1 Two spheres, one of mass 1.5 kg and the other of mass 4.5 kg, are moving towards each other, travelling in the same straight line, with constant speeds of 0.4 m s^{-1} and 0.1 m s^{-1} respectively.

a Marsha predicts that the collision between these two spheres will be perfectly elastic. What velocity does Marsha expect each sphere to have after the collision? **(3 marks)**

Marsha now experimentally measures the velocity of each sphere after the collision. Her results show that these velocities are equal in magnitude but opposite in direction.

b Using Newton's experimental law, express the velocities of both spheres after the collision, in terms of e, and hence find the value for the coefficient of restitution suggested by Marsha's experimental results. **(4 marks)**

c State the range of values for the coefficient of restitution for which these two spheres will always travel in opposite directions after the collision. **(2 marks)**

2 A small rubber ball is dropped from a height of h_0 metres above a fixed horizontal plane. The ball falls freely under gravity until it collides with the plane's surface. The ball then rebounds vertically upwards.

a If e is the coefficient of restitution between the ball and the plane, show that the greatest height achieved by the ball after first bouncing, h_1, is given by:

$$h_1 = e^2 h_0$$ **(4 marks)**

b Deduce an expression for the height reached after n bounces, hn, in terms of e and h_0. **(2 marks)**

c The maximum height achieved by the ball after six bounces is half of the maximum height achieved by the ball after two bounces. Find e, correct to three decimal places, and thus comment on the elasticity of the collisions. **(3 marks)**

3 A sphere of mass 2 kg is moving on a smooth horizontal surface, with speed 6 m s^{-1}, towards a fixed vertical wall. The coefficient of restitution in the collision between the sphere and the wall is e.

a Find an expression for the kinetic energy lost in this collision with the wall, E_1. **(2 marks)**

After the collision with the wall the sphere travels towards another sphere of mass 4 kg, which is initially stationary. The coefficient of restitution when these two spheres collide is also e.

b Show that the kinetic energy lost in this further collision, E_2, is given by:

$$E_2 = 24\, e^2 (1 - e^2)$$ **(6 marks)**

The total loss in kinetic energy from both collisions is known to be 16 joules.

c Find the value of e correct to three decimal places. **(3 marks)**

4 Three particles, A, B and C, lie at rest in a straight line on a perfectly smooth horizontal plane. B lies between A and C. The particles A, B and C have masses in kilograms of 2 m, 4 m and 6 m respectively.

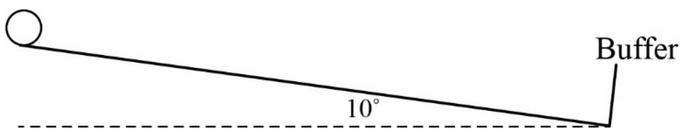
Particle A is projected towards particle B with constant speed u m s^{-1}. The coefficient of restitution between each pair of particles is e.

a After the initial collision between A and B, there is a further collision between B and C. Show that the velocity of particle B after this further collision, v_B, is given by:

$$v_B = \frac{1}{15}u(2 - e - 3e^2)$$ **(6 marks)**

b Particles A and B are known to collide for a second time when they are both travelling in the same direction as C. Determine the possible range of values for e. **(5 marks)**

5 A sphere of mass 1 kg is released from rest at the top of a plane that is inclined at 10º to the horizontal. The plane is 1 metre long and at its bottom end is a buffer that lies at right angles to the plane. This buffer acts as wall against which the sphere will directly collide. The coefficient of restitution between the sphere and the buffer is e. The coefficient of friction between the sphere and the inclined plane is ke^2, where k is a constant. The constant of acceleration due to gravity is denoted by g.



a Show that the total energy lost, L, between the sphere being first released, to the instant directly after its first collision with the buffer is given by the equation,

$$L = e^4\, kg \cos(10) + (1 - e^2)\, g \sin(10)$$ **(7 marks)**

It is known experimentally that L represents 50% of the sphere's initial energy.

b Find the maximum possible value for constant, k, correct to three decimal places. **(3 marks)**

