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Heinemann Modular Maths for Edexcel AS and A-level

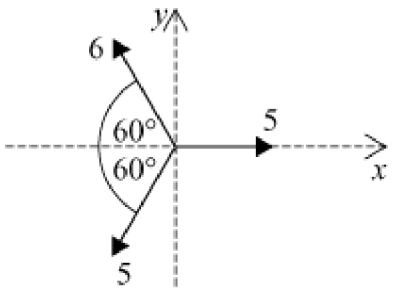
4 Forces

Exercise C, Question 1

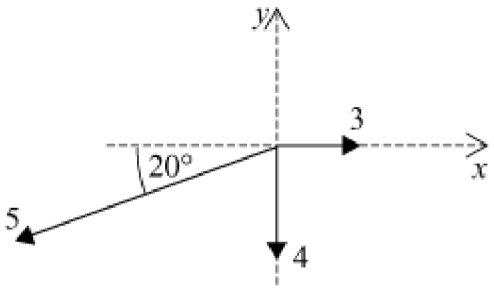
Question:

Find the magnitude of the resultant of the following sets of forces, by forming a quadrilateral of forces.

(a)

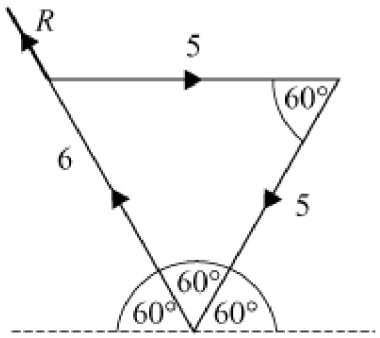


(b)

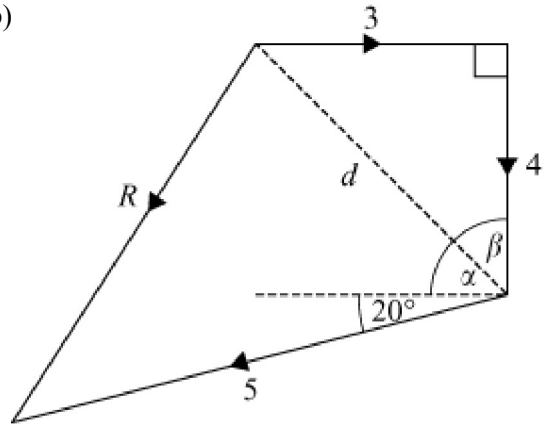


Solution:

(a) The sketch shows that the magnitude of R is 1N (since the basic triangle is equilateral).



(b)



By Pythagoras' theorem

$d = \sqrt{3^2 + 4^2}$

$\therefore d = \sqrt{25}$

i.e. $d = 5$

then $\tan \beta = \frac{3}{4}$ gives $\beta = 36.869...°$

$\therefore \alpha = 90° - \beta$

$= 90° - 36.869°$

i.e. $\alpha = 53.130...°$

$\therefore$ in the lower triangle, the angle between d and 5 is $20° + \alpha = 20 + 53.130° = 73.130...°$ then the cosine rule gives

$R^2 = d^2 + 5^2 - 2 \times d \times 5 \times \cos 73.130°$, but $d = 5$

$\therefore R^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 73.130°$

i.e. $R^2 = 35.490...$

$\therefore R = 5.9573...$

i.e. $R = 5.96$ N (3 s.f.)

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4 Forces

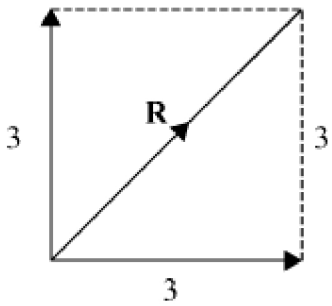
Exercise C, Question 2

Question:

Find the magnitude of the resultant of the forces $(2i + j)$ N, $(3i - 2j)$ N and $(-2i + 4j)$ N.

Solution:

$$\begin{aligned} R &= (2i + j) + (3i - 2j) + (-2i + 4j) \\ R &= 3i + 3j \\ \therefore\ |R| &= \sqrt{3^2 + 3^2} \\ |R| &= \sqrt{18} \\ |R| &= 4.2426... \\ \text{i.e. } |R| &= 4.24 \text{ N (3 s.f.)} \end{aligned}$$



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4 Forces

Exercise C, Question 3

Question:

The resultant of the forces $(2\mathbf{i} + \mathbf{j})$ N , $3\mathbf{j}$ N, $(2\mathbf{i} + 4\mathbf{j})$ N , $(6\mathbf{i} + b\mathbf{j})$ N and $(a\mathbf{i} + \mathbf{j})$ N is $(3\mathbf{i} + 4\mathbf{j})$ N , where **i** and **j** are perpendicular unit vectors. Find a and b.

Solution:

$(2\mathbf{i} + \mathbf{j}) + (3\mathbf{j}) + (2\mathbf{i} + 4\mathbf{j}) + (6\mathbf{i} + b\mathbf{j}) + (a\mathbf{i} + \mathbf{j})$	$= 3\mathbf{i} + 4\mathbf{j}$
$\therefore (a + 10)\mathbf{i} + (b + 9)\mathbf{j}$	$= 3\mathbf{i} + 4\mathbf{j}$
comparing i components $a + 10$	$= 3$
$\therefore a$	$= -7$
comparing j components $b + 9$	$= 4$
b	$= -5$

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Heinemann Modular Maths for Edexcel AS and A-level

4 Forces

Exercise C, Question 4

Question:

Three forces (3i + 5j) N , (4i + 11j) N , (2i + j) N act at a point. Given that **i** and **j** are perpendicular unit vectors find:

(a) the resultant of the forces in the form ai + bj,

(b) the magnitude of this resultant,

(c) the angle that the resultant makes with the unit vector **i**. [A]

Solution:

(a) $\mathrm{R} = (3\mathrm{i} + 5\mathrm{j}) + (4\mathrm{i} + 11\mathrm{j}) + (2\mathrm{i} + \mathrm{j})$

$\therefore \mathrm{R} = 9\mathrm{i} + 17\mathrm{j}$ N

(b) $|\mathrm{R}| = \sqrt{9^2 + 17^2}$

$|\mathrm{R}| = \sqrt{370}$

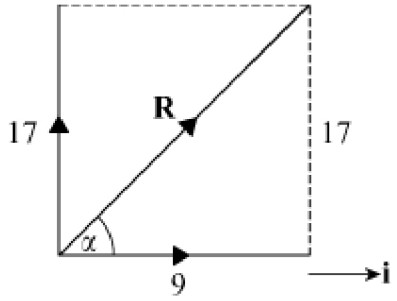
$|\mathrm{R}| = 19.235...$

i.e. $|\mathrm{R}| = 19.2$ N (3 s.f.)

(c) $\tan \alpha = \frac{17}{9}$

i.e. $\alpha = 62.102...°$

i.e. at 62.1° with the **i** direction (3 s.f.)



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4 Forces

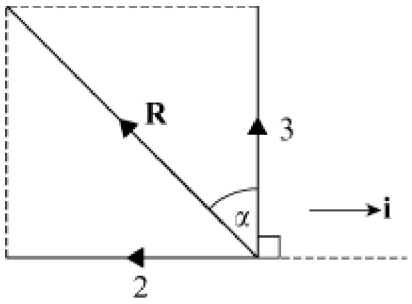
Exercise C, Question 5

Question:

Two forces (3i + 2j) N and (− 5i + j) N act at a point. Find the magnitude of the resultant of these forces and determine the angle which the resultant makes with the unit vector **i**. [A]

Solution:

R	= (3i + 2j) + (− 5i + j)
R	= − 2i + 3j
| R |	$= \sqrt{(-2)^2 + 3^2}$
	$= \sqrt{13}$
	= 3.61 N (3 s.f.)
then tan α	$= \frac{2}{3}$
gives α	= 33.690... °
∴ angle with i direction	= 90 + α
	= 90 + 33.690... °
	= 123.690... °
	= 124 ° (3 s.f.)



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4 Forces

Exercise C, Question 6

Question:

Two forces $\begin{bmatrix} 7 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -6 \\ 3 \end{bmatrix}$ act at a point. Calculate the magnitude of the resultant of these two forces.

Solution:

$$\mathbf{R} = \begin{bmatrix} 7 \\ 1 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix}$$

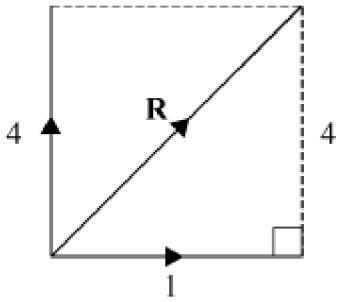
$$\text{i.e. } \mathbf{R} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\therefore |\mathbf{R}| = \sqrt{1^2 + 4^2}$$

$$|\mathbf{R}| = \sqrt{17}$$

$$|\mathbf{R}| = 4.1231...$$

$$\text{i.e. } |\mathbf{R}| = 4.12 \text{ N (3 s.f.)}$$



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4 Forces

Exercise C, Question 7

Question:

The forces $\mathbf{F}_1 = \begin{bmatrix} a \\ -2a \end{bmatrix}$ and $\mathbf{F}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ act on a particle. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ has magnitude 4 N. Find the possible values of a.

Solution:

$$\mathbf{R} = \begin{bmatrix} a \\ -2a \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\text{i.e. } \mathbf{R} = \begin{bmatrix} a+2 \\ 4-2a \end{bmatrix} \quad \text{but } |\mathbf{R}| = 4$$

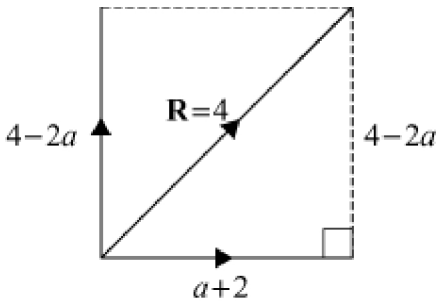
$$\therefore (a+2)^2 + (4-2a)^2 = 4^2$$

$$a^2 + 4a + 4 + 16 - 16a + 4a^2 = 16$$

$$\therefore 5a^2 - 12a + 4 = 0$$

$$\therefore (5a-2)(a-2) = 0$$

$$a = \frac{2}{5} \text{ or } a = 2$$



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Heinemann Modular Maths for Edexcel AS and A-level

4 Forces

Exercise C, Question 8

Question:

Three forces $(i + j)$ N, $(-5i + 3j)$ N and $\lambda \mathbf{i}$ N, where **i** and **j** are perpendicular unit vectors, act at a point. Express the resultant in the form $(ai + bj)$ and find its magnitude in terms of λ. Given that the resultant has magnitude 5 N, find the two possible values of λ.
Take the larger value of λ and find the tangent of the angle between the resultant and the unit vector **i**. [A]

Solution:

$$R = (i + j) + (-5i + 3j) + \lambda i$$

$$\text{i.e. } R = [\lambda - 4] i + 4j$$

$$\text{and } | R | = \sqrt{(\lambda - 4)^2 + 4^2}$$

$$\text{i.e. } | R | = \sqrt{(\lambda^2 - 8\lambda + 16) + 16}$$

$$\text{i.e. } | R | = \sqrt{(\lambda^2 - 8\lambda + 32)}$$

$$\therefore \text{ if } | R | = 5, \text{ then } \sqrt{\lambda^2 - 8\lambda + 32} = 5$$

$$\lambda^2 - 8\lambda + 32 = 5^2$$

$$\lambda^2 - 8\lambda + 7 = 0$$

$$(\lambda - 7) (\lambda - 1) = 0$$

$$\text{i.e. } \lambda = 7 \text{ or } \lambda = 1$$

$$\therefore \text{ when } \lambda = 7 \quad R = (7 - 4) i + 4j$$

$$\text{i.e. } R = 3i + 4j$$

$$\text{i.e. } \tan \alpha = \frac{4}{3}$$