Solutionbank M1

Heinemann Modular Maths for Edexcel AS and A-level

5 Newton's laws of motion

Exercise B, Question 19

Question:

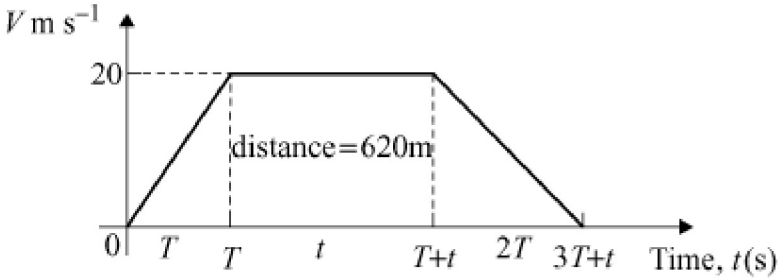
A car, of mass 900 kg, is initially at rest. On a short journey the car
I. accelerates uniformly for T seconds to a speed of 20 m s^{-1},
II. then travels at this speed for a period of time,
III. then decelerates uniformly for $2T$ seconds before coming to rest.

(a) In one journey the car moves for a total of 40 seconds and travels a total of 620 m. Using this information:
(i) Sketch a velocity-time graph and hence, or otherwise, find T.
(ii) Calculate the magnitude of the resultant force on the car during each stage of the journey.
(iii) Sketch a graph to show how the resultant force acting on the car varies with time.
(iv) Find the speed of the car after it has travelled 20 m.

(b) In the case when $T = 5$, find the time that it would take the car to complete a 1000 m journey. [A]

Solution:

(a) (i)



$$\left[\ \tfrac{1}{2} \times T \times 20\ \right] + \left[\ t \times 20\ \right] + \left[\ \tfrac{1}{2} \times 2T \times 20\ \right] = 620$$

and $3T + t = 40$

$t = 40 - 3T$

$\therefore\ 10T + (\ 40 - 3T\ ) \times 20 + 20T = 620$

$\therefore\ 10T + 800 - 60T + 20T = 620$

$\therefore\ 800 - 620 = 60T - 20T - 10T$

$180 = 30T$

$\therefore\ T = 6$ seconds.

1st stage, acceleration $= \frac{20}{6}$

$= 3\frac{1}{3}$ m s^{-2}

(ii) 2nd stage, acceleration $= 0$

3rd stage, acceleration $= \frac{-20}{12}$

$= -1\frac{2}{3}$ m s^{-2} (i.e. a retardation)

$\therefore$ by Newton's 2nd Law

1st stage, resultant force $= 900 \times 3\frac{1}{3}$

$= 3000$ N in direction of motion

2nd stage, resultant force $= 900 \times 0$

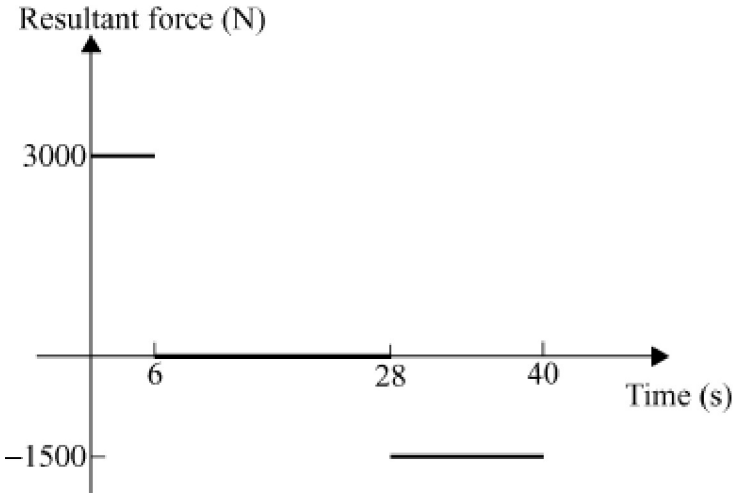
$= 0$ N

3rd stage, resultant force $= 900 \times \left(-1\frac{2}{3}\right)$

$= -1500$ N

$\therefore$ in 3rd stage resultant force has magnitude 1500 N

(iii)



(iv) In 1st stage it travels $\frac{1}{2} \times 6 \times 20 = 60$ m

$\therefore$ 20 metres happens in the first stage.

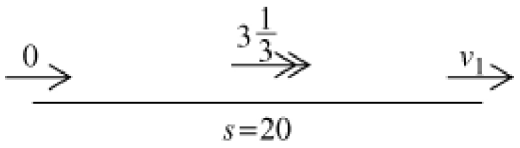
$v^2 = u^2 + 2as$

$v_1^2 = 0^2 + 2\left(3\tfrac{1}{3}\right) \times 20$

$v_1^2 = 133\tfrac{1}{3}$

$\therefore v_1 = 11.547$

$v_1 = 11.5 \text{ m s}^{-1}$ (3 s.f.)



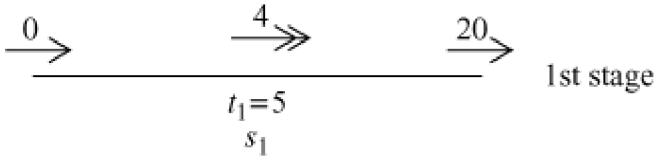
(b) When $T = 5$, $a = \frac{20}{5} = 4 \text{ m s}^{-2}$

1st stage

$s = ut + \frac{1}{2}at^2$

$s_1 = 0 \times 5 + \frac{1}{2} \times 4 \times 5^2$

$s_1 = 50$



1st stage

3rd stage

$s = ut + \frac{1}{2}at^2$

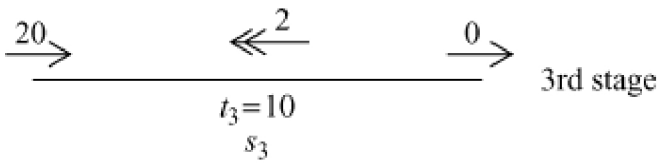
$s_3 = 20 \times 10 + \frac{1}{2} \times (-2) \times 10^2$

$s_3 = 100$

$s_1 + s_2 + s_3 = 1000$

$\therefore s_2 = 1000 - 50 - 100$

$s_2 = 850$ metres



3rd stage

2nd stage

$s = ut \therefore 850 = 20 \times t_2$

$t_2 = 42.5$ seconds

$\therefore$ total time $= 5 + 42.5 + 10$

$= 57.5$ seconds