

Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

7 Projectiles

Exercise B, Question 14

Question:

Take $g = 10 \text{ m s}^{-2}$ in this question.

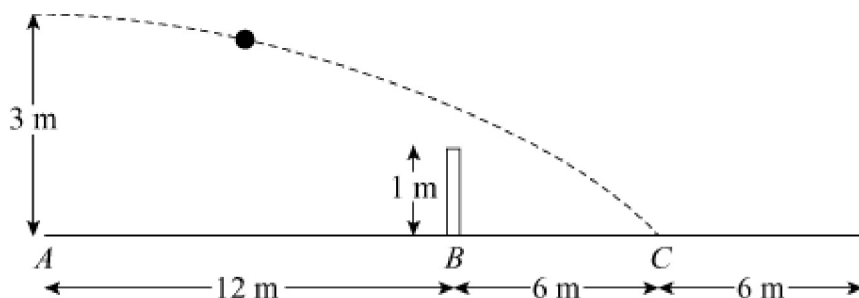
When a tennis ball is served, it is hit from the baseline A . It must pass over the net, at B , and land **between** the net and service line on the other side, at C . The diagram shows the positions of these lines and the height of the net.

A tennis ball is served so that it initially moves horizontally at a speed of $v \text{ m s}^{-1}$ from a point 3 m above the baseline A . Assume that there are no resistance forces and that the ball moves in a vertical plane at right angles to the net.

(a) (i) Show that v must be greater than $\sqrt{360} \text{ m s}^{-1}$ if the ball is to go over the net.

(ii) What is the maximum value of v if the ball is to land in the area between B and C ?

(b) A tennis player hits a tennis ball so that it initially moves at a speed of 30 m s^{-1} and at an angle of 5° below the horizontal. Is this serve successful? [A]



Solution:

(a) The position of the tennis ball, relative to the point it is hit, is given by

$$x = vt \quad [1]$$

$$y = -\frac{1}{2}gt^2 \quad [2]$$

(i) If the ball passes over the net, $y > -2$ when $x = 12$

$$[1] x = 12 \Rightarrow 12 = vt \text{ or } t = \frac{12}{v}$$

$$[2] y = -\frac{1}{2}g \left(\frac{12}{v} \right)^2 > -2$$

$$72g < 2v^2$$

$$\text{or } v^2 > 360$$

$$\therefore v > \sqrt{360} \text{ m s}^{-1} \text{ if the ball is to go over the net.}$$

(ii) If the ball lands between B and C , $y < -3$ when $x = 18$

$$[1] x = 18 \Rightarrow 18 = vt \text{ or } t = \frac{18}{v}$$

$$[2] y = -\frac{1}{2}g \left(\frac{18}{v} \right)^2 < -3$$

$$\begin{aligned} 1620 &> 3v^2 \\ v &< \sqrt{540} \end{aligned}$$

The maximum value of v is $\sqrt{540} \text{ m s}^{-1}$

(b) The new position of the ball, relative to the point it is hit, is given by

$$x = 30 \cos 5^\circ t \quad [1]$$

$$y = -30 \sin 5^\circ t - \frac{1}{2}gt^2 \quad [2]$$

To see whether it passes over the net;

$$\begin{aligned} \text{When } x = 12, \quad [1] \Rightarrow t &= \frac{12}{30 \cos 5^\circ} \\ &= 0.4015 \text{ s} \end{aligned}$$

From [2],

$$\begin{aligned} y &= -30 \sin 5^\circ \times 0.4015 - \frac{1}{2}g \times (0.4015)^2 \\ &= -1.8398 \end{aligned}$$

\therefore The ball passes over the net.

To see whether it hits the ground before C .

$$\begin{aligned} \text{When } x = 18, \quad [1] \Rightarrow t &= \frac{18}{30 \cos 5^\circ} \\ &= 0.60229 \end{aligned}$$

$$\begin{aligned} \text{From [2], } y &= -30 \sin 5^\circ \times 0.60229 - \frac{1}{2}g \times (0.60229)^2 \\ &= -3.352 \end{aligned}$$

Hence the ball hits the ground before C .

\therefore The serve is successful.