

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
<b>1a</b>	The sample mean...	<b>B1</b>	1.2	4th
	...is (approximately) normally distributed with mean $\mu$ and variance $\frac{\sigma^2}{n}$	<b>B1</b>	1.2	Be able to quote the central limit theorem
		<b>(2)</b>		
<b>1b</b>	The sample must be random	<b>B1</b>	1.2	5th Know the conditions for the use of the central limit theorem
		<b>(1)</b>		
				<b>(3 marks)</b>
<b>Notes</b>				
<b>1a</b>	<b>B1</b> must state <i>sample</i> mean (accept $\bar{X}$ )			
	<b>B1</b> parameters must be given. Accept standard deviation rather than variance			
<b>1b</b>	<b>B1</b> for sample is random			

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
2a	$\bar{X} \sim N\left(12, \frac{9}{10}\right)$	M1	3.1a	7th Apply the central limit theorem to a range of probability distributions
	$P(\bar{X} > 13) = P\left(Z > \frac{13-12}{\frac{3}{\sqrt{10}}}\right)$	M1	1.1b	
	= 0.1469	A1	1.1b	
		(3)		
2b	No: The population is normally distributed so the sample mean is also normally distributed.	B1	2.4	5th Know the conditions for the use of the central limit theorem
		(1)		
<b>(4 marks)</b>				
<b>Notes</b>				
2a	M1 for attempt to find distribution of $\bar{X}$ using given parameters, must divide by 10			
	M1 for attempt to find $P(\bar{X} > 13)$			
	A1 awrt 0.147			
2b	B1 for 'No' with correct reason			

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
3a	$\bar{X} \sim N\left(21, \frac{1.1^2}{50}\right)$	M1	3.1a	7th Apply the central limit theorem to a range of probability distributions
	$P(\bar{X} < 20.7) = P\left(Z < \frac{20.7 - 21}{\frac{1.1}{\sqrt{50}}}\right)$	M1	1.1b	
	= 0.0268	A1	1.1b	
		(3)		
3b	Require $P\left(Z < \frac{20.7 - 21}{\frac{1.1}{\sqrt{n}}}\right) < 0.01$	M1	3.1a	8th Recognise and apply the central limit theorem in contextualised situations
	$Z = (-)2.3263$	B1	1.1a	
	$\frac{-0.3}{\frac{1.1}{\sqrt{n}}} < -2.3263 \Rightarrow \frac{0.3\sqrt{n}}{1.1} > 2.3263$	M1	1.1b	
	$\sqrt{n} > 8.5297... \Rightarrow n > 72.7...$	M1	1.1b	
	$n = 73$	A1	3.2a	
		(5)		
				<b>(8 marks)</b>
<b>Notes</b>				
<p><b>3a</b> M1 for attempt to find distribution of <math>\bar{X}</math> using given parameters, must divide by 50  M1 for attempt to find <math>P(\bar{X} &lt; 20.7)</math>  A1 awrt 0.027</p> <p><b>3b</b> M1 for probability statement in terms of <math>Z</math> and using <math>\frac{1.1}{\sqrt{n}}</math>  B1 for correct <math>Z</math> value (from tables); accept + or – here  M1 for attempt to solve resulting equation using <i>their</i> <math>Z</math> ( must now be –)  M1 for value for <math>\sqrt{n}</math> leading to value for <math>n</math>  A1ft <i>their</i> <math>Z</math> value if all M marks awarded (answer must be an integer)</p>				

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
4a	$0.1 + k + 2k + 0.3 + 0.24 = 1$	M1	1.1b	1st
	Hence $k = 0.12$	A1	1.1b	Understand how to find unknowns from a probability mass function
		(2)		
4bi	$E(X) = 1 \times 0.1 + 2 \times 0.12 + 4 \times 0.24 + 5 \times 0.3 + 6 \times 0.24$	M1	1.1b	2nd
	$= 4.24$	A1	1.1b	Calculate the mean of a discrete random variable
		(2)		
4bii	$\text{Var}(X) = 1^2 \times 0.1 + \dots + 6^2 \times 0.24 - 4.24^2$	M1	1.1b	3rd
	$= 2.5824$	A1	1.1b	Calculate the variance of a discrete random variable
		(2)		
4c	CLT states that the sample mean is approximately normally distributed.	B1	1.2	6th
		(1)		Recognise when the central limit theorem is required
4d	$\bar{X} \approx \sim N\left(4.24, \frac{2.5824}{80}\right)$	M1	3.1a	7th
	$P(\bar{X} < 4.5) = P\left(Z < \frac{4.5 - 4.24}{\sqrt{\frac{2.5824}{80}}}\right) (= P(Z < 1.447))$	M1	1.1b	Apply the central limit theorem to a range of probability distributions
	$= 0.9265$	A1	1.1b	
		(3)		

4e	Estimate is fairly accurate since $n$ is large.	<b>B1</b>	2.4	5th Know the conditions for the use of the central limit theorem
		<b>(1)</b>		

**(11 marks)**

**Notes**

- 4a **M1** for attempt to add probabilities and equate to 1  
**A1** cao
- 4bi **M1A1ft** from *their k*
- 4bii **M1A1ft** from *their k*
- 4c **B1** for completely correct reason
- 4d **M1** for attempt to find distribution of  $\bar{X}$  using calculated parameters, must divide by 80  
**M1** for attempt to find  $P(\bar{X} < 4.5)$   
**A1ft** awrt 0.927 (ft *their 4bi* and *4bii*)
- 4e **B1** for correct statement including  $n$  is large

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
5	$\bar{X} \sim N\left(2.6, \frac{2.6}{50}\right)$	M1	3.1a	7th Apply the central limit theorem to a range of probability distributions
		A1	1.1b	
	$P(\bar{X} > 2.7) = P\left(Z > \frac{2.7 - 2.6}{\sqrt{\frac{2.6}{50}}}\right) (= P(Z > 0.4385\dots))$	M1	1.1b	
	= 0.3300	A1	1.1b	<b>(4 marks)</b>
<p><b>Notes</b></p> <p>1st M1 for use of CLT with given Poisson parameters</p> <p>1st A1 for completely correct normal distribution</p> <p>2nd M1 for attempt to find <math>P(\bar{X} &gt; 2.7)</math> using <i>their</i> normal distribution</p> <p>2nd A1 awrt 0.33</p>				

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
6a	Mean: $\frac{1}{0.2} = 5$	B1	1.1b	6th Calculate the mean/variance of a geometric distribution
	Variance: $\frac{1-0.2}{0.2^2} = 20$	B1	1.1b	
		(2)		
6b	$\bar{X} \square N\left(5, \frac{20}{20}\right)$	M1 A1	3.1a 1.1b	7th Apply the central limit theorem to a range of probability distributions
	$P(\bar{X} > 6) = P\left(Z > \frac{6-5}{\sqrt{\frac{20}{20}}}\right) (= P(Z > 1))$	M1	1.1b	
	= 0.1587	A1	1.1b	
		(4)		
<b>(6 marks)</b>				
<b>Notes</b>				
<p><b>6a</b> B1 for correct mean B1 for correct variance</p> <p><b>6b</b> 1st M1 for use of CLT with <i>their</i> Geo parameters 1st A1 for completely correct normal distribution 2nd M1 for attempt to find <math>P(\bar{X} &gt; 6)</math> using <i>their</i> normal distribution 2nd A1 awrt 0.159</p>				

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
7	Model is Negative B(8, 0.3)	<b>B1</b>	3.3	8th Recognise and apply the central limit theorem in contextualised situations
	Expectation = $\frac{8}{0.3} = \frac{80}{3}$	<b>B1</b>	1.1b	
	Variance = $\frac{8(1-0.3)}{0.3^2} = \frac{560}{9}$	<b>B1</b>	1.1b	
	$\bar{X} \square N\left(\frac{80}{3}, \frac{560}{9}\right)$	<b>M1</b> <b>A1</b>	3.1a 1.1b	
	$P(\bar{X} < 25) = P\left(Z < \frac{25 - \frac{80}{3}}{\sqrt{\frac{560}{9}}}\right) (= P(Z < -1.0564\dots))$	<b>M1</b>	1.1b	
	= 0.145	<b>A1</b>	2.1	
<b>(7 marks)</b>				
<b>Notes</b>				
<p><b>B1</b> for correct model</p> <p><b>B1</b> for correct expectation</p> <p><b>B1</b> for correct variance</p> <p>1st <b>M1</b> for use of CLT with <i>their</i> Negative B parameters</p> <p>1st <b>A1</b> for completely correct normal distribution</p> <p>2nd <b>M1</b> for attempt to find <math>P(\bar{X} &lt; 25)</math> using <i>their</i> normal distribution</p> <p>2nd <b>A1</b> cao</p>				



Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
8a	$P(\bar{X} < 72) = P\left(Z < \frac{72 - 75}{\frac{6}{\sqrt{24}}}\right) (= P(Z < -2.449\dots))$	M1	3.1b	7th Apply the central limit theorem to a range of probability distributions
	= 0.007	A1	1.1b	
		(2)		
8b	The population is normally distributed.	B1	2.4	5th Know the conditions for the use of the central limit theorem
		(1)		
8c	Model is B(24, 0.2)	B1	3.1b	8th Recognise and apply the central limit theorem in contextualised situations
	Mean: $24 \times 0.2 = 4.8$ Variance: $4.8 \times 0.8 = 3.84$	B1	1.1b	
	$P(\bar{X} < 4) = P\left(Z < \frac{4 - 4.8}{\sqrt{\frac{3.84}{15}}}\right) (= P(Z < -1.581\dots))$	M1	1.1b	
	= 0.057	A1	2.1	
		(4)		
				(7 marks)
<b>Notes</b>				
<p><b>8a</b> M1 for attempt to find <math>P(\bar{X} &lt; 72)</math> A1 awrt 0.007</p> <p><b>8b</b> B1 must state normally distributed</p> <p><b>8c</b> B1 correct model B1 mean and variance both correct M1 for attempt to find <math>P(\bar{X} &lt; 4)</math> using <i>their</i> model and mean/variance A1 cao</p>				