would move to the side with the 7 kg particle; thus the acceleration would increase as time goes by.

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Exercise MM1B, Question 4

Question:

A car, of mass 1000 kg, skids for 20 m and then hits a car of mass 1200 kg that is moving at 10 $\mathrm{m\ s^{-1}}$ in the same direction. After the collision, both cars move together at a speed of 12 $\mathrm{m\ s^{-1}}$. The coefficient of friction between the tyres and the road for the first car is 0.8. Assume that all the motion takes place on a horizontal surface.

(a) Use conservation of momentum to find the speed of the first car, just before the collision. (2 marks)

(b) Find the speed of the first car when it begins to skid. (4 marks)

Solution:

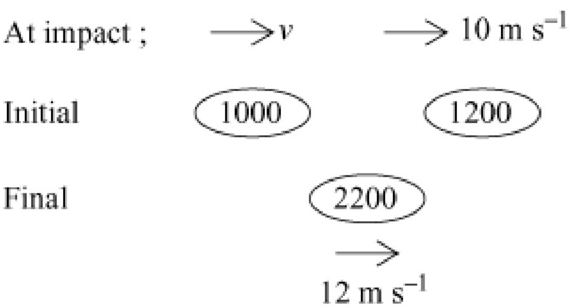
(a) Using conservation of momentum

$$1000v + 1200 \times 10 = 2200 \times 12$$

$$1000v = 14400$$

$$v = 14.4$$

$\therefore$ speed of first car was $14.4\ \mathrm{m\ s^{-1}}$



(b) For the first car;

resolve vertically $R = 1000g$

Using $F = \mu R$

$$F = 0.8 \times 1000g$$

$$= 7840\ \mathrm{N}$$

Using $F = ma$

$$-7840 = 1000a$$

$$a = -7.84\ \mathrm{m\ s^{-2}}$$

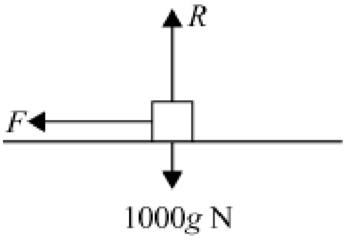
To find the speed when the car started to slide, using $v^2 = u^2 + 2as$

$14.4^2 = u^2 - 2 \times 7.84 \times 20$

$u^2 = 520.96$

$u = 22.82$

Speed was 22.8 m s^{-1}.



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Exercise MM1B, Question 5

Question:

An aeroplane heads due north at a speed of 200 m s^{-1}. A wind is blowing north east and has speed 50 m s^{-1}.

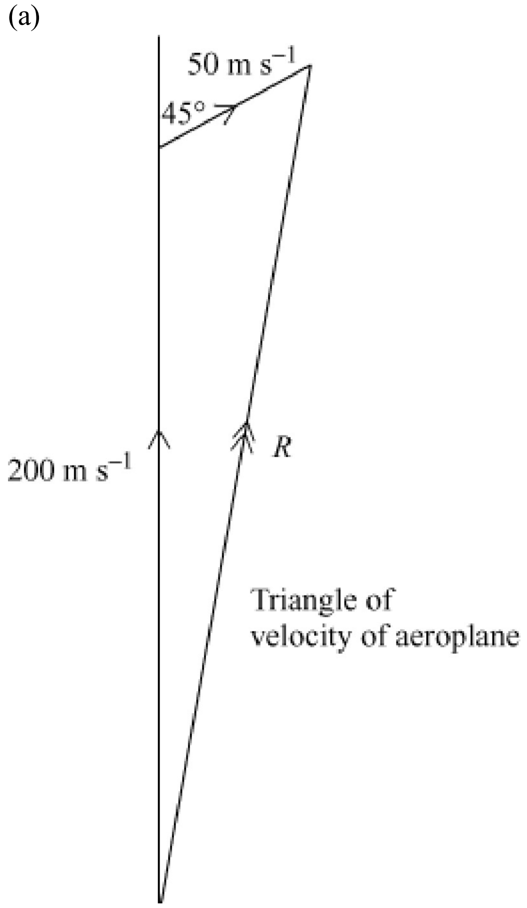
(a) Sketch a velocity triangle. (1 mark)

(b) Calculate the resultant speed of the aeroplane. (4 marks)

(c) Find the bearing on which the aeroplane actually moves. Give your answer to the nearest 0.1°. (4 marks)

Solution:

(a)



(b)

By cosine rule $R^2 = 200^2 + 50^2 - 2 \times 50 \times 200 \cos 135°$

$= 42500 + 14142$

$= 56642$

$R = 237.995$

Resultant speed is 238 m s^{-1}

(c) In the triangle shown,

$P = 50 \sin 45°$

$q = 200 + 50 \cos 45°$

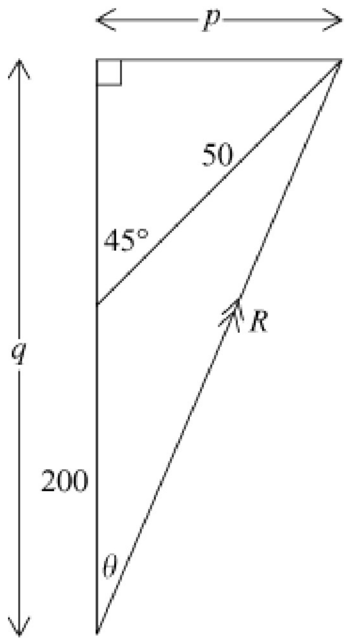
$\tan \theta = \frac{p}{q}$

$= \frac{50 \sin 45°}{200 + 50 \cos 45°}$

$= 0.15022$

$\therefore \theta = 8.54°$

$\therefore$ Bearing is 008.5° .



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Exercise MM1B, Question 6

Question:

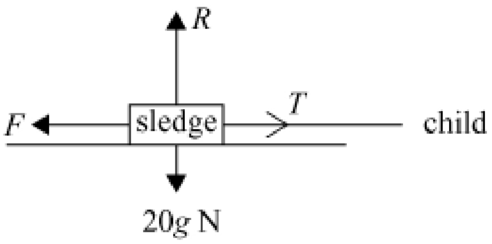
A child pulls a sledge, of mass 20 kg, along a snow-covered surface. The child uses a rope that remains horizontal as he pulls. The coefficient of friction between the ground and the sledge is 0.2. The tension in the rope is T N.

(a) The sledge is pulled along a horizontal surface.
(i) Show that the magnitude of the friction force acting on the sledge is 39.2 N. (2 marks)
(ii) Find the tension in the rope if the sledge accelerates at 0.05 m s^{-2} on a horizontal surface. (4 marks)
(iii) State the tension needed to keep the sledge moving at a constant speed. (2 marks)

(b) The child then pulls the sledge up a slope inclined at an angle of 10° to the horizontal. The rope is parallel to the slope. Find the tension if the sledge moves at a constant speed. (4 marks)

Solution:

(a) (i)



For the sledge; resolve vertically $R = 20g$
$F = \mu R$
$F = 0.2 \times 20g$
$F = 4g$
$\therefore$ Magnitude of the friction force is 39.2 N

(ii) For the sledge, using $F = ma$ horizontally
$T - F = 20 \times 0.05$
$T = 40.2$
Tension is 40.2 N

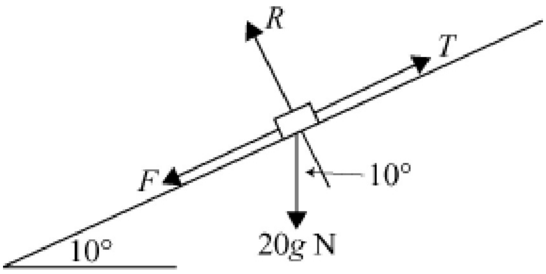
(iii) If the sledge is to move at a constant speed,
$a = 0$ and $T = F$ $\therefore$ Tension is 39.2 N

Resolve perpendicular to the slope $R = 20g \cos 10°$
$= 193.02$

(b) Using $F = \mu R$,
$F = 0.2 \times 193.02$
$= 38.6$

Resolve along the slope (constant speed ⇒ body in equilibrium) $T = F + 20g \sin 10°$
$= 38.6 + 34.03$

Tension is 72.6 N.



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Exam style practice papers

Exercise MM1B, Question 7

Question:

A particle is initially at rest and has position vector $(30\mathbf{i} + 400\mathbf{j})$ m, where the unit vectors **i** and **j** are east and north, respectively. The particle moves with a constant acceleration so that 10 seconds later its position is $(100\mathbf{i} + 450\mathbf{j})$ m.

(a) (i) Show that the acceleration of the particle is $(1.4\mathbf{i} + \mathbf{j})$ m s^{-2}. (3 marks)
(ii) Find the velocity of the particle after 10 s. (2 marks)
(iii) Find an expression for the position of the particle at time t s. (2 marks)

(b) After the first 10 seconds the particle stops accelerating and moves with a constant velocity. Find the position vector of the particle when it has been moving for 25 s. (4 marks)

Solution:

(a) (i)

$$\text{Displacement in 10 seconds} = 100\mathbf{i} + 450\mathbf{j} - (30\mathbf{i} + 400\mathbf{j}) = 70\mathbf{i} + 50\mathbf{j}$$

$$\text{Using } s = ut + \frac{1}{2}at^2,$$

$$70\mathbf{i} + 50\mathbf{j} = \frac{1}{2} \times a \times (10)^2$$

$$a = 1.4\mathbf{i} + \mathbf{j}$$

$$\therefore \text{Acceleration is } (1.4\mathbf{i} + \mathbf{j}) \text{ m s}^{-2}$$

(ii)

$$\text{To find } v \text{ use } v = u + at$$

$$\therefore v = 10 \times (1.4\mathbf{i} + \mathbf{j})$$

$$v = 14\mathbf{i} + 10\mathbf{j} \text{ m s}^{-1}$$

(iii) Using $s = ut + \frac{1}{2}at^2$,

Position vector at time t is given by

$$s = ut + \frac{1}{2}at^2 + 30\mathbf{i} + 400\mathbf{j} \text{ since particle starts at } 30\mathbf{i} + 400\mathbf{j}$$

$$s = \frac{1}{2}(1.4\mathbf{i} + \mathbf{j})t^2 + 30\mathbf{i} + 400\mathbf{j}$$

$$s = (30 + 0.7t^2)\mathbf{i} + (400 + 0.5t^2)\mathbf{j}$$

(b) When the particle has moved for 25 seconds,

position vector of particle, r	= position vector (when $t = 10$) + 15 × velocity (when $t = 10$)
r	= 100i + 450j + 15 (14i + 10j)
∴ r	= 310i + 600j

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Heinemann Modular Maths for Edexcel AS and A-level

Exam style practice papers
Exercise MM1B, Question 8

Question:

A shot is thrown with an initial velocity 10 m s^{-1}, at an angle of 40° above the horizontal.

(a) A simple model assumes that the height of release of the shot is zero. Calculate the range of the shot. (5 marks)

(b) In fact the shot is released at a height of 1.8 m. What would happen if the shot was thrown in this way inside a hall of length 20 m and height 3.5 m? (6 marks)

Solution:

(a) The position of the shot, relative to the point of throwing, is given by

$$x = 10 \cos 40° t$$
$$y = 10 \sin 40° t - \frac{1}{2}gt^2$$

At the range, its height, y, is zero.

$$\therefore 10 \sin 40° t - \frac{1}{2}gt^2 = 0$$

$$t = 0 \text{ (not required) or } t = \frac{10 \sin 40°}{\frac{1}{2}g}$$

$$t = 1.3118s$$
$$\text{Range, } x = 10 \cos 40° \times 1.3118$$
$$= 10.049$$
$$\therefore \text{Range is } 10.0 \text{ m}$$

(b) For the maximum height of the shot,

$$\text{Using } v^2 = u^2 + 2as,$$
$$0 = (10 \sin 40°)^2 - 2gs$$
$$s = \frac{(10 \sin 40°)^2}{2g}$$
$$s = 2.108 \text{ m}$$

$\therefore$ The shot reaches a height of
$2.108 + 1.8 = 3.908$ m above the floor.
Hence it will hit the roof of the hall as the shot will rise to a height of 3.91 m.