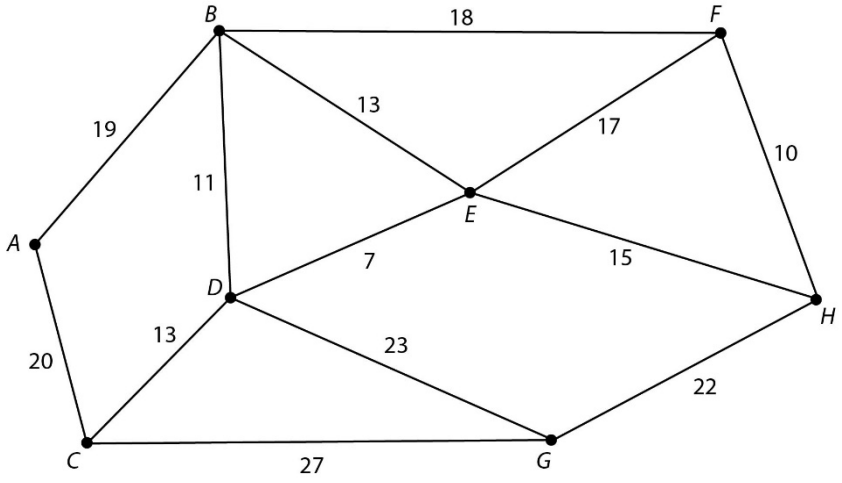
1 **a** Define the term *minimum spanning tree*. **(1 mark)**

b State two differences between Kruskal's algorithm and Prim's algorithm, to find a minimum spanning tree. **(2 marks)**

2

Figure 1



a Complete Matrix 1 to represent the network shown in Figure 1. **(2 marks)**

b Starting at A, use Prim's algorithm and your completed Matrix 1 to find a minimum spanning tree.

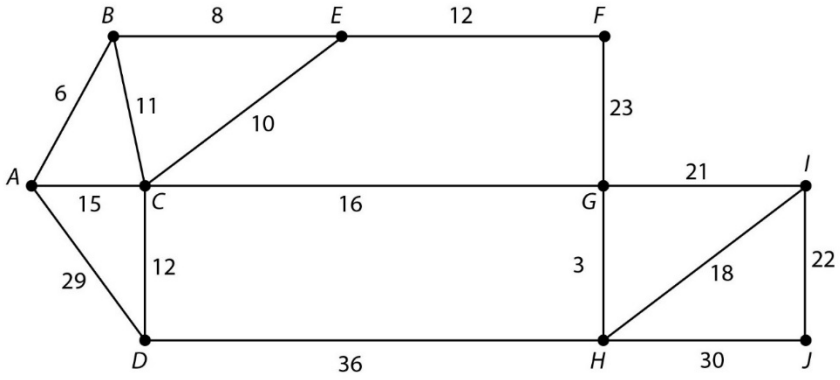
Clearly state the order in which you selected the arcs for your tree.

State the weight of the minimum spanning tree. **(4 marks)**

3 The network in Figure 2 shows a plan of possible paths to be built between buildings in a school. The numbers on each arc are lengths in metres.

The paths are to form a network along the arcs, using the least possible length.

Figure 2



a Find a minimum spanning tree for the network, showing clearly the order in which you selected the arcs for your tree using

i Kruskal's algorithm **(3 marks)**

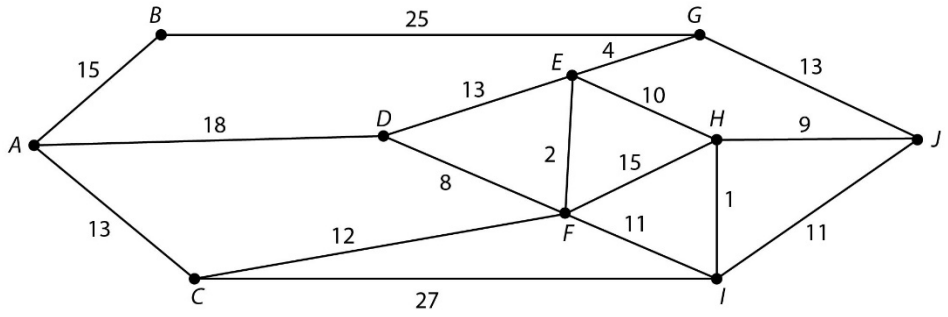
ii Prim's algorithm, starting from A. **(3 marks)**

Footpaths have already been built along AD and AB and so should be included in the spanning tree.

b Explain which algorithm you would choose to complete the tree, and how the method should be adapted. (You do **not** have to find the tree.) **(2 marks)**

4 Figure 3 shows a network of roads between towns. The number on each arc represents the length of the road in km.

Figure 3

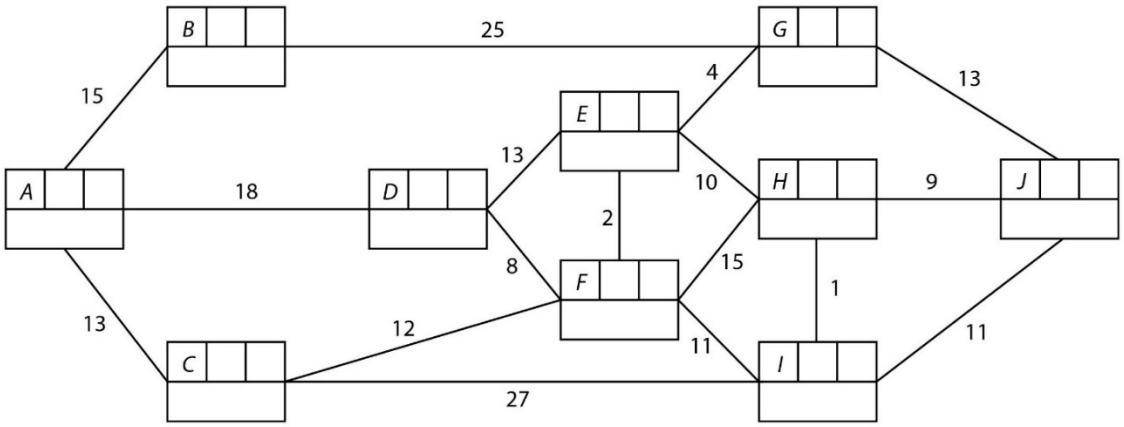


Shiv lives in town *A* and works in town *J*.

a Complete Figure 4 using Dijkstra's algorithm to find the shortest route from *A* to *J*.

State your shortest route and its length. **(6 marks)**

Figure 4



b Explain how you determined the shortest route from your labelled diagram. **(2 marks)**

The route from *C* to *F* will be closed for repairs on Wednesday.

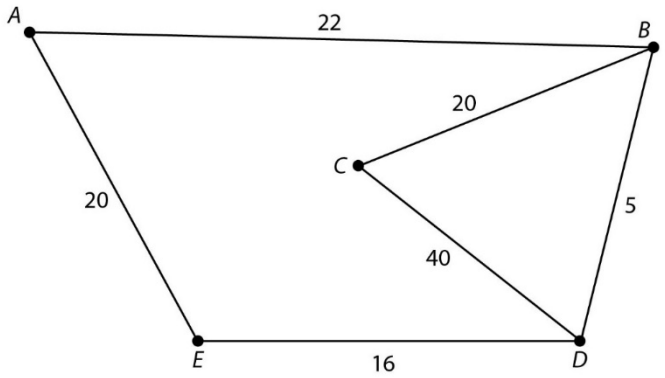
c Find the shortest route for Shiv from *A* to *J* avoiding *CF* and state its length. **(3 marks)**

d On another day, Shiv needs to collect her dry-cleaning from town, *I*, on her way to *J*.

Find the shortest route that includes *I* and state its length. **(2 marks)**

5 Figure 5 shows roads connecting five towns. The numbers show distances in kilometres.

Figure 5



These are the distance and route matrices after the third iteration of Floyd's algorithm:

	A	*B*	*C*	*D*	*E*
A	-	22	42	15	15
B	22	-	20	5	23
C	42	20	-	25	43
D	15	5	25	-	16
E	15	23	43	16	-

	A	*B*	*C*	*D*	*E*
A	-	B	B	D	E
B	A	-	C	D	E
C	B	B	-	B	B
D	A	B	B	-	E
E	A	B	B	D	-

a Perform the fourth iteration. **(4 marks)**

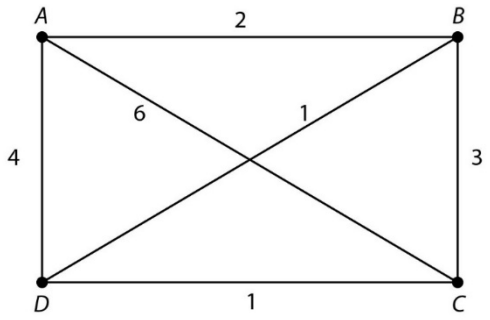
There are no changes on the fifth iteration.

b Explain how to find the shortest distance and route from town *C* to town *A* using your answer from part **a**.

State the route and the distance. **(4 marks)**

6 Figure 6 shows a network with the weights on the arcs representing distances.

Figure 6



a Apply Floyd's algorithm to find the complete network of shortest distances.

You should show both the distance table and the route table after each iteration. **(9 marks)**

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
B				
C				
D				

b Explain how to use your final matrix to find the shortest route from vertex A to vertex C.

State this distance. **(3 marks)**