

Further Statistics 1 Unit Test 7: Central limit theorem

- 1 A sample of size n is taken from a population with mean μ and variance σ^2
- a Explain, in this context, what the central limit theorem is. **(2 marks)**
- b State one condition that must be met to use the central limit theorem. **(1 mark)**

- 2 A random sample of size 10 is taken from a population that is normally distributed with mean 12 and standard deviation 3
- a Find the probability that the sample mean is greater than 13 **(3 marks)**
- b State, with a reason, whether your answer is an approximation. **(1 mark)**

- 3 The width of washers produced in a factory has an unknown distribution with mean 21 mm and standard deviation 1.1 mm.
- A sample of 50 washers is taken.
- a Estimate the probability that the mean width of the washers is less than 20.7 mm. **(3 marks)**
- A second sample is taken. The probability that the mean of this sample is less than 20.7 mm needs to be less than 1%.
- b Find the minimum sample size required. **(5 marks)**

- 4 The random variable X has the probability distribution shown in the table.

x	1	2	4	5	6
$P(X = x)$	0.1	k	$2k$	0.3	0.24

- a Find the value of k **(2 marks)**
- b Find
- i the expectation of X **(2 marks)**
- ii the variance of X **(2 marks)**

A random sample of 80 observations of X is taken.

- c Explain how the central limit theorem can be used to estimate the probability that the mean of these observations is less than 4.5 **(1 mark)**
- d Find an estimate for the probability that the mean of these observations is less than 4.5 **(3 marks)**
- e Comment on the accuracy of your estimate. **(1 mark)**

Further Statistics 1 Unit Test 7: Central limit theorem

- 5 A random sample of 50 observations is taken from a Poisson distribution with mean 2.6
Estimate the probability that the sample mean is greater than 2.7 **(4 marks)**
- 6 A random sample of 20 observations is taken from a random variable $X \sim \text{Geo}(0.2)$
- a Find the mean and variance of X **(2 marks)**
- b Estimate the probability that the sample mean is greater than 6, using the central limit theorem. **(4 marks)**
- 7 Russell is an amateur snooker player. He practises a particular pot several times and the probability of making the pot each time is 0.3. In any one practice session, his goal is to make the pot 8 times. You may assume that the probability of him making each pot is constant and each attempt at making the pot is independent.
In one week, he practises the pot in 25 practice sessions and achieves his goal each time.
Show that the probability that his average number of shots in each session was less than 25 is 0.145, correct to three significant figures. **(7 marks)**
- 8 The mass of duck eggs is normally distributed with mean 75 g and standard deviation 6 g.
Becky fills baskets with 24 randomly chosen eggs.
- a Calculate the probability that the mean mass of a duck egg in a randomly chosen basket is less than 72 g. **(2 marks)**
- b Explain why your answer to part a is not an estimate. **(1 mark)**
- The probability that a duck egg is a rare purple egg is 0.2. A sample of 15 baskets is taken.
- c Show that the probability that the sample of baskets will contain fewer than 4 purple eggs on average is 0.057, correct to three decimal places. **(4 marks)**