Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2K: Decision Mathematics 1 and Decision Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2K

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If a pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.
- Do not return this question paper with the answer book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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SECTION A

Answer ALL questions. Write your answers in the answer book provided.

1.

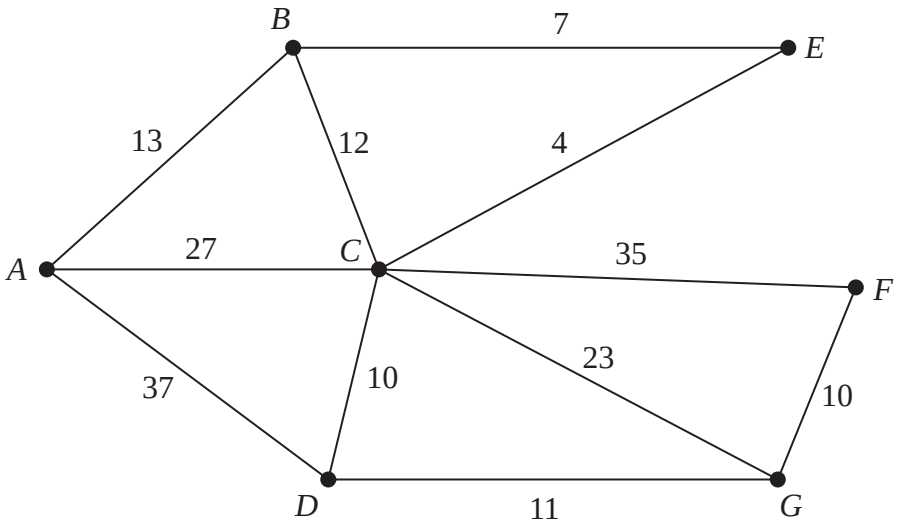

Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from *A* to *F*. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at *A*, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, *BG*, is added to the network. A route of minimum length that traverses each pipe, including *BG*, needs to be found. The route must start and finish at *A*.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of *BG*.

(d) Calculate the length of *BG*. You must show your working.

(2)

(Total for Question 1 is 12 marks)

2.

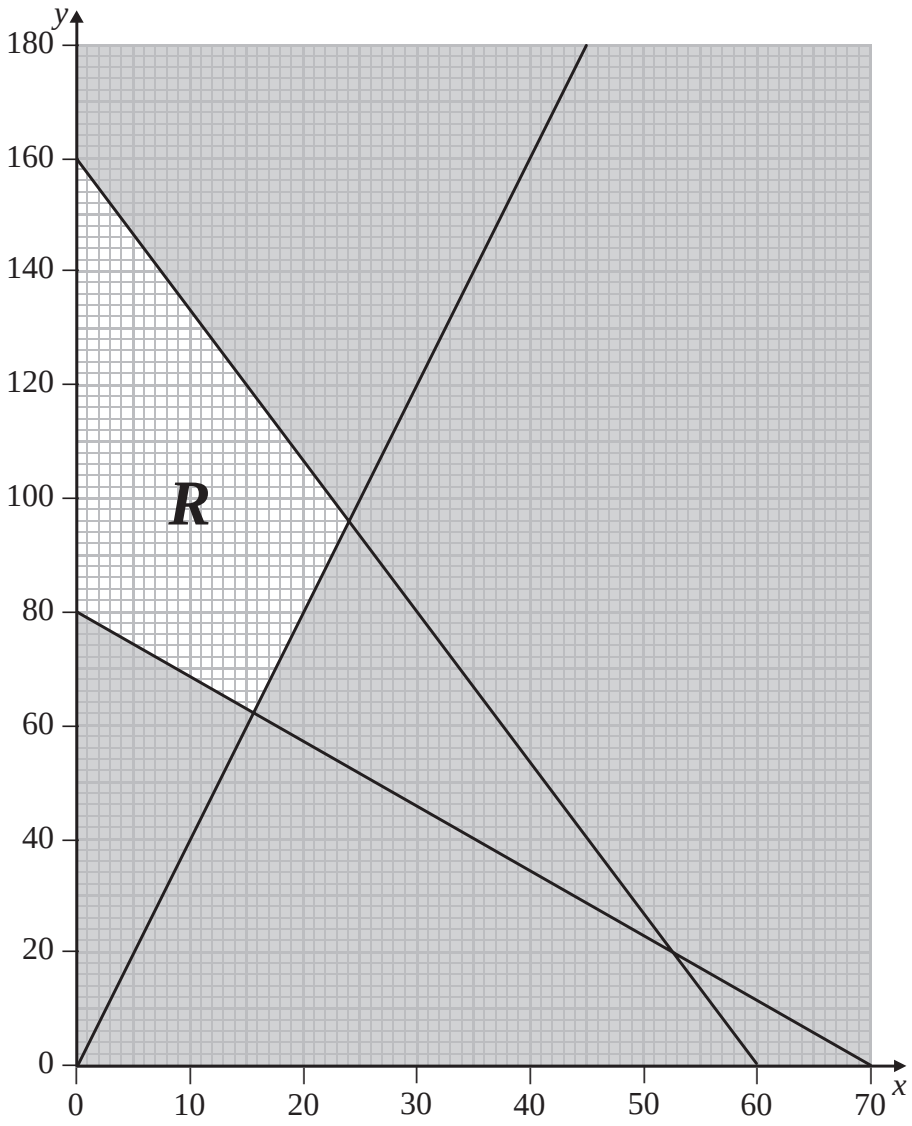

Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$
$$8x + 7y \geqslant 560$$
$$y \geqslant 4x$$
$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 2 is 7 marks)

3.

Activity	Time taken (days)	Immediately preceding activities
A	5	–
B	7	–
C	3	–
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

(ii) Identify the critical activities.

(4)

(Total for Question 3 is 7 marks)

4. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 4 is 9 marks)

5. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

6. Six workers, A, B, C, D, E and F, are to be assigned to five tasks, P, Q, R, S and T.

Each worker can be assigned to at most one task and each task must be done by just one worker.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

(Total for Question 6 is 9 marks)

7. In two-dimensional space, lines divide a plane into a number of different regions.

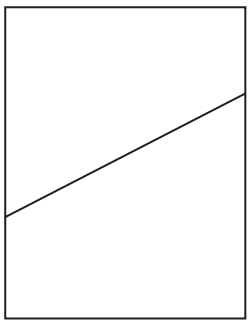

Figure 1

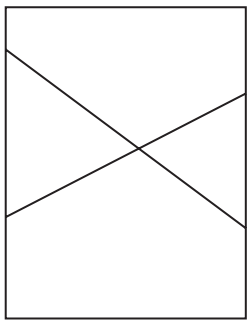

Figure 2

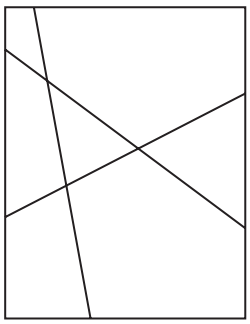

Figure 3

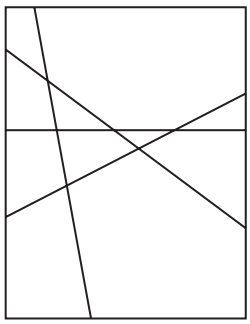

Figure 4

It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1
- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2
- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3
- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

(b) Find, in terms of u_n, the recurrence relation for u_{n+1}, the maximum number of regions when a plane is divided by $(n + 1)$ lines where $n \geqslant 1$

(1)

(c) (i) Solve the recurrence relation for u_n

(ii) Hence determine the maximum number of regions created when 200 lines divide a plane.

(3)

(Total for Question 7 is 5 marks)

8.

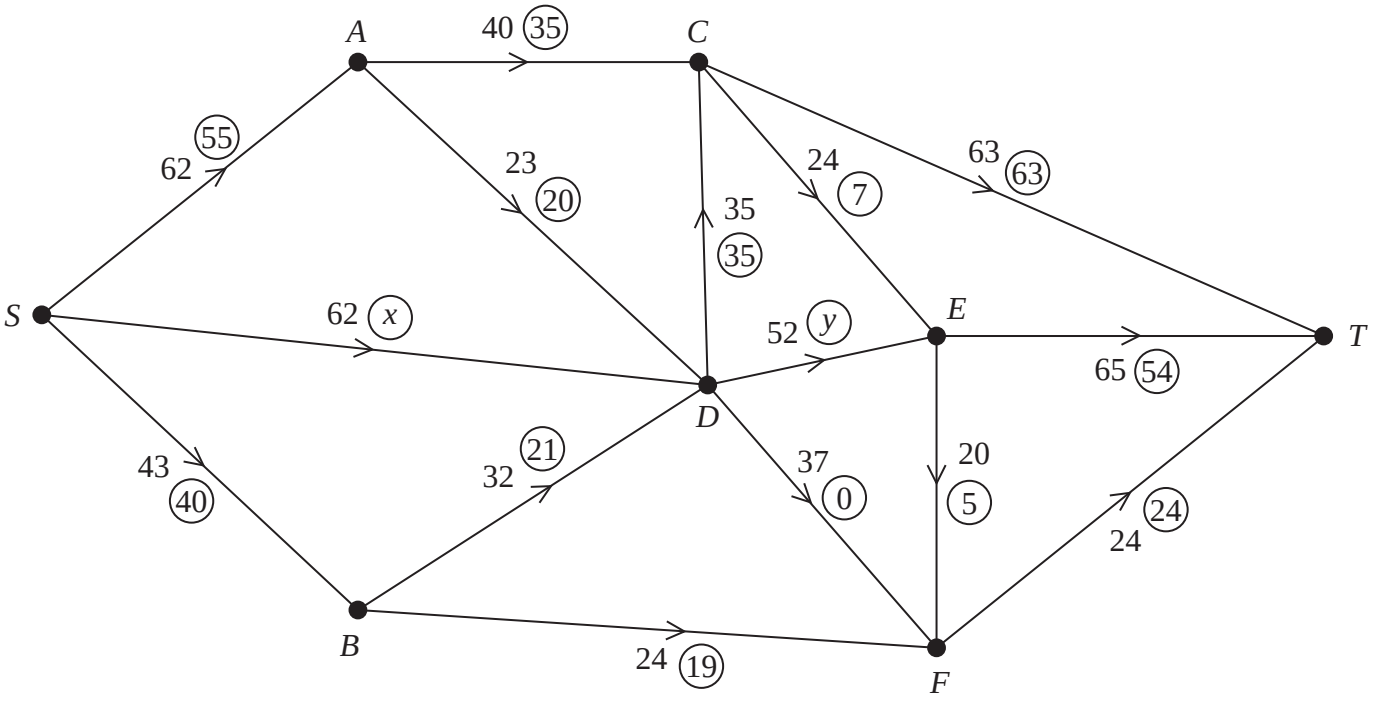

Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time. At 11 am on Friday morning, all students leave the hall (S) after assembly and travel to the cybercafé (T). The numbers in circles represent the initial flow of students recorded at 11 am one Friday.

(a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network.

(1)

(b) Find the value of x and the value of y, explaining your reasoning.

(3)

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafé at 11 am. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

(c) (i) List all the flow augmenting routes from S to T that increase the flow by at least 5

(ii) State which route the new students should take, giving a reason for your answer.

(3)

(d) Use the answer to part (c) to find a maximum flow pattern for this network and draw it on Diagram 1 in the answer book.

(1)

(e) Prove that the answer to part (d) is optimal.

(3)

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

(f) State, explaining your reasoning,

(i) which corridor they should widen,

(ii) the resulting increase of flow through the network.

(3)

(Total for Question 8 is 14 marks)

9. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	***B* plays 1**	***B* plays 2**	***B* plays 3**
***A* plays 1**	4	1	2
***A* plays 2**	2	4	3

(a) Verify that there is no stable solution.

(3)

(b) (i) Find the best strategy for player *A*.

(ii) Find the value of the game to her.

(9)

(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | | | | | |

Candidate Number | | | | |

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Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

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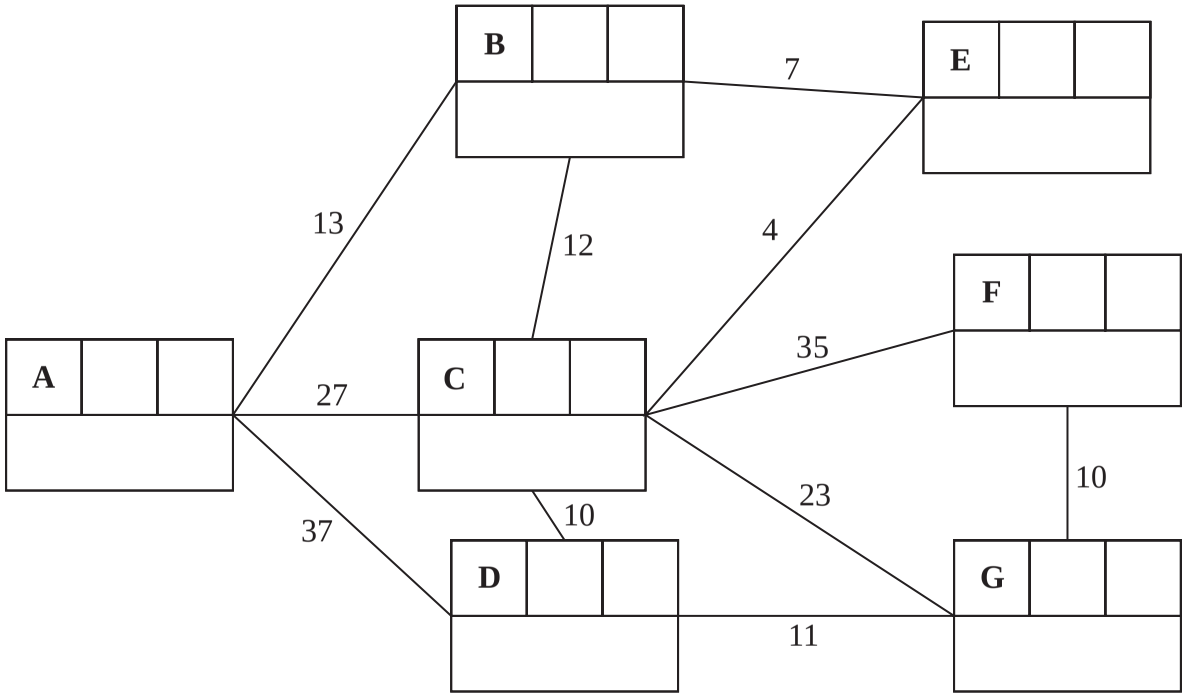
SECTION A

1.



Key:

Vertex	Order of labelling	Final values
Working value		

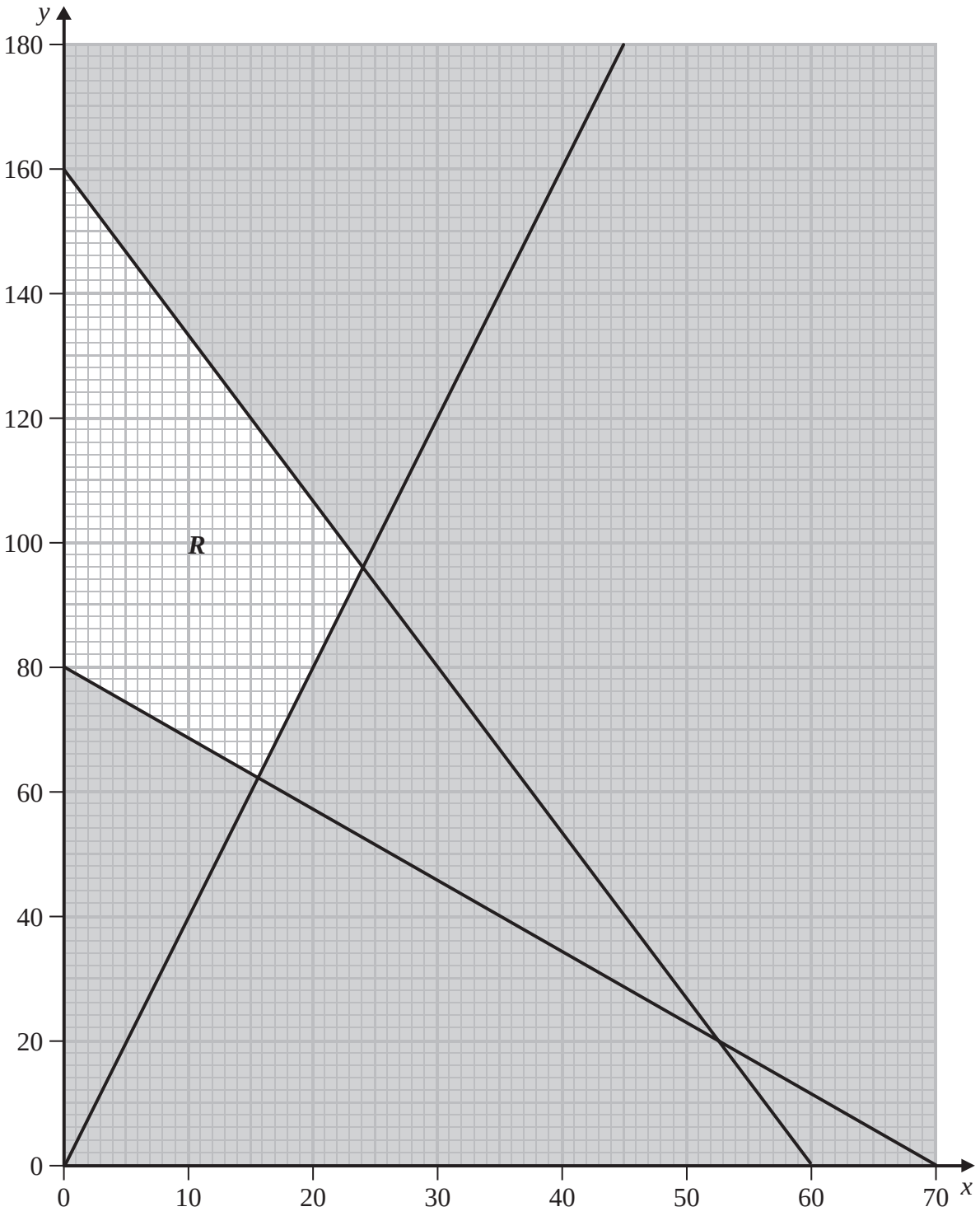
Shortest path: _____

Length of shortest path: _____

Question 1 continued

(Total for Question 1 is 12 marks)

2.



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Question 2 continued

(Total for Question 2 is 7 marks)

3. (a) and (b)

(Total for Question 3 is 7 marks)

4.

(Total for Question 4 is 9 marks)

5.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

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SECTION B

6.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

	P	Q	R	S	T	
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(Total for Question 6 is 9 marks)

7. (a)

Number of lines	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Maximum number of regions	2	4	7	11			

(Total for Question 7 is 5 marks)

8.

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Question 8 continued

(d)

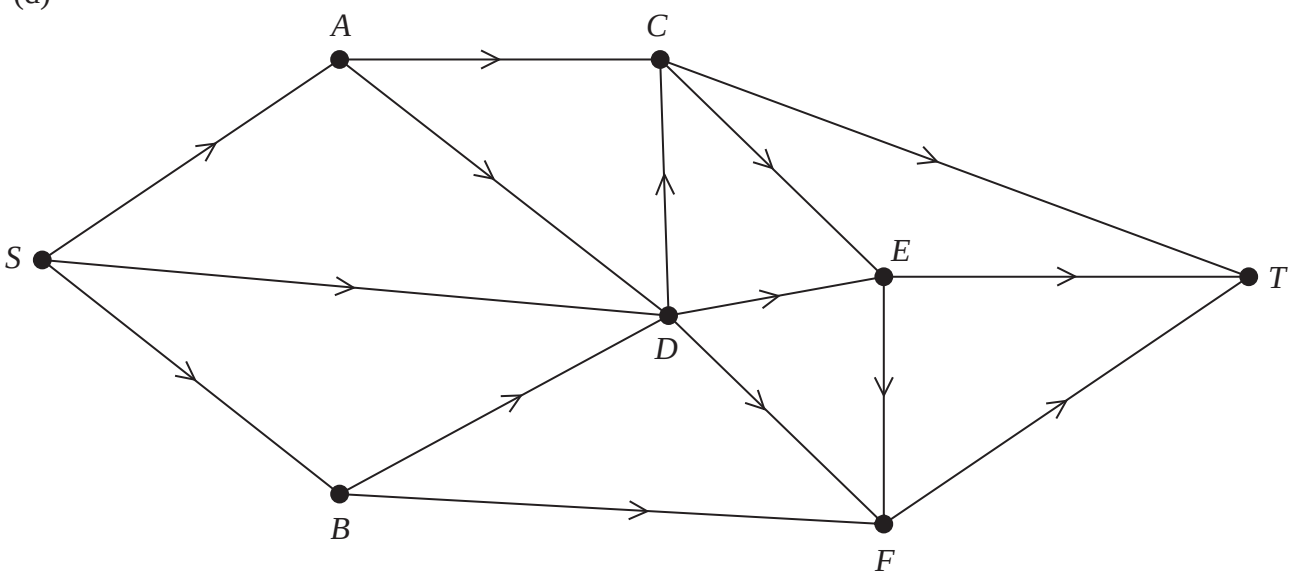

Diagram 1

(Total for Question 8 is 14 marks)

9.

	***B* plays 1**	***B* plays 2**	***B* plays 3**
***A* plays 1**	4	1	2
***A* plays 2**	2	4	3

Question 9 continued

(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS