1 A shop stocks hats in sizes small, medium and large.

It is decided that the total number of medium and large hats should be at most half of the number of small hats.

The number of large hats should be at most 40% of the total number of hats.

The maximum number of hats that can be stored is 200.

Small, medium and large hats give a profit of £3, £5 and £6 respectively.

Let the number of small, medium and large hats be x, y and z respectively.

The shop wishes to maximise its profit, £C, for the hats.

Formulate this situation as a linear programming problem, simplifying your inequalities so all coefficients are integers. **(5 marks)**

2 A linear programming problem in x and y is described as follows.

Minimise $C = 2x + 3y$

Subject to

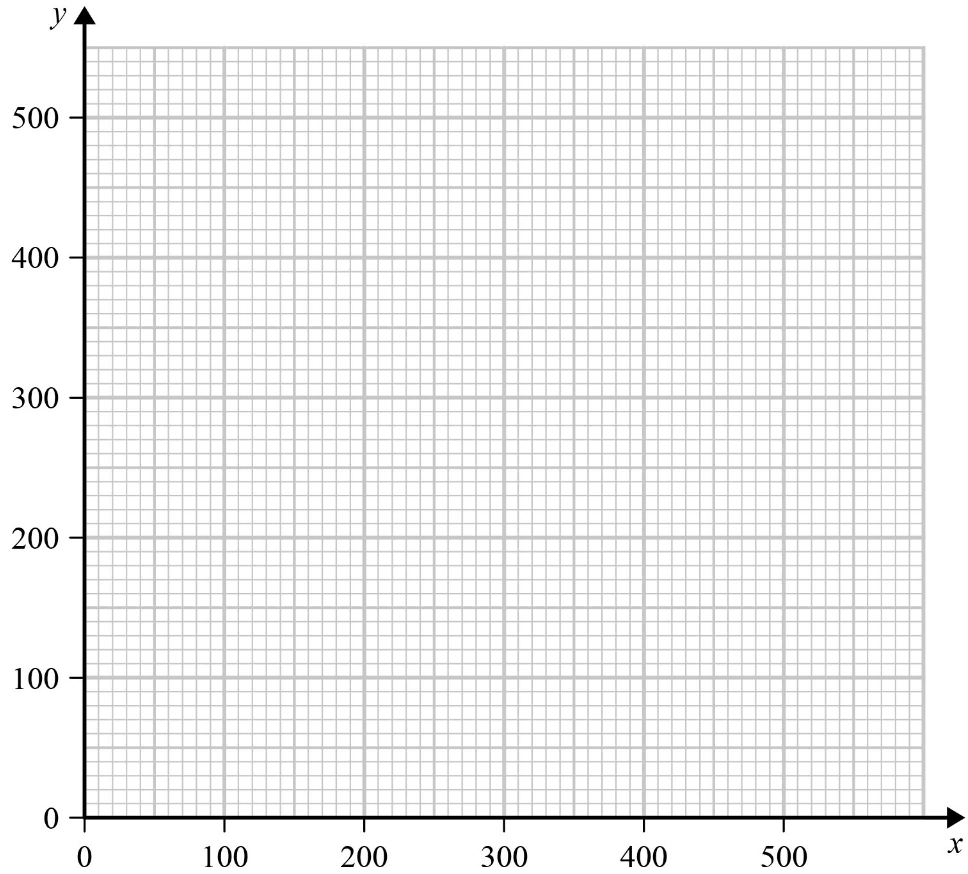
$5x + 4y \geqslant 2000$

$y \leqslant 2x$

$y \geqslant x - 150$

$x, y \geqslant 0$

a Represent these inequalities on the graph below, including shading. **(3 marks)**



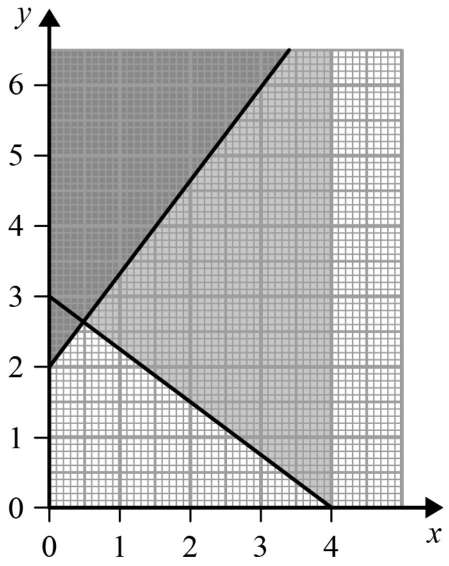
b Show the feasible region, labelling it R. **(1 mark)**

c Use **point testing** to determine the exact coordinates of the optimal point P.
You must show your working. **(5 marks)**

The first constraint is changed to $x + y \geqslant k$ for some value of k.

d Determine the greatest value of k for which P is still optimal. **(2 marks)**

3 The constraints of a linear programming problem are represented by the graph below.



The feasible region is the unshaded region, including its boundaries.

a Write down four inequalities that define the feasible region. **(3 marks)**

The objective is to maximise $P = x + 2y$

b Use the graph and a profit line to obtain the coordinates of the vertices of the feasible region. Hence find the values of x and y that maximise P and the corresponding maximum value of P. **(5 marks)**

4 A youth hostel is to be built with a maximum of 60 beds allowed due to fire restrictions. Rooms will have either one bed, two beds or four beds. The maximum amount of space available for bedrooms is 700 m^2.

The table below shows the floor area and profit per **room** per night.

Number of beds in room	**Variable**	**Floor area (m^2)**	**Profit per room per night (£)**
1	x	15	5
2	y	25	9
4	z	40	15

a Formulate a linear programming problem for the above information to maximise the profit per night. **(4 marks)**

b Use the simplex algorithm to solve the problem. **(5 marks)**

c Interpret your solution in terms of what rooms are to be built and the number of beds they will contain. **(2 marks)**

5 The following linear programming problem is to be solved using the two-stage simplex method.

Maximise $P = 2x + 3y + z$

Subject to: $x + y + z \leqslant 40$

$2x + 2y - z \geqslant 10$

$x - y \geqslant 8$

$x, y, z \geqslant 0$

a Write the constraints as equations using slack, surplus and artificial variables. **(4 marks)**

b Write the initial tableau in standard form. **(2 marks)**

DO NOT ATTEMPT TO SOLVE THE PROBLEM.

6 The Big-M method is to be used to solve the following linear programming problem.

Maximise $P = 2x + y$

Subject to: $x + y \leqslant 10$

$-x + y \geqslant 2$

$x, y \geqslant 0$

with the following initial tableau in standard form.

Basic variable	***x***	***zy***	s_1	s_2	a_1	**Value**
s_1	1	1	S1	0	0	10
a_1	–1	1	0	–1	1	2
P						

a Complete the table to express P in terms of M. **(3 marks)**

b Carry out iterations to solve the problem. State the value of each variable in your solution. **(6 marks)**