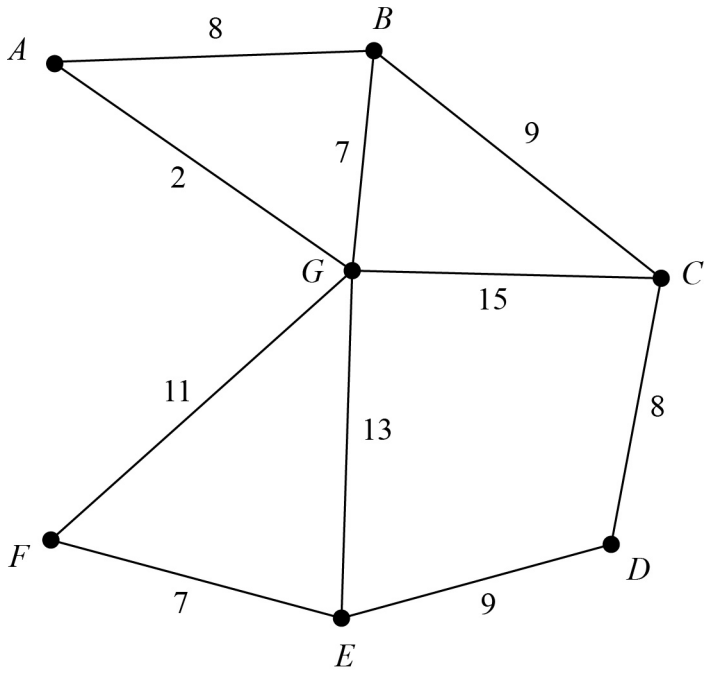
1 The diagram represents a network of pipes. Each pipe needs to be cleaned. The only access to the pipes is at B.



a Use the Route Inspection Algorithm to find which paths, if any, need to be traversed twice. State the length and a possible route. **(6 marks)**

It is decided to make another access point at E. The pipes can now be entered at one access point and exited at the other.

b Determine the changes to the route from your answer in **a** if the route is to be minimised.

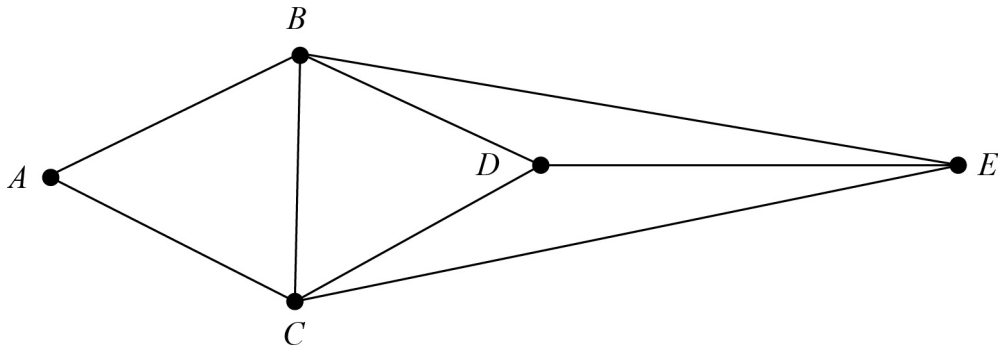
Explain your reasoning.

How much shorter is the length of the new route? **(3 marks)**

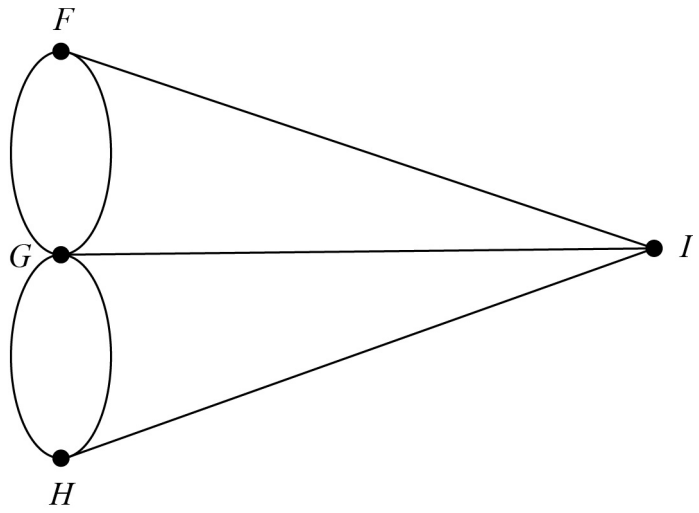
2 a Use the valency of each vertex to determine if the following networks are traversable.

Explain your reasoning.

i



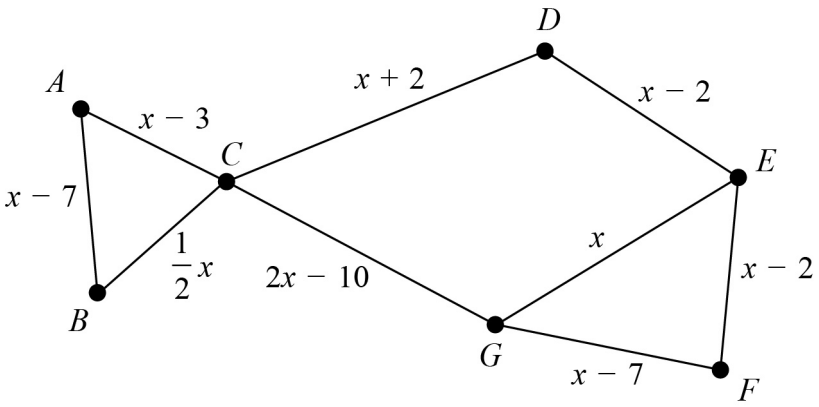
ii



(2 marks)

b If the network in **a** is traversable, state a possible route. **(1 mark)**

3 **a** Explain why it is impossible to draw a network with exactly three odd vertices. **(2 marks)**



The Route Inspection Algorithm is used on the above network. The length of the route is found to be 100.

b Determine the value of x. You must show all your working clearly. **(4 marks)**

4 The table shows the distances, in km, between six towns.

	A	***B***	***C***	***D***	***E***	***F***
A	–	80	105	170	103	95
B	80	–	33	170	155	88
C	105	33	–	143	151	68
D	170	170	143	–	105	79
E	103	155	151	105	–	87
F	95	88	68	79	87	–

a Use Prim's algorithm, starting from A, to find a minimum spanning tree. You must make the order of the arc selection clear. Draw your minimum spanning tree. **(4 marks)**

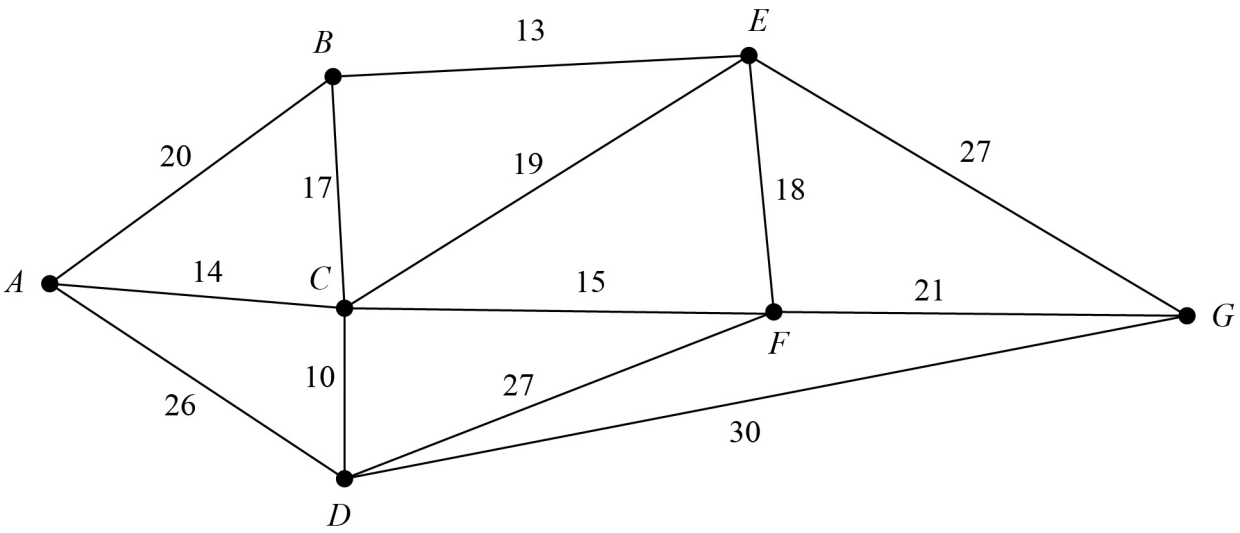
b **i** Hence find an initial upper bound for the travelling salesman problem. **(1 mark)**

ii Using short cuts, find two routes that reduce the upper bound to a value less than 640 km. **(5 marks)**

c By deleting vertex F, find a lower bound to the travelling salesman problem for this network. **(3 marks)**

d Using your answers to **b** and **c**, write down the smallest interval that must contain the length of the optimal route between the towns. **(2 marks)**

5 The network in the diagram shows the distances, in km, between seven wind turbines. All the turbines need to be serviced by an engineer.



By deleting C a lower bound for the length of the route is found to be 122 km.

a By deleting F, find another lower bound of the route. State which is the better lower bound of the two. **(5 marks)**

b By inspection, complete the table of least distances below. **(3 marks)**

	A	***B***	***C***	***D***	***E***	***F***	***G***
A	–	20	14		33	29	
B	20	–	17		13	31	40
C	14	17	–	10	19	15	
D			10	–			30
E	33	13	19		–	18	27
F	29	31	15		18	–	21
G		40		30	27	21	–

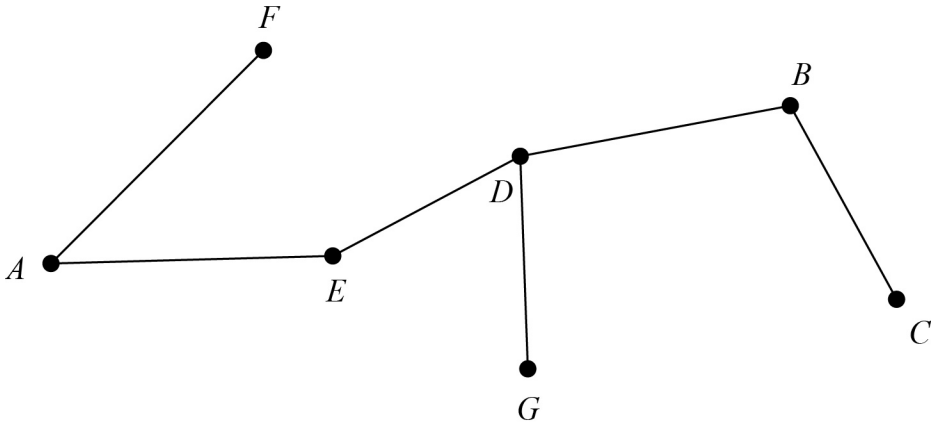
c Starting at F use the nearest neighbour algorithm with the completed table to obtain an upper bound for the length of the route. State your route. **(3 marks)**

6

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	4	5	3	2	5	6
B	4	–	1	2	4	7	6
C	5	1	–	3	4	6	7
D	3	2	3	–	2	6	4
E	2	4	4	2	–	6	6
F	5	7	6	6	6	–	10
G	6	6	7	4	6	10	–

The table shows the distance, in km, between seven houses.

Prim's algorithm is used on the above table to find a minimum spanning tree. The order in which the arcs are chosen is *AE ED BD BC DG AF* with the associated tree.



Melissa delivers parcels. She has a parcel to deliver to each of the seven houses.

The distance between the parcel depot and each house, in km, is shown in the table below.

House	***A***	***B***	***C***	***D***	***E***	***F***	***G***
Distance from depot P (km)	11	15	16	16	12	17	18

Melissa wants to travel the shortest distance from the depot to each house before returning to the depot.

a Using the given information, calculate a lower bound for the length of Melissa'sroute. You must show all your working clearly. **(3 marks)**

b Use the nearest neighbour algorithm, starting from the parcel depot (P), to find a route for Melissa to deliver all her parcels. Hence find an upper bound for the length of Melissa's route. **(3 marks)**