

Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

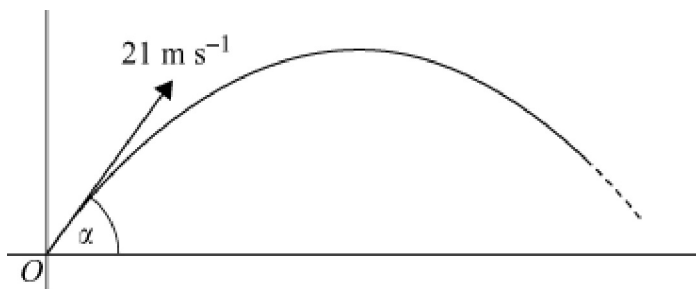
7 Projectiles

Exercise A, Question 9

Question:

Paul throws a ball from a point O with velocity 21 m s^{-1} at an angle of α to the horizontal, where $\sin \alpha = 0.7$. The ball subsequently moves freely under gravity in a vertical plane, as shown in the diagram.

(a) Show that the time taken for the ball to reach its greatest height above O is 1.5 seconds.

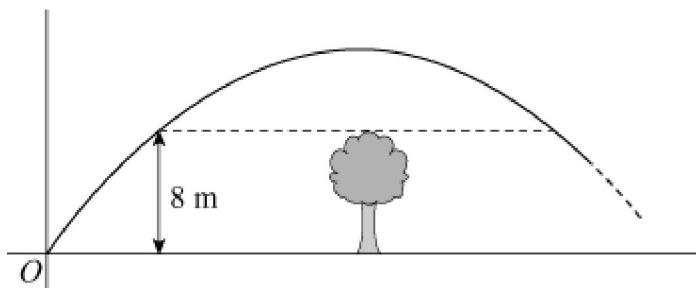


(b) When the ball reaches its greatest height, it passes over a tree of vertical height 8 metres, as shown in the diagram.

(i) Find the vertical distance between the ball and the top of the tree at this time.

(ii) Find the time between the ball leaving O and first reaching the horizontal level of the top of the tree. Give your answer to two decimal places.

(iii) Find the length of time for which the ball is above the horizontal level of the top of the tree. [A]



Solution:

(a) The equations of motion of a projectile give

$$x = 21 \cos \alpha t$$

$$y = 21 \sin \alpha t - \frac{1}{2}gt^2$$

For the greatest height, $v = u + at$ vertically gives

$$0 = 21 \sin \alpha - gt$$

$$t = \frac{21 \sin \alpha}{g}$$

$$= 1.5 \text{ s (with } \sin \alpha = 0.7 \text{)}$$

When $t = 1.5$, height of ball, y is $21 \sin \alpha \times 1.5 - \frac{1}{2}g (1.5)^2$

$$\begin{aligned} \text{(b) (i)} \quad &= 21 \times 0.7 \times 1.5 - \frac{1}{2}g (1.5)^2 \\ &= 11.025 \text{ m} \end{aligned}$$

\therefore Vertical distance between the ball and the top of the tree is $11.025 - 8 = 3.025 \text{ m}$

(ii) When vertical height, $y = 8$

$$\begin{aligned} 8 &= 21 \sin \alpha t - \frac{1}{2}gt^2 \\ 4.9t^2 - 14.7t + 8 &= 0 \\ t &= \frac{14.7 \pm \sqrt{14.7^2 - 4 \times 4.9 \times 8}}{2 \times 4.9} \\ &= 0.714 \text{ s and } 2.2857 \text{ s} \end{aligned}$$

\therefore Time to first reach the level of top of the tree is 0.714 s

(iii) The length of time the ball is above the top of the tree is $2.2857 - 0.714 = 1.57 \text{ s}$.