

Write your name here

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Other names

**Pearson Edexcel**  
**Level 3 GCE**

Centre Number

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Candidate Number

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# Further Mathematics

**Advanced**

**Further Mathematics Option 1**

**Paper 3: Further Pure Mathematics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 30 minutes**

Paper Reference

**9FM0/3A**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

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**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1. Use Simpson's Rule with 6 intervals to estimate

$$\int_1^4 \sqrt{1+x^3} \, dx \qquad (5)$$

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**Question 1 continued**

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**(Total for Question 1 is 5 marks)**

2. Given  $k$  is a constant and that

$$y = x^3 e^{kx}$$

use Leibnitz theorem to show that

$$\frac{d^n y}{dx^n} = k^{n-3} e^{kx} (k^3 x^3 + 3nk^2 x^2 + 3n(n-1)kx + n(n-1)(n-2)) \quad (4)$$



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- 3. A vibrating spring, fixed at one end, has an external force acting on it such that the centre of the spring moves in a straight line. At time  $t$  seconds,  $t \geq 0$ , the displacement of the centre  $C$  of the spring from a fixed point  $O$  is  $x$  micrometres.

The displacement of  $C$  from  $O$  is modelled by the differential equation

$$t^2 \frac{d^2x}{dt^2} - 2t \frac{dx}{dt} + (2 + t^2)x = t^4 \tag{I}$$

- (a) Show that the transformation  $x = tv$  transforms equation (I) into the equation

$$\frac{d^2v}{dt^2} + v = t \tag{II}$$

(5)

- (b) Hence find the general equation for the displacement of  $C$  from  $O$  at time  $t$  seconds.

(7)

- (c) (i) State what happens to the displacement of  $C$  from  $O$  as  $t$  becomes large.

- (ii) Comment on the model with reference to this long term behaviour.

(2)

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**Question 3 continued**

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**(Total for Question 3 is 14 marks)**

4.

$$\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + y = 0 \quad (I)$$

(a) Show that

$$\frac{d^5y}{dx^5} = ax \frac{d^4y}{dx^4} + b \frac{d^3y}{dx^3}$$

where  $a$  and  $b$  are integers to be found.

(4)

(b) Hence find a series solution, in ascending powers of  $x$ , as far as the term in  $x^5$ ,of the differential equation (I) where  $y = 0$  and  $\frac{dy}{dx} = 1$  at  $x = 0$ 

(5)

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5. The normal to the parabola  $y^2 = 4ax$  at the point  $P(ap^2, 2ap)$  passes through the parabola again at the point  $Q(aq^2, 2aq)$ .

The line  $OP$  is perpendicular to the line  $OQ$ , where  $O$  is the origin.

Prove that  $p^2 = 2$

(9)

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6. A tetrahedron has vertices  $A(1, 2, 1)$ ,  $B(0, 1, 0)$ ,  $C(2, 1, 3)$  and  $D(10, 5, 5)$ .

Find

- (a) a Cartesian equation of the plane  $ABC$ . (3)

- (b) the volume of the tetrahedron  $ABCD$ . (3)

The plane  $II$  has equation  $2x - 3y + 3 = 0$

The point  $E$  lies on the line  $AC$  and the point  $F$  lies on the line  $AD$ .

Given that  $II$  contains the point  $B$ , the point  $E$  and the point  $F$ ,

- (c) find the value of  $k$  such that  $\vec{AE} = k\vec{AC}$ . (3)

Given that  $\vec{AF} = \frac{1}{9}\vec{AD}$

- (d) show that the volume of the tetrahedron  $ABCD$  is 45 times the volume of the tetrahedron  $ABEF$ . (2)

**Question 6 continued**

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**(Total for Question 6 is 11 marks)**

7.  $P$  and  $Q$  are two distinct points on the ellipse described by the equation  $x^2 + 4y^2 = 4$

The line  $l$  passes through the point  $P$  and the point  $Q$ .

The tangent to the ellipse at  $P$  and the tangent to the ellipse at  $Q$  intersect at the point  $(r, s)$ .

Show that an equation of the line  $l$  is

$$4sy + rx = 4 \tag{8}$$

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8.

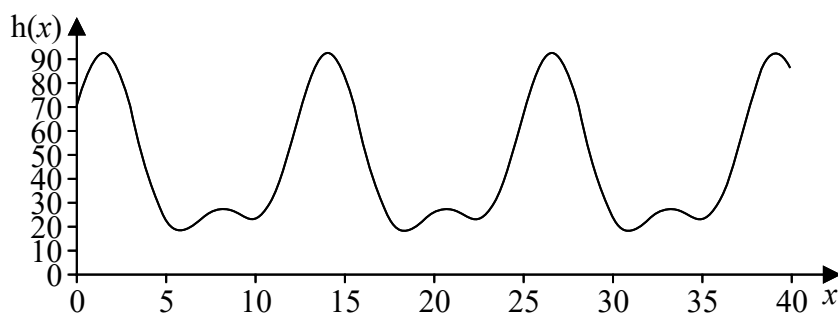


Figure 1

Figure 1 shows the graph of the function  $h(x)$  with equation

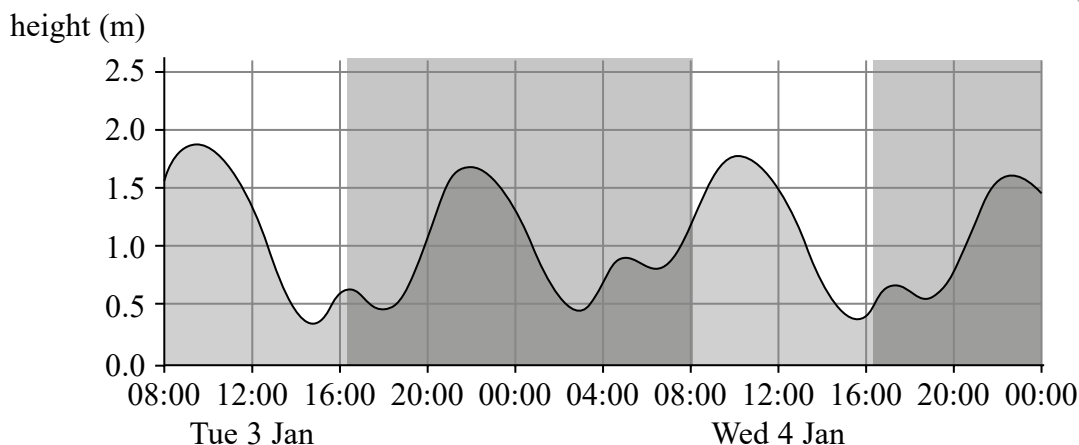
$$h(x) = 45 + 15 \sin x + 21 \sin\left(\frac{x}{2}\right) + 25 \cos\left(\frac{x}{2}\right) \quad x \in [0, 40]$$

(a) Show that

$$\frac{dh}{dx} = \frac{(t^2 - 6t - 17)(9t^2 + 4t - 3)}{2(1 + t^2)^2}$$

where  $t = \tan\left(\frac{x}{4}\right)$ .

(6)



Source: <sup>1</sup>Data taken on 29th December 2016 from <http://www.ukho.gov.uk/easytide/EasyTide>

Figure 2

Figure 2 shows a graph of predicted tide heights, in metres, for Portland harbour from 08:00 on the 3rd January 2017 to the end of the 4th January 2017<sup>1</sup>.

The graph of  $kh(x)$ , where  $k$  is a constant and  $x$  is the number of hours after 08:00 on 3rd of January, can be used to model the predicted tide heights, in metres, for this period of time.

(b) (i) Suggest a value of  $k$  that could be used for the graph of  $kh(x)$  to form a suitable model.

(ii) Why may such a model be suitable to predict the times when the tide heights are at their peaks, but not to predict the heights of these peaks?

(3)

(c) Use Figure 2 and the result of part (a) to estimate, to the nearest minute, the time of the highest tide height on the 4th January 2017.

(6)



**Question 8 continued**

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