1 The random variable X has a Poisson distribution $X \sim \text{Po}(5)$. A sample is taken, and it is desired to test H_0: $\lambda = 5$ against H_1: $\lambda > 5$, using a 5% level of significance.

a Find the critical region for this test. **(2 marks)**

b **i** Define a Type I error. **(1 mark)**

ii State the probability of a Type I error. **(1 mark)**

2 The random variable Y is binomially distributed. A sample of size 10 is taken and it is desired to test H_0: $p = 0.35$ against H_1: $p \neq 0.35$, using a 5% level of significance.

a Calculate the probability of a Type I error **(4 marks)**

Given that true probability is later found to be 0.4

b **i** define a Type II error. **(1 mark)**

ii calculate the probability of a Type II error. **(4 marks)**

3 A manager of a canning factory claims that the amount of soup in a can, measured in ml, is normally distributed with a mean of 550 ml and a variance of 12 ml^2

A random sample of 20 cans is taken to check if there has been a decrease in the mean amount of soup per can. You can assume that the variance has remained unchanged.

a Find, at the 5% level of significance, the critical region for the test statistic $\overline{X}$, the mean amount of soup in a sample of 20 cans. **(4 marks)**

Given that the actual mean amount has decreased to 549 ml,

b find the probability of a Type II error. **(2 marks)**

The manager would like to see the probability of a Type II error reduced.

She considers changing the significance level.

c State, giving a reason, what she should do to the significance level to achieve her aim. **(1 mark)**

4 Explain briefly what you understand by

a the size of a hypothesis test **(1 mark)**

b the power of a hypothesis test. **(1 mark)**

The random variable X has a Poisson distribution. A sample is taken, and it is desired to test H_0: $\lambda = 7$ against H_1: $\lambda < 7$, using a 10% level of significance.

c Find

i the critical region for the test and state the size of the test. **(3 marks)**

ii the power of the test, given that λ is in fact 7.5. **(2 marks)**

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5 It is thought that a biased coin lands on heads with probability p. The coin is tossed 15 times and the random variable Y represents the number of heads thrown.

In a test of H_0: $p = 0.4$ against H_1: $p < 0.4$, the null hypothesis is rejected if the number of successes is less than 3

a Show that the power function for this test is given by

$$(1-p)^{13}(1+13p+91p^2)$$ **(4 marks)**

b Find the power of this test when $p = 0.3$ **(2 marks)**

6 Jack and George are playing a computer game where they play the role of a cat who has to catch a virtual mouse. They are testing to see if the probability of catching the mouse is less than 0.15

They both use a 5% significance level.

Jack plays the game until he catches the mouse and he records the number of attempts.

a Find the critical region for Jack's test. **(5 marks)**

b Find the size of Jack's test. **(1 mark)**

c Find the power function for Jack's test. **(1 mark)**

George plays the game 31 times and records the number of times he catches the mouse.

d Find the size of George's test. **(3 marks)**

e Show that the power function for George's test is

$$(1-p)^{30}(1+30p)$$ **(3 marks)**

f With reference to your answers to parts **b**, **c**, **d** and **e**, state, giving reasons, whether you would recommend Jack's test or George's test when $p = 0.12$ **(4 marks)**

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