

Further Statistics 1 Unit Test 9: Probability generating functions

- 1 The probability generating function of a discrete random variable X is given by

$$G_X(t) = k(1 + 3t + 2t^2)^2$$

- a Show that $k = \frac{1}{36}$ (2 marks)
- b Write down the probability distribution of X (3 marks)

- 2 $Y \sim \text{Geo}(0.6)$

Show, from first principles, that the probability generating function for Y is

$$G_Y(t) = \frac{0.6t}{1 - 0.4t} \quad (5 \text{ marks})$$

- 3 Paulette writes the following probability generating function for a discrete random variable X

$$G_X(t) = 0.2(3t + 4t^2 + 3t^3)$$

- a Explain why this is not a probability generating function. (1 mark)
- Given that the bracketed expression is correct, find
- b $G_X(t)$ (2 marks)
- c $P(X = 2)$ (2 marks)

- 4 The random variable X has the probability distribution shown in the table.

x	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.1	0.3	0.3

- a Show that the probability generating function for X can be written as

$$G_X(t) = \frac{1}{10}(t + 2t^2 + t^3 + 3t^4 + 3t^5) \quad (2 \text{ marks})$$

- b Use the probability generating function to show that $E(X) = 3.5$ (3 marks)

- 5 A discrete random variable X has a probability generating function, given by

$$G_X(t) = \frac{at}{b + t^2} \quad \text{where } a \text{ and } b \text{ are positive integers.}$$

- Given that the mean of X is $\frac{2}{3}$, find the values of a and b (6 marks)

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- 6 The probability generating function of a discrete random variable Y is given as

$$G_Y(t) = k(t + 2t^2 + t^3)^2$$

- a** Show that $k = \frac{1}{16}$ **(2 marks)**
b Find $P(Y = 4)$ **(2 marks)**
c Show that $\text{Var}(Y) = 1$ **(8 marks)**
d Find the probability generating function of $3Y + 1$ **(2 marks)**

- 7 A discrete random variable X has a probability generating function given by

$$G_X(t) = \frac{1}{6} + \frac{1}{3}t + \frac{1}{2}t^3$$

- a** Show that the standard deviation of X is $\frac{\sqrt{53}}{6}$ **(5 marks)**
b Find the probability generating function of $4X + 3$ **(2 marks)**

A second discrete random variable $Y \sim B(2, 0.4)$

The discrete random variable $Z = X + Y$

- c** Show that the probability generating function for Z is given by

$$G_Z(t) = \frac{1}{150}(9 + 30t + 28t^2 + 35t^3 + 36t^4 + 12t^5) \quad \mathbf{(3 \text{ marks})}$$