

## Paper 2 Option E

### Further Statistics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs																	
<b>1(a)</b>	$H_0$ : There is no association between language and gender	B1	1.2																	
		(1)																		
<b>(b)</b>	$\frac{54 \times 85}{150} = 30.6$ *	B1*cs0	1.1b																	
		(1)																		
<b>(c)</b>	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2" rowspan="2">Expected frequencies</th> <th colspan="3">Language</th> </tr> <tr> <th>French</th> <th>Spanish</th> <th>Mandarin</th> </tr> </thead> <tbody> <tr> <th rowspan="2">Gender</th> <th>Male</th> <td>26.43...</td> <td>23.4</td> <td>15.16...</td> </tr> <tr> <th>Female</th> <td>34.56...</td> <td>[30.6]</td> <td>19.83...</td> </tr> </tbody> </table>	Expected frequencies		Language			French	Spanish	Mandarin	Gender	Male	26.43...	23.4	15.16...	Female	34.56...	[30.6]	19.83...	M1	2.1
	Expected frequencies			Language																
			French	Spanish	Mandarin															
	Gender	Male	26.43...	23.4	15.16...															
Female		34.56...	[30.6]	19.83...																
$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \dots + \frac{(15-19.83)^2}{19.83}$	M1	1.1b																		
Awrt <b>3.6/3.7</b>	A1	1.1b																		
		(3)																		
<b>(d)</b>	Degrees of freedom $(3-1)(2-1) \rightarrow$ Critical value $\chi_{2,0.01}^2 = 9.210$	M1	3.1b																	
	As $\sum \frac{(O-E)^2}{E} < 9.210$ , the null hypothesis is not rejected	A1	2.2b																	
		(2)																		
<b>(e)</b>	Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi_{2,0.1}^2 = 4.605$	B1	2.4																	
		(1)																		
<b>(8 marks)</b>																				
Notes:																				
<b>(a)</b>																				
<b>B1:</b> For correct hypothesis in context																				
<b>(b)</b>																				
<b>B1*:</b> For a correct calculation leading to the given answer and no errors seen																				
<b>(c)</b>																				
<b>M1:</b> For attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$ to find expected frequencies																				
<b>M1:</b> For applying $\sum \frac{(O-E)^2}{E}$																				
<b>A1:</b> awrt 3.6 or 3.7																				
<b>(d)</b>																				
<b>M1:</b> For using degrees of freedom to set up a $\chi^2$ model critical value																				
<b>A1:</b> For correct comparison and conclusion																				
<b>(e)</b>																				
<b>A1ft:</b> For correct conclusion with supporting reason																				

Question	Scheme	Marks	AOs
<b>2(a)</b>	$-4 = 2 - 5E(X)$	M1	3.1a
	$E(X) = 1.2$		
	$-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ <span style="float:right">[1]</span>		
	$P(Y \geq -3) = 0.45$ gives $P(2 - 5X \geq -3) = 0.45$ i.e. $P(X \leq 1) = 0.45$	M1	2.1
	$2a + c = 0.45$ <span style="float:right">[2]</span>		
	$2a + b + 2c = 1$ <span style="float:right">[3]</span>	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $[3] - [2] \Rightarrow b + c = 0.55$ sub. $2(b + c)$ into $[1] \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
	(7)		
<b>(b)</b>	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> $59$	M1	1.1a
	$[\text{Var}(Y) = 5^2 \text{Var}(X) \text{ implies}] \text{Var}(X) = 2.36$	A1	1.2
		(2)	
<b>(c)</b>	$P(Y > X) = P(2 - 5X > X) \rightarrow P(X < \frac{1}{3})$	M1	3.1a
	$P(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
<b>(11 marks)</b>			
Notes:			
<p><b>(a)</b></p> <p><b>M1:</b> For using given information to find an expression for <math>E(X)</math> i.e. use of <math>E(Y) = 2 - 5E(X)</math></p> <p><b>M1:</b> For use of <math>\sum xP(X = x) = '1.2'</math></p> <p><b>M1:</b> For use of <math>P(Y \geq -3) = 0.45</math> to set up the argument for solving by forming an equation in <math>a</math> and <math>c</math></p> <p><b>M1:</b> For use of <math>\sum P(X = x) = 1</math></p> <p><b>M1:</b> For solving their 3 linear equations (matrix or elimination)</p> <p><b>A1:</b> For any 2 of <math>a, b</math> or <math>c</math> correct</p> <p><b>A1:</b> For all 3 correct values</p>			

Question 2 notes continued:

**Another method for part (a) is:**

**M1:** For using given information to find the probability distribution for  $Y$  leading to an expression for  $E(Y)$

**M1:** For use of  $\sum yP(Y = y) = -4$

**M1:** For use of  $P(Y \geq -3) = 0.45$  to set up the argument for solving by forming an equation in  $a$  and  $c$

**M1:** For use of  $\sum P(Y = y) = 1$

**M1:** For solving their 3 linear equations (matrix or elimination)

**A1:** For any 2 of  $a$ ,  $b$  or  $c$  correct

**A1:** For all 3 correct values

**(b)**

**M1:** For use of  $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$  (may be implied by a correct answer)

**A1:** For use of  $\text{Var}(aX) = a^2 \text{Var}(X)$  to reach 2.36 or exact equivalent

**(c)**

**M1:** For rearranging to the form  $P(X < k)$

**A1ft:** '0.1' + '025' (provided their  $a$  and  $c$  and their  $a + c$  are all probabilities)

**Another method for part (c) is:**

**M1:** For comparing distribution of  $X$  with distribution of  $Y$  to identify  $X = -1$  and  $X = 0$

**A1ft:** '0.1' + '025' (provided their  $a$  and  $c$  and their  $a + c$  are all probabilities)

Question	Scheme	Marks	AOs
<b>3(a)</b>	$X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= P(X=2) \times P(Y=2) = 0.25104\dots \times 0.21685\dots$	M1	3.3
	$= 0.05444\dots$ awrt <b><u>0.0544</u></b>	A1	1.1b
		(2)	
<b>(b)</b>	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$P(W = 3) = 0.20458\dots$ awrt <b><u>0.205</u></b>	A1	1.1b
		(2)	
<b>(c)</b>	$T \sim \text{Po}((2.6+1.2) \times 2)$	M1	3.3
	$P(T < 9) = 0.64819\dots$ awrt <b><u>0.648</u></b>	A1	1.1b
		(2)	
<b>(d)</b>	<b>(i)</b> Mean = $np = \underline{2.4}$	B1	1.1b
	<b>(ii)</b> Variance = $np(1 - p) = 2.3904$ awrt <b><u>2.39</u></b>	B1	1.1b
		(2)	
<b>(e)</b>	<b>(i)</b> [ $D \sim \text{Po}(2.4) \quad P(D \leq 4)$ ] $= 0.9041\dots$ awrt <b><u>0.904</u></b>	B1	1.1b
	<b>(ii)</b> Since $n$ is large and $p$ is small/mean is approximately equal to variance	B1	2.4
		(2)	
<b>(10 marks)</b>			
Notes:			
<b>(a)</b> <b>M1:</b> For $P(X=2) \times P(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer) <b>A1:</b> awrt <b>0.0544</b>			
<b>(b)</b> <b>M1:</b> For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) <b>A1:</b> awrt <b>0.205</b>			
<b>(c)</b> <b>M1:</b> For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) <b>A1:</b> awrt <b>0.648</b>			
<b>(d)(i)</b> <b>B1:</b> For <b>2.4</b>			
<b>(d)(ii)</b> <b>B1:</b> For awrt <b>2.39</b>			
<b>(e)(i)</b> <b>B1:</b> For awrt <b>0.904</b>			
<b>(e)(ii)</b> <b>B1:</b> For a correct explanation to support use of Poisson approximation in this case			

Question	Scheme	Marks	AOs
<b>4(a)</b>	(i) $P(X = 1) = 0.34523\dots$ awrt <b>0.345</b>	B1	1.1b
	(ii) $P(X \leq 4) = 0.98575\dots$ awrt <b>0.986</b>	B1	1.1b
		<b>(2)</b>	
<b>(b)</b>	$\frac{(0 \times 10) + 1 \times 16 + 2 \times 7 + 3 \times 4 + 4 \times 2 + (5 \times 0) + 6 \times 1}{40} = 1.4^*$	B1*cs0	1.1b
		<b>(1)</b>	
<b>(c)</b>	$r = 40 \times '0.34523\dots'$ $s = 40 \times '1 - 0.986\dots'$	M1	3.4
	$r = \underline{\mathbf{13.81}}$ $s = \underline{\mathbf{0.57}}$	A1ft	1.1b
		<b>(2)</b>	
<b>(d)</b>	$H_0$ : The Poisson distribution is a suitable model $H_1$ : The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(10 - 9.86)^2}{9.86} + \dots + \frac{(7 - (4.51 + 1.58 + 0.57))^2}{(4.51 + 1.58 + 0.57)}$	M1	1.1b
	awrt <b>1.1</b>	A1	1.1b
	Degrees of freedom = 4 - 1 - 1 = 2	B1	3.1b
	(Do not reject $H_0$ since $1.10 < \chi_{2,(0.05)}^2 = 5.991$ ). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		<b>(6)</b>	
<b>(11 marks)</b>			
Notes:			
<b>(a)(i)</b> <b>B1:</b> awrt 0.345			
<b>(a)(ii)</b> <b>B1:</b> awrt 0.986			
<b>(b)</b> <b>B1*:</b> For a fully correct calculation leading to given answer with no errors seen			
<b>(c)</b> <b>M1:</b> For attempt at $r$ or $s$ (may be implied by correct answers) <b>A1ft:</b> For both values correct (follow through their answers to part (a))			
<b>(d)</b> <b>B1:</b> For both hypotheses correct (lambda should not be defined so correct use of the model) <b>M1:</b> For understanding the need to combine cells before calculating the test statistic (may be implied) <b>M1:</b> For attempt to find the test statistic using $\chi^2 = \sum \frac{(O - E)^2}{E}$ <b>A1:</b> awrt 1.1 <b>B1:</b> For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^2(0.05) = 5.991$ <b>A1:</b> Concluding that a Poisson model is suitable for the number of mortgages approved each week			

Further Mechanics 1 Mark Scheme (Section **B**)

Question	Scheme	Marks	AOs
<b>5(a)</b>	Using the model and $v^2 = u^2 + 2as$ to find $v$	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{4.8g}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find $u$	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{1.2g}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{1.2g} / \sqrt{4.8g} = 0.5$ *	A1*	1.1b
	<b>(6)</b>		
<b>(b)</b>	Using the model and $e = \text{sep. speed} / \text{app. speed}$ , $v = 0.5\sqrt{1.2g}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		<b>(3)</b>	
<b>(c)</b>	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		<b>(1)</b>	
<b>(10 marks)</b>			
Notes:			
<p><b>(a)</b>  <b>M1:</b> For a complete method to find <math>v</math>  <b>A1:</b> For a correct value (may be numerical)  <b>M1:</b> For a complete method to find <math>u</math>  <b>A1:</b> For a correct value (may be numerical)  <b>M1:</b> For finding both <math>v</math> and <math>u</math> and use of Newton's Law of Restitution  <b>A1*:</b> For the given answer</p>			
<p><b>(b)</b>  <b>M1:</b> For use of Newton's Law of Restitution to find rebound speed  <b>M1:</b> For a complete method to find <math>h</math>  <b>A1:</b> For 0.15 (m) oe</p>			
<p><b>(c)</b>  <b>B1:</b> For a clear description including reference to a quarter</p>			

Question	Scheme	Marks	AOs
<b>6(a)</b>	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5 g \times 20$	A1	1.1b
	$= 58.25 = 58 \text{ (J) or } 58.3 \text{ (J)}$	A1	1.1b
		<b>(3)</b>	
<b>(b)</b>	Using work-energy principle, $20 R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9 \text{ or } 2.91$	A1ft	1.1b
		<b>(2)</b>	
<b>(c)</b>	Make resistance variable (dependent on speed)	B1	3.5c
		<b>(1)</b>	
<b>(6 marks)</b>			
Notes:			
<b>(a)</b> <b>M1:</b> For a difference in KE and PE <b>A1:</b> For a correct expression <b>A1:</b> For either 58 (2sf) or 58.3(3sf)			
<b>(b)</b> <b>M1:</b> For use of work-energy principle <b>A1ft:</b> For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)			
<b>(c)</b> <b>B1:</b> For variable resistance oe			

Question	Scheme	Marks	AOs
<b>7(a)</b>	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force $\times$ Speed = 30 $\times$ 4	M1	1.1b
	= 120 W	A1 ft	1.1b
		(3)	
<b>(b)</b>	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force $\times$ Speed = 70 $\times$ 3	M1	1.1b
	= 210 W	A1 ft	1.1b
		(5)	
<b>(8 marks)</b>			
Notes:			
<p><b>(a)</b>  <b>B1:</b> For force = 30 seen  <b>M1:</b> For use of <math>P = Fv</math>  <b>A1ft:</b> For 120 (W), follow through on their '30'</p>			
<p><b>(b)</b>  <b>M1:</b> For resolving parallel to the slope with correct no. of terms and 60g resolved  <b>A1:</b> For a correct equation  <b>A1:</b> For <math>F = 70</math>  <b>M1:</b> For use of <math>P = Fv</math>  <b>A1ft:</b> For 210 (W), follow through on their '70'</p>			



Question	Scheme	Marks	AOs
<b>8(a)</b>	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		<b>(6)</b>	
<b>(b)</b>	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		<b>(2)</b>	
<b>(c)</b>	Solving for $w$	M1	2.1
	$w = \frac{u}{4}(1 + 9e)^*$	A1 *	1.1b
		<b>(2)</b>	
<b>(d)</b>	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f \frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		<b>(6)</b>	

**(16 marks)**

Notes:

**(a)**

**M1:** For use of CLM, with correct no. of terms, condone sign errors

**A1:** For a correct equation

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**A1:** For a correct equation

**M1:** For setting up *two* equations and solving their equations for  $v$

**A1:** For a correct expression for  $v$

**(b)**

**M1:** For use of an appropriate inequality

**A1:** For a complete range of values of  $e$

**(c)**

**M1:** For solving their equations for  $w$

**A1:** For the given answer

Question **8** notes continued:

**(d)**

**M1:** For substituting  $e = \frac{5}{9}$  into their  $v$  and  $w$

**A1:** For correct expressions for  $v$  and  $w$

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**M1:** For use of appropriate inequality

**A1:** For a correct inequality

**A1:** For a correct range