

Further Statistics 1 Unit Test 6: Hypothesis testing

- 1** On a stretch of road, drivers are caught speeding at an average rate of 5 per day. A new speed camera is installed and on the day after, it is found that 2 drivers are caught speeding. The random variable X represents the number of drivers caught speeding on a particular day.
- a** State the distribution that can be used to model X **(1 mark)**
 - b** Write down suitable hypotheses that can be used to test the claim that the installation of the camera has reduced the number of drivers caught speeding. **(2 marks)**
 - c** Test, at the 5% significance level, whether or not the mean number of drivers caught speeding has reduced. **(3 marks)**
- 2** Steph owns an industrial oven manufacturing company that prides itself on the reliability of its products. The company website advertises that breakdowns occur on average at a rate of 0.25 per month.
- a** Suggest a suitable model for the random variable X , the number of breakdowns in a month. **(1 mark)**
- A survey is carried out and over a four-month period it was found that four machines broke down. Karen, a long-standing customer, claims that the figure quoted on the website is incorrect.
- b** Test, at the 5% level of significance, whether there is evidence to support Karen's claim. State your hypotheses clearly. **(5 marks)**
- 3** Wendy, an apple wholesaler, packages apples into crates of 200. It is found that 2% of the apples have imperfections on their skin.
- a** State a suitable approximating distribution to model the number of imperfect apples in each crate and explain why the distribution is suitable. **(3 marks)**
 - b** Using your distribution in part **a**, estimate the probability that a randomly chosen crate contains
 - i** exactly three imperfect apples **(1 mark)**
 - ii** at least six imperfect apples. **(2 marks)**
- Wendy changes suppliers and in the first crate checked after the change there are six imperfect apples.
- c** Is there evidence to suggest, at the 10% level of significance, that the change in supplier has increased the number of imperfect apples?
State your hypotheses clearly. **(3 marks)**

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- 4** Dennis, a manufacturer of fine silk, believes that faults in the cloth occur at an average rate of 0.15 per metre. To test this, he takes a sample of 50 metres of cloth.
- a** Use a 5% level of significance to find the critical region of the hypothesis that the faults in the cloth occur at a rate of less than 0.15 per metre. The probability should be as close to 5% as possible. **(3 marks)**
- b** Write down the actual significance level of the test. **(1 mark)**
- The actual number of faults in the 50-metre length of cloth was 2
- c** Comment on this observation in light of your critical region. **(2 marks)**
- 5** A single observation, y , is taken from a Poisson distribution with parameter λ . This observation is used to test, at the 5% level of significance, $H_0: \lambda = k$ against $H_1: \lambda \neq k$, where k is a positive integer. The probability in each tail is less than 0.025
- Given that the critical region for this test is $Y \leq 3$ and $Y \geq 16$, find, with justification, the value of k and the actual significance level of the test. **(5 marks)**
- 6** Conor is a keen skateboarder. When learning a new trick, the probability that he succeeds in landing the trick on each attempt is 0.3. The random variable X represents the number of attempts needed until he lands the trick.
- a** Write down a suitable distribution to model X and give two conditions that are necessary for the model to be valid. **(2 marks)**
- b** Calculate the probability that Conor lands his first trick
- i** on his fifth attempt **(2 marks)**
- ii** in six or fewer attempts. **(2 marks)**
- Conor has some coaching. He claims that the coaching improves his likelihood of landing the trick in any one attempt to 0.45. On his next day of practice, it took eight attempts to achieve his first successful landing of the trick.
- c** Is there any evidence, at the 5% level of significance that Conor is overstating the probability of landing the trick on any given attempt?
- State your hypotheses clearly. **(5 marks)**
- 7** Tom is a circus entertainer. He throws knives at a target while wearing a blindfold. He claims that the probability of him hitting the target with each throw is 0.25. Emma, his partner, believes Tom is overstating his ability to hit the target. Tom throws knives until he hits the target.
- a** Find the critical region, at the 10% level of significance, for the number of knives thrown by Tom to accept Emma's claim. **(5 marks)**
- b** Find the actual significance level for this test. **(2 marks)**