Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 1

Question:

Two particles, *A* and *B*, collide. After the collision they move together as a single particle. The mass of *A* is 12 kg and the mass of *B* is 8 kg. Before the collision, the velocity of *A* is $(4\mathbf{i} - 6\mathbf{j})$ m s^{-1} and the velocity of *B* is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. Find the velocity and the speed of the combined particle after the collision.

Solution:

Using conservation of momentum,

$$\begin{aligned} 12(4\mathbf{i} - 6\mathbf{j}) + 8(3\mathbf{i} + 2\mathbf{j}) &= 20\mathbf{v} \\ 48\mathbf{i} - 72\mathbf{j} + 24\mathbf{i} + 16\mathbf{j} &= 20\mathbf{v} \\ 72\mathbf{i} - 56\mathbf{j} &= 20\mathbf{v} \\ \mathbf{v} &= 3.6\mathbf{i} - 2.8\mathbf{j} \text{ m s}^{-1} \end{aligned}$$

Where **v** is the velocity of the combined particle.

Speed of the combined particle is

$$\sqrt{3.6^2 + (-2.8)^2} = 4.56 \text{ m s}^{-1}.$$

Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 2

Question:

Particle *A* has mass 2 kg and velocity $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ m s^{-1} when it collides with particle *B*, which has mass 3 kg and velocity $\begin{bmatrix} 10 \\ 8 \end{bmatrix}$ m s^{-1}. After the collision the particles move together. Find the velocity of the combined particles after the collision.

Solution:

The combined particles have mass 5 kg and travel with final velocity **v**.
Using conservation of momentum

$$2\begin{bmatrix} 6 \\ -2 \end{bmatrix} + 3\begin{bmatrix} 10 \\ 8 \end{bmatrix} = 5\mathbf{v}$$

$$\begin{bmatrix} 12 \\ -4 \end{bmatrix} + \begin{bmatrix} 30 \\ 24 \end{bmatrix} = 5\mathbf{v}$$

$$5\mathbf{v} = \begin{bmatrix} 42 \\ 20 \end{bmatrix}$$

$$\therefore \mathbf{v} = \begin{bmatrix} 8.4 \\ 4 \end{bmatrix} \text{ m s}^{-1}$$

Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 3

Question:

Particle A has mass 5 kg and velocity $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ m s^{-1} when it collides with particle B, which has mass 4 kg. After the collision the particles move together with velocity $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ m s^{-1}. Find the velocity of B before the collision.

Solution:

Using conservation of momentum,
with $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ being the velocity of B before the collision,

$$5\begin{bmatrix} 3 \\ 7 \end{bmatrix} + 4\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 9\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 15 + 4u_1 \\ 35 + 4u_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 36 \end{bmatrix}$$

Using i components; $15 + 4u_1 = 9 \quad \Rightarrow \quad u_1 = -1.5$

Using j components; $35 + 4u_2 = 36 \quad \Rightarrow \quad u_2 = 0.25$

$\therefore$ Velocity of B was $\begin{bmatrix} -1.5 \\ 0.25 \end{bmatrix}$ m s^{-1}.

Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 4

Question:

Two particles both have mass 5 kg and are moving with velocities $(9\mathbf{i} - 6\mathbf{j})$ m s^{-1} and $\left(6\mathbf{i} + 6\mathbf{j}\right)$ m s^{-1} when they collide and coalesce. Describe the direction in which they move after the collision and their speed.

Solution:

Using conservation of momentum with mass of coalesced particle as 10 kg;

$$\begin{aligned} 5(9\mathbf{i} - 6\mathbf{j}) + 5(6\mathbf{i} + 6\mathbf{j}) &= 10\mathbf{v} \\ 75\mathbf{i} &= 10\mathbf{v} \\ \therefore \mathbf{v} &= 7.5\mathbf{i} \end{aligned}$$

$\therefore$ The coalesced particles move parallel to the unit vector **i** at 7.5 m s^{-1}.

Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 5

Question:

A particle has mass 2 kg and velocity $(7\mathbf{i} + 2\mathbf{j})$ m s^{-1} when it collides with a particle of mass m, which is moving parallel to the unit vector **j**. After the collision the two particles move together with velocity $(2\mathbf{i} + \mathbf{j})$ m s^{-1}.

(a) Find m.

(b) Find the velocity of B before the collision.

Solution:

Let the original velocity of the particle of mass m to be $k\mathbf{j}$.
Using conservation of momentum;

$$2\begin{bmatrix} 7 \\ 2 \end{bmatrix} + m\begin{bmatrix} 0 \\ k \end{bmatrix} = (2 + m)\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 14 \\ 4 + mk \end{bmatrix} = \begin{bmatrix} 4 + 2m \\ 2 + m \end{bmatrix}$$

(a) Using i components; $14 = 4 + 2m$
$\therefore m = 5$

(b) Using j components; $4 + mk = 2 + m$
since $m = 5$; $4 + 5k = 7$
$\therefore k = \frac{3}{5}$

$\therefore$ Velocity of B before the collision was $0.6\mathbf{j}$ m s^{-1}.

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 6

Question:

Two particles, A and B, collide. The table gives the masses and velocities of the particles, where α and β are constants.

Particle	Mass	Velocity before collision	Velocity after collision
A	4 kg	$(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}	$(\mathbf{i} + \alpha\mathbf{j})$ m s^{-1}
B	6 kg	$(-2\mathbf{i} + 6\mathbf{j})$ m s^{-1}	$(\beta\mathbf{i} + 2\mathbf{j})$ m s^{-1}

Find α and β.

Solution:

Using conservation of momentum;

$$4(3\mathbf{i} + 5\mathbf{j}) + 6(-2\mathbf{i} + 6\mathbf{j}) = 4(\mathbf{i} + \alpha\mathbf{j}) + 6(\beta\mathbf{i} + 2\mathbf{j})$$

$$12\mathbf{i} + 20\mathbf{j} - 12\mathbf{i} + 36\mathbf{j} = 4\mathbf{i} + 4\alpha\mathbf{j} + 6\beta\mathbf{i} + 12\mathbf{j}$$

$$56\mathbf{j} = (4 + 6\beta)\mathbf{i} + (4\alpha + 12)\mathbf{j}$$

Using i components; $0 = 4 + 6\beta$

$$\beta = -\frac{2}{3}$$

Using j components; $56 = 4\alpha + 12$

$$44 = 4\alpha$$

$$\therefore \alpha = 11$$

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 7

Question:

When two particles collide they are both brought to rest. One particle has mass 5 kg and had velocity $\begin{bmatrix} 8 \\ -9 \end{bmatrix}$ m s^{-1} before the collision. The other particle has mass 2 kg. Find the velocity of this particle before the collision.

Solution:

By conservation of momentum,

$5\begin{bmatrix} 8 \\ -9 \end{bmatrix} + 2u = 0$ where u is the original velocity of the 2 kg mass.

$\therefore \begin{bmatrix} 40 \\ -45 \end{bmatrix} + 2u = 0$

$u = \begin{bmatrix} -20 \\ 22.5 \end{bmatrix}$

$\therefore$ Velocity was $\begin{bmatrix} -20 \\ 22.5 \end{bmatrix}$ m s^{-1}.

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 8

Question:

Two particles, *A* and *B*, collide. The table gives the masses and velocities of the particles.

Particle	Mass	Velocity before collision	Velocity after collision
A	1 kg	$(8\mathbf{i} + 15\mathbf{j})$ m s^{-1}	$(\mathbf{i} + 6\mathbf{j})$ m s^{-1}
B	4 kg	$(-\mathbf{i} - 2\mathbf{j})$ m s^{-1}	$\mathbf{v}$ m s^{-1}

Find **v**.

Solution:

Using conservation of momentum

$$\begin{aligned} 1(8\mathbf{i} + 15\mathbf{j}) + 4(-\mathbf{i} - 2\mathbf{j}) &= 1(\mathbf{i} + 6\mathbf{j}) + 4\mathbf{v} \\ 4\mathbf{i} + 7\mathbf{j} &= \mathbf{i} + 6\mathbf{j} + 4\mathbf{v} \\ 3\mathbf{i} + \mathbf{j} &= 4\mathbf{v} \\ \therefore \mathbf{v} &= 0.75\mathbf{i} + 0.25\mathbf{j} \text{ m s}^{-1}. \end{aligned}$$

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 9

Question:

A sphere has mass 7 kg and velocity $(5\mathbf{i} - 8\mathbf{j})$ m s^{-1}. It collides with a stationary sphere of mass 5 kg. After the collision the 7 kg sphere has velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. Find the velocity of the other sphere after the collision.

Solution:

Let the velocity of the other sphere after the collision be **v**.
Using conservation of momentum

$$\begin{aligned} 7(5\mathbf{i} - 8\mathbf{j}) &= 7(2\mathbf{i} - 3\mathbf{j}) + 5\mathbf{v} \\ 35\mathbf{i} - 56\mathbf{j} &= 14\mathbf{i} - 21\mathbf{j} + 5\mathbf{v} \\ 21\mathbf{i} - 35\mathbf{j} &= 5\mathbf{v} \\ \therefore \mathbf{v} &= 4.2\mathbf{i} - 7\mathbf{j} \end{aligned}$$

$\therefore$ Velocity was $(4.2\mathbf{i} - 7\mathbf{j})$ m s^{-1}.

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 10

Question:

Two particles, *A* of mass 7 kg and *B* of mass 9 kg, collide and coalesce. Before the collision the velocity of *A* was **v** and the velocity of *B* was 2**v**. After the collision their velocity is $\begin{bmatrix} \frac{3}{2} \end{bmatrix}$ m s^{-1}.

Find **v**.

Solution:

After particles coalesce, combined mass is 16 kg.
Using conservation of momentum

$$7v + 9 \times 2v = 16 \begin{bmatrix} \frac{3}{2} \end{bmatrix}$$

$$25v = \begin{bmatrix} \frac{48}{32} \end{bmatrix}$$

$$v = \begin{bmatrix} \frac{1.92}{1.28} \end{bmatrix} \text{ m s}^{-1}.$$

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 11

Question:

A particle A, of mass 0.3 kg, is moving with velocity $\begin{bmatrix} 7 \\ 4 \end{bmatrix}$ m s^{-1} when it collides with a stationary particle, B, of mass 0.5 kg. Immediately after the collision, B moves with velocity $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$ m s^{-1}.

(a) Find the velocity of A immediately after the collision.

(b) Find the speed of A immediately after the collision.

(c) State which of A and B moves faster after the collision. [A]

Solution:

(a) Let **v** be the velocity of A after the collision.
Using conservation of momentum,

$$0.3 \begin{bmatrix} 7 \\ 4 \end{bmatrix} = 0.3\mathbf{v} + 0.5 \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2.1 \\ 1.2 \end{bmatrix} = 0.3\mathbf{v} + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$0.3\mathbf{v} = \begin{bmatrix} -0.9 \\ 1.2 \end{bmatrix}$$

$$\therefore \mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

Velocity of A immediately after the collision is $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$ m s^{-1}

(b) The speed of A immediately after the collision is
$\sqrt{(-3)^2 + 4^2} = 5$ m s^{-1}

(c) After the collision, the speed of B is 6 m s^{-1}

$\therefore$ B is moving faster.

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Heinemann Modular Maths for Edexcel AS and A-level

8 Momentum

Exercise B, Question 12

Question:

A particle, P, moves with constant velocity.

(a) The particle passes through the points with position vectors $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$ at times $t = 0$ and $t = 3$, respectively. The units of distance and time are metres and seconds, respectively. Show that the velocity of P is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ m s^{-1}.

(b) When $t = 3$, the particle P, of mass 0.2 kg, collides with a particle Q. The particle Q has mass 0.1 kg and, immediately before the collision, it is moving with velocity $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$ m s^{-1}. As a result of the collision, P and Q coalesce into a single particle, R. Find the velocity of R immediately after the collision.

(c) Assuming that the velocity of R remains constant, find the position vector of R three seconds after the collision. [A]

Solution:

(a) The particle moves from $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ to $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$ which is a

displacement of $\begin{bmatrix} 2 \\ 8 \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

This takes 3 seconds;

$\therefore$ The velocity of P is $\frac{1}{3} \times \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ m s^{-1}

(b) Using conservation of momentum with **v** as the velocity of R immediately after the collision,

$0.2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0.1 \begin{bmatrix} -5 \\ 5 \end{bmatrix} = 0.3\mathbf{v}$

$\begin{bmatrix} -0.3 \\ 0.9 \end{bmatrix} = 0.3\mathbf{v}$

$\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\therefore$ Velocity of R immediately after the collision is $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ m s^{-1}

(c) Three seconds after the impact, R will have moved a displacement

$3 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$

It was, at impact, at $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$

$\therefore$ Its position vector, three seconds after impact,

is $\begin{bmatrix} -3 \\ 9 \end{bmatrix} + \begin{bmatrix} 2 \\ 8 \end{bmatrix} = \begin{bmatrix} -1 \\ 17 \end{bmatrix}$