Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

7 Projectiles

Exercise A, Question 1

Question:

A rugby ball is kicked from ground level so that its initial velocity is 18 m s^{-1} and at an angle of 45° above the horizontal. Find:

(a) the time of flight,

(b) the range,

(c) the maximum height of the ball.

Solution:

(a)
$$x = v\cos\theta\ t \quad \Rightarrow \quad x = 18\cos 45°\ t$$
$$y = v\sin\theta\ t - \tfrac{1}{2}gt^2 \quad \Rightarrow \quad y = 18\sin 45°\ t - \tfrac{1}{2}gt^2$$

For time of flight, $y = 0 \quad \Rightarrow \quad 18\sin 45°\ t - \tfrac{1}{2}gt^2 = 0$

$t = 0$ (not required) or $t = \dfrac{18\sin 45°}{\frac{1}{2}g}$

$= 2.60$ s

(b) Range, x, is $18\cos 45° \times 2.60$

$= 33.1$ m

(c) At maximum height, vertical component of velocity is zero

$\therefore v = u + at \quad \Rightarrow \quad 0 = 18\sin 45° - gt$

$t = 1.30$ sec

Maximum height, $y = 18\sin 45° \times 1.30 - \tfrac{1}{2}g(1.30)^2$

$= 8.27$ m.

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Exercise A, Question 2

Question:

David kicks a ball, from ground level, with a speed of 15 m s^{-1} at an angle of 30° to the horizontal. How far away from him does the ball land?

Solution:

$x = 15 \cos 30° t$
$y = 15 \sin 30° t - \frac{1}{2}gt^2$
} equations of motion of projectile

Ball lands when $y = 0 \Rightarrow$

$15 \sin 30° t - \frac{1}{2}gt^2 = 0$

$t = \frac{15 \sin 30°}{\frac{1}{2}g}$ or 0 (not required)

$= 1.5306$ s

$\therefore$ Distance, x, is $15 \cos 30° \times 1.5306$

$= 19.9$ m.

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Exercise A, Question 3

Question:

A bushbaby makes hops with a take-off speed of 6 m s^{-1} and at an angle of 30° to the horizontal. How far does it go in each hop?

Solution:

Equations of motion for a projectile give

$$x = 6 \cos 30° t$$

$$y = 6 \sin 30° t - \frac{1}{2}gt^2$$

Bushbaby lands when $y = 0$

$$\Rightarrow \quad 6 \sin 30° t - \frac{1}{2}gt^2 = 0$$

$$t = \frac{6 \sin 30°}{\frac{1}{2}g} \text{ or } 0 \text{ (not required)}$$

$$= 0.6122 \text{ s}$$

$$\therefore \text{Distance, } x = 6 \cos 30° \times 0.6122$$

$$= 3.18 \text{ m}.$$

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Exercise A, Question 4

Question:

A ball is thrown with initial speed 20 m s^{-1} at an angle of 60° to the horizontal. Assume that the ball is initially at ground level. How high does it rise? How far has it then travelled horizontally?

Solution:

Equations of motion for a projectile give

$$x = 20 \cos 60° t$$

$$y = 20 \sin 60° t - \frac{1}{2}gt^2$$

At maximum height, vertical speed is zero,

$v = u + at$ (vertically) gives

$0 = 20 \sin 60° - gt$

$\therefore t = \frac{20 \sin 60°}{g}$

$= 1.767$ s

Vertical distance, $y = 20 \sin 60° \times 1.767 - \frac{1}{2}g \times 1.767^2$

$= 15.3$ m

Horizontal distance, $x = 20 \cos 60° \times 1.767$

$= 17.7$ m.

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Exercise A, Question 5

Question:

A stunt motorcyclist takes off at a speed of 35 m s^{-1} up a ramp of 30° to the horizontal to clear a river 50 m wide. Does the cyclist succeed in doing this? Does the motorcyclist have to worry about air resistance?

Solution:

Equations of motion for a particle give

$x = 35 \cos 30° t$

$y = 35 \sin 30° t - \frac{1}{2}gt^2$

$y = 0 \Rightarrow t = 0$ (not required) or $\frac{35 \sin 30°}{\frac{1}{2}g} = 3.571$ s

The horizontal distance, $x = 35 \cos 30° \times 3.571$

$= 108$ m

Yes; the model predicts 108 m.
Air resistance is unlikely to cause a problem as the range is so much greater than 50 m.

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7 Projectiles

Exercise A, Question 6

Question:

A ball is thrown with a speed of 12 m s^{-1} at an angle of 30° to the horizontal. It is initially at ground level.

(a) Find the time of flight, the maximum height to which the ball rises and the range.

(b) Find also the speed and direction of flight of the ball after 0.5 s and 1.0 s.

Solution:

(a) The equations of motion of a projectile give

$$x = 12 \cos 30° \, t$$

$$y = 12 \sin 30° \, t - \frac{1}{2} g t^2$$

For the time of flight, $y = 0 \Rightarrow t = \dfrac{12 \sin 30°}{\frac{1}{2} g}$ or 0 (not required)

$$\therefore t = 1.22 \text{ s}$$

To find the maximum height,

use $v^2 = u^2 + 2as$ vertically

$$0 = (12 \sin 30°)^2 - 2gs$$

$$s = \frac{(12 \sin 30°)^2}{2g}$$

$$= 1.84 \text{ m}$$

The range, x at the time of flight, is $12 \cos 30° \times 1.22 = 12.7$ m

(b) After 0.5 sec, the horizontal speed is $12 \cos 30° = 10.392$ m s^{-1}

Using $v = u + at$, the vertical speed is $12 \sin 30° - g \times \frac{1}{2} = 1.1$ m s^{-1}

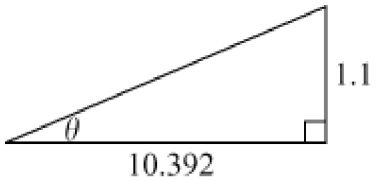
Speed is $\sqrt{10.392^2 + 1.1^2} = 10.45005$

Speed is 10.5 m s^{-1}

$$\tan \theta = \frac{1.1}{10.392}$$

$$\theta = 6.04$$

i.e. speed is 10.5 m s^{-1} at an angle 6.04° above horizontal.



After 1 sec, horizontal speed is $12 \cos 30° = 10.392 \text{ m s}^{-1}$

vertical speed ($v = u + at$) is $12 \sin 30° - g \times 1 = -3.8 \text{ m s}^{-1}$

$\therefore$ Speed is $\sqrt{10.392^2 + 3.8^2}$

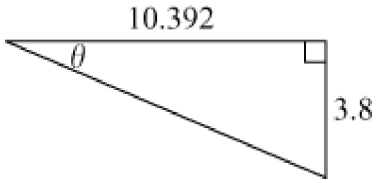
$= 11.06$

$\therefore$ Speed is 11.1 m s^{-1}

$\tan \theta = \frac{3.8}{10.392}$

$\Rightarrow \quad \theta = 20.08°$

$\therefore$ Speed is 11.1 m s^{-1} at 20.1° below the horizontal.



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Heinemann Modular Maths for Edexcel AS and A-level

7 Projectiles

Exercise A, Question 7

Question:

A golf ball is struck at a point O on the ground and moves with an initial velocity of $20\ \text{m}\ \text{s}^{-1}$ at an angle of 53° to the horizontal. The ball subsequently lands at a point X which is on the same horizontal level as Q.

(a) Show that the time taken by the ball to reach the point X is approximately 3.26 seconds.

(b) Calculate the distance OX.

(c) State:
(i) the least speed of the ball during its flight from O to X,
(ii) the direction of motion of the ball when this least speed occurs. [A]

Solution:

(a) The equations of motion of a projectile give

$$x = 20\cos 53° t$$

$$y = 20\sin 53° t - \frac{1}{2}gt^2$$

Ball reaches X when $y = 0$,

$$\therefore t = 0 \text{ (not required) or } \frac{20\sin 53°}{\frac{1}{2}g} = 3.26\ \text{s}$$

(b) OX, $x = 20\cos 53° \times 3.26$
$= 39.2\ \text{m}$

(c) (i) The horizontal speed does not change.
$\therefore$ The least speed of the ball is when the vertical velocity is zero,
i.e. the least speed is $20\cos 53° = 12.0\ \text{m}\ \text{s}^{-1}$.
(ii) The ball is moving horizontally.

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7 Projectiles

Exercise A, Question 8

Question:

A golfer hits a ball, from ground level on a horizontal surface. The initial velocity of the ball is 21 m s^{-1} at an angle of 60° above the horizontal. Assume that the ball is a particle and that no resistance forces act on the ball.

(a) Find the maximum height of the ball.

(b) Find the range of the ball.

(c) Find the speed of the ball at its maximum height. [A]

Solution:

(a) The equations of motion of a projectile give

$$x = 21 \cos 60° t$$

$$y = 21 \sin 60° t - \frac{1}{2}gt^2$$

At the maximum height, $v = u + at$ vertically, gives

$$0 = 21 \sin 60° - gt$$

$$t = \frac{21 \sin 60°}{g}$$

$$= 1.855 \text{ s}$$

$$\text{Maximum height, } y = 21 \sin 60° \times 1.855 - \frac{1}{2}g(1.855)^2$$

$$= 16.9 \text{ m}$$

(b) For the time of flight,

$$y = 0 \Rightarrow 21 \sin 60° t - \frac{1}{2}gt^2 = 0$$

$$t = 0 \text{ (not required) or } t = \frac{21 \sin 60°}{\frac{1}{2}g}$$

$$= 3.7115 \text{ s}$$

$$\text{Range of the ball, } x = 21 \cos 60° \times 3.7115$$

$$= 39.0 \text{ m}$$

(c) At the maximum height, the vertical speed of the ball is zero.
$\therefore$ The speed of the ball is its horizontal speed which is $21 \cos 60°$ or 10.5 m s^{-1}.

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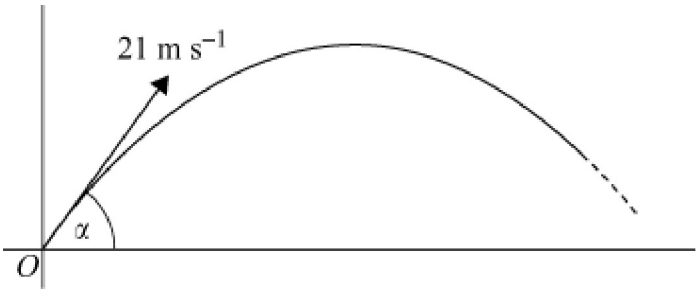
7 Projectiles

Exercise A, Question 9

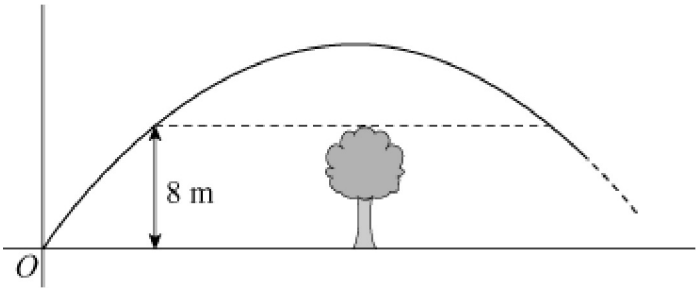
Question:

Paul throws a ball from a point O with velocity 21 m s^{-1} at an angle of α to the horizontal, where $\sin \alpha = 0.7$. The ball subsequently moves freely under gravity in a vertical plane, as shown in the diagram.

(a) Show that the time taken for the ball to reach its greatest height above O is 1.5 seconds.



(b) When the ball reaches its greatest height, it passes over a tree of vertical height 8 metres, as shown in the diagram.
(i) Find the vertical distance between the ball and the top of the tree at this time.
(ii) Find the time between the ball leaving O and first reaching the horizontal level of the top of the tree. Give your answer to two decimal places.
(iii) Find the length of time for which the ball is above the horizontal level of the top of the tree. [A]



Solution:

(a) The equations of motion of a projectile give

$$x = 21 \cos \alpha \; t$$

$$y = 21 \sin \alpha \; t - \tfrac{1}{2} g t^2$$

For the greatest height, $v = u + at$ vertically gives

$$0 = 21 \sin \alpha - gt$$

$$t = \frac{21 \sin \alpha}{g}$$

$$= 1.5 \text{ s} \quad (\text{with } \sin \alpha = 0.7)$$

When $t = 1.5$, height of ball, y is $21 \sin \alpha \times 1.5 - \frac{1}{2}g(1.5)^2$

(b) (i)

$$= 21 \times 0.7 \times 1.5 - \frac{1}{2}g(1.5)^2$$

$$= 11.025 \text{ m}$$

$\therefore$ Vertical distance between the ball and the top of the tree is $11.025 - 8 = 3.025$ m

(ii) When vertical height, $y = 8$

$$8 = 21 \sin \alpha \; t - \frac{1}{2}gt^2$$

$$4.9t^2 - 14.7t + 8 = 0$$

$$t = \frac{14.7 \pm \sqrt{14.7^2 - 4 \times 4.9 \times 8}}{2 \times 4.9}$$

$$= 0.714 \text{ s and } 2.2857 \text{ s}$$

$\therefore$ Time to first reach the level of top of the tree is 0.714 s

(iii) The length of time the ball is above the top of the tree is $2.2857 - 0.714 = 1.57$ s .

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7 Projectiles

Exercise A, Question 10

Question:

A particle P is projected at time $t = 0$ in a vertical plane from a point O with speed u at an angle α above the horizontal. Write down expressions for the horizontal and vertical components of:

(a) the velocity of P at time t,

(b) the displacement, at time t, of P from O.

(c) Given that the particle strikes the horizontal plane through O at time T show that $T = \frac{2u \sin \alpha}{g}$.

Find, in terms of g and T, the maximum height that P rises above the horizontal plane through O. [A]

Solution:

(a)
$$v_x = u \cos \alpha$$
$$v_y = u \sin \alpha - gt$$

(b)
$$x = u \cos \alpha \, t$$
$$y = u \sin \alpha \, t - \frac{1}{2} g t^2$$

(c) The particle strikes the horizontal plane through O when

$$y = 0 \quad \Rightarrow \quad 0 = u \sin \alpha \, t - \frac{1}{2} g t^2$$

$$\therefore t = 0 \text{ (not required) or } t = \frac{u \sin \alpha}{\frac{1}{2} g}$$

$$\therefore T = \frac{2u \sin \alpha}{g}$$

For the maximum height,

$$v = u + at \text{ vertically} \quad \Rightarrow \quad 0 = u \sin \alpha - gt$$

$$t = \frac{u \sin \alpha}{g}$$

$$\text{The vertical height, } y = u \sin \alpha \cdot \frac{u \sin \alpha}{g} - \frac{1}{2} g \left(\frac{u \sin \alpha}{g} \right)^2$$

$$= \frac{u^2 \sin^2 \alpha}{2g}$$