Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

Exam style practice papers

Exercise MM1A, Question 1

Question:

A ball is dropped from a height of 8 metres. The ball falls vertically until it hits the ground.

(a) Find the speed of the ball when it hits the ground. (2 marks)

(b) Find the time that it takes the ball to reach the ground. (2 marks)

(c) State the key assumption that you have made about the motion of the ball. (1 mark)

(d) Sketch a velocity-time graph for the motion of the ball. (2 marks)

Solution:

(a)

Using $v^2 = u^2 + 2as$,

$v^2 = 0 + 2 \times g \times 8$

$v^2 = 16g$

$v = 12.52$

Speed of the ball is 12.5 m s^{-1}

(b)

Using $s = ut + \frac{1}{2}at^2$,

$8 = \frac{1}{2}gt^2$

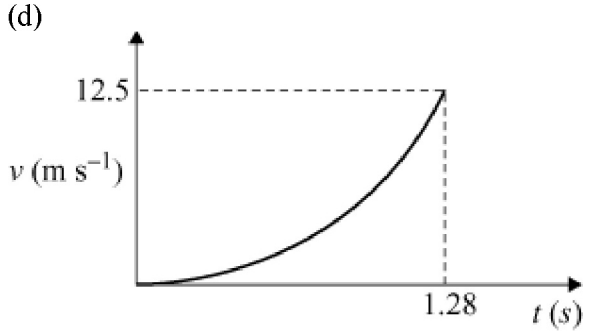
$t = \sqrt{\frac{8}{\frac{1}{2}g}}$

$t = 1.2777$

Time is 1.28 s

(c) No air resistance.

(d)



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Exercise MM1A, Question 2

Question:

Two particles, *A* and *B*, are moving on a horizontal surface when they collide. Before the collision the velocity of *A* is $\begin{bmatrix} \frac{3}{6} \end{bmatrix}$ m s^{-1} and the velocity of *B* is $\begin{bmatrix} \frac{-5}{4} \end{bmatrix}$ m s^{-1}. After the collision, both particles move together with velocity **v**.

(a) If the mass of *A* is 2 kg and the mass of *B* is 3 kg, find **v**. (3 marks)

(b) If the particles have the same mass, find **v**. (3 marks)

Solution:

(a) Using conservation of momentum

$$2\left[\frac{3}{6}\right] + 3\left[\frac{-5}{4}\right] = 5\mathrm{v}$$

$$\left[\frac{6}{12}\right] + \left[\frac{-15}{12}\right] = 5\mathrm{v}$$

$$\left[\frac{-9}{24}\right] = 5\mathrm{v}$$

$$\therefore \mathrm{v} = \left[\frac{-1.8}{4.8}\right] \text{ m s}^{-1}$$

(b) Let both particles have mass *m* kg; using conservation of momentum

$$m\left[\frac{3}{6}\right] + m\left[\frac{-5}{4}\right] = 2m\ \mathrm{v}$$

$$\left[\frac{-2}{10}\right] = 2\mathrm{v}$$

$$\mathrm{v} = \left[\frac{-1}{5}\right] \text{ m s}^{-1}$$

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Exercise MM1A, Question 3

Question:

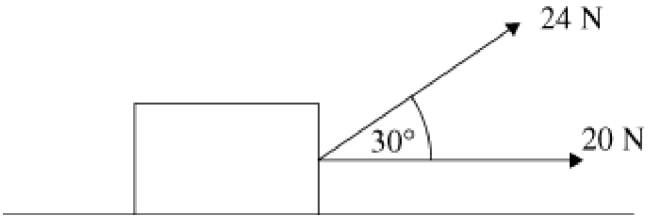
A box of mass 20 kg is at rest on a rough horizontal surface. Two ropes are attached to the box and exert forces as shown in the diagram.
Model the box as a particle.

(a) Draw a diagram to show all of the forces acting on the box. (1 mark)

(b) Show that the magnitude of the normal reaction force acting on the box is 184 N. (3 marks)

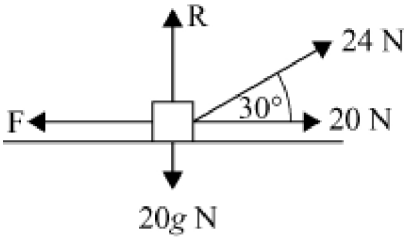
(c) Calculate the magnitude of the friction force acting on the box. (2 marks)

(d) The coefficient of friction between the box and the surface is μ. Find an inequality that μ must satisfy. (3 marks)



Solution:

(a)



Resolve vertically $R + 24 \sin 30° = 20g$

(b) $\therefore R = 184$

Magnitude of normal reaction force is 184 N .

Resolve horizontally $F = 20 + 24 \cos 30°$

(c) $= 40.78$

Magnitude of friction force is 40.8 N .

(d) Particle is stationary,

$$
\begin{aligned}
\text{thus } F &\leq \mu R \\
40.78 &\leq 184\mu \\
\mu &\geq \frac{40.78}{184} \\
\mu &\geq 0.2216 \\
\therefore \mu &\geq 0.222
\end{aligned}
$$

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Exercise MM1A, Question 4

Question:

A boat is to cross a river. In still water the boat would travel at 3 m s^{-1}. A current flows at a speed of 1 m s^{-1} in the river.

(a) If the velocity of the boat relative to the water is perpendicular to the bank, find the angle between the resultant velocity of the boat and the bank. (4 marks)

(b) If the resultant velocity of the boat is perpendicular to the bank, find the angle between the bank and the velocity of the boat. (4 marks)

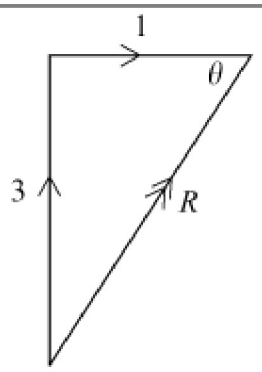
Solution:

(a) θ is the angle between the resultant velocity and the bank.

$\tan \theta = \frac{3}{1}$

$\theta = 71.56$

Angle is 71.6 °

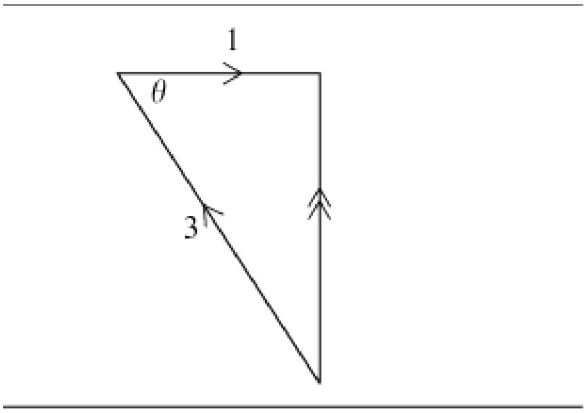


(b) In this scenario

$\cos \theta = \frac{1}{3}$

$\theta = 70.52$

$\therefore$ Angle is 70.5 °



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Exercise MM1A, Question 5

Question:

The diagram shows two particles that are connected by a light, inelastic string that passes over a smooth, light pulley. One of the particles is on a rough horizontal surface and the other is hanging freely.
The particle on the surface has mass 9 kg and the other particle has mass 5 kg. The coefficient of friction between the particle and the surface is 0.2. The particles are released from rest.

(a) Find the acceleration of the particles. (5 marks)

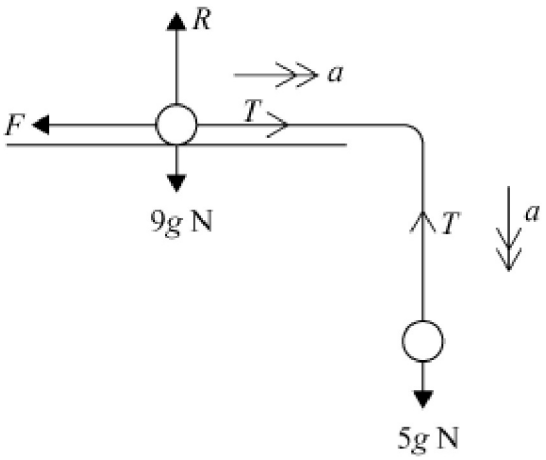
(b) Find the tension in the string. (2 marks)

(c) When the particles have been moving for 1 second the string breaks and the particle on the surface slows down and stops before it reaches the pulley. Find the total distance travelled by this particle. (5 marks)

Solution:

(a) For 9 kg particle, resolve vertically

$R = 9g$
Using $F = \mu R$,
$F = 0.2 \times 9g$
$= 17.64$



Using $F = ma$, for 5 kg particle; $5g - T = 5a$ [1]
for 9 kg particle; $T - F = 9a$
Adding $5g - F = 14a$
$14a = 5g - 17.64$
$14a = 31.36$
$a = 2.24$

Acceleration is 2.24 m s^{-2}

(b) From [1] $T = 5g - 5a$
$= 37.8$

$\therefore$ Tension is 37.8 N

(c) To find the speed of the particles just before the string breaks,

use $v = u + at$
$v = 2.24 \times 1$
$= 2.24$ m s^{-1}

Distance travelled in this 1 second,

using $s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2} \times 2.24 \times 1^2 = 1.12$ m

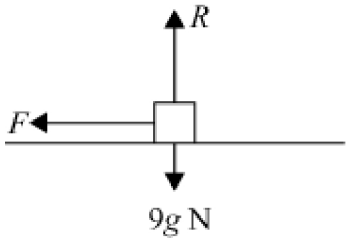
Particle of mass 9 kg moves under frictional force
(R and $9g$ N cancel each other)

Using $F = ma$,
$-17.64 = 9a$
$a = -1.96$ m s^{-2}

To find the distance moved after the string breaks,

$v^2 = u^2 + 2as$
$2.24^2 = 2 \times 1.96 \times s$
$s = \frac{2.24^2}{2 \times 1.96}$
$s = 1.28$ m

$\therefore$ Total distance travelled is 1.12 + 1.28
= 2.4 m



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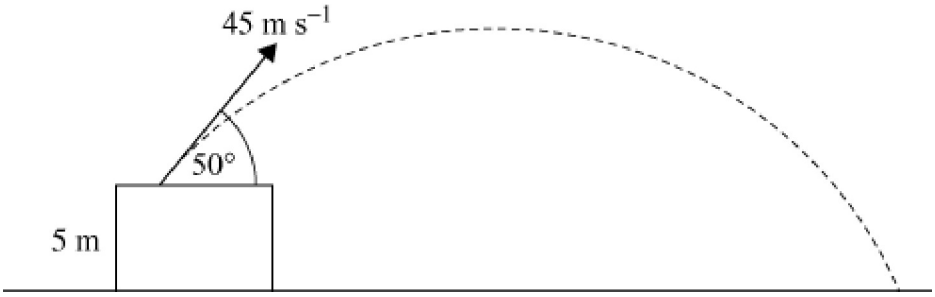
Exercise MM1A, Question 6

Question:

A golfer hits a ball from a raised platform. The ball initially moves at 45 m s^{-1} and at an angle of 50° above the horizontal. The ball lands on a horizontal surface that is 5 m lower than the platform. The diagram shows the path of the ball.

(a) Show that the time that the ball is in the air is 7.18 seconds, correct to three significant figures. (5 marks)

(b) Find the horizontal distance travelled by the ball. (2 marks)



Solution:

(a) The position of the ball relative to its point of projection is given by

$$x = 45 \cos 50° t$$

$$y = 45 \sin 50° t - \frac{1}{2}gt^2$$

When the particle hits the ground, $y = -5$

$$\therefore -5 = 45 \sin 50° t - \frac{1}{2}gt^2$$

$$4.9t^2 - 34.47t - 5 = 0$$

$$t = \frac{34.47 \pm \sqrt{34.47^2 + 4 \times 4.9 \times 5}}{2 \times 4.9}$$

$$t = 7.177 \text{ s}$$

Time that the ball is in the air is 7.18 s (to 3 s.f.)

(b) Horizontal distance, x is $45 \cos 50° \times 7.177$

$= 207.59$ m

$\therefore$ Horizontal distance is 208 m.

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Exercise MM1A, Question 7

Question:

A speed boat moves with a constant acceleration of $(-\mathbf{i}+\mathbf{j})$ m s^{-2}. Initially the boat is at the origin and has velocity $4\mathbf{i}$ m s^{-1}. The unit vectors **i** and **j** are directed east and north, respectively. The velocity of the boat at time t seconds is $\mathbf{v}$ m s^{-1}.

(a) Find **v** in terms of t. (2 marks)

(b) At time T seconds, where $T > 0$, the speed of the boat is 20 m s^{-1}.
(i) Write down an equation that T must satisfy. (3 marks)
(ii) Hence, find T. (2 marks)
(iii) Find the distance of the boat from the origin at this time. (4 marks)

Solution:

Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$,

(a) $\mathbf{v} = 4\mathbf{i} + (-\mathbf{i}+\mathbf{j})t$

$= (4-t)\mathbf{i} + t\mathbf{j}$

(b) (i) If the speed is 20 m s^{-1}, and $t = T$;

$20 = \sqrt{(4-T)^2 + T^2}$

$400 = 16 - 8T + T^2 + T^2$

$2T^2 - 8T - 384 = 0$

$T^2 - 4T - 192 = 0$

(ii) $(T-16)(T+12) = 0$

$\therefore T = 16$ (-12, not required)

Using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$,

$\mathbf{s} = 4\mathbf{i}T + \frac{1}{2}(-\mathbf{i}+\mathbf{j})T^2$

(iii) $\mathbf{s} = 64\mathbf{i} + 128(-\mathbf{i}+\mathbf{j})$ (using $T = 16$)

$\mathbf{s} = -64\mathbf{i} + 128\mathbf{j}$

Distance is $\sqrt{64^2 + 128^2}$

$= 143.1$

Distance is 143 m.