1 The probability density function of the random variable X is defined as

$$f(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x}{72} & 0 \leqslant x \leqslant b \\ 0 & x > b \end{cases}$$

a Find the value of b. **(2 marks)**

b Calculate $P(X < 8)$. **(1 mark)**

2 The probability density function of the random variable X is defined as

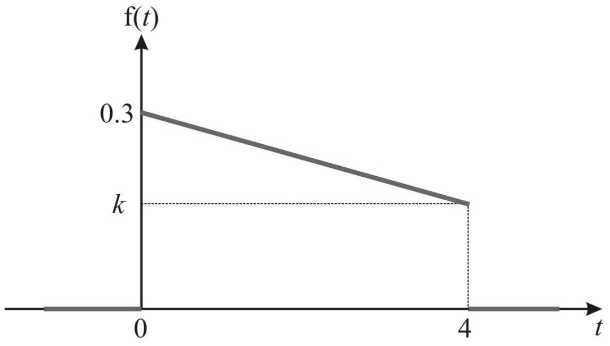
$$f(x) = \begin{cases} \dfrac{1}{6} & 0 \leqslant x \leqslant 4 \\ \dfrac{1}{24}(8 - x) & 4 \leqslant x \leqslant 8 \\ 0 & \text{otherwise.} \end{cases}$$

a Find $P(X > 6)$. **(3 marks)**

b Find the median value of X. **(1 mark)**

c Given that $E(X) = \dfrac{28}{9}$, find the value of k such that $P(X > E(kX - 1)) = 0.5$ **(2 marks)**

3 The probability density function of the continuous random variable T is sketched in the graph below, where $P(T < 0) = P(T > 4) = 0$.



a Show that $k = 0.2$ **(1 mark)**

b Given that $E(T) = \frac{28}{15}$, use integration to find the variance of T. **(3 marks)**

The cumulative distribution function for T is denoted by $F(t)$ and is given by

$$F(t) = \begin{cases} 0 & t < 0 \\ 0.3t - 0.0125t^2 & 0 \le t \le 4 \\ 1 & t > 4 \end{cases}$$

c Use $F(t)$ to find $P(T > 3.2)$ **(2 marks)**

d Find the value of b such that $P(1.2 < T < b) = 0.4$ **(3 marks)**

e Assuming that all professional track athletes can run 1000 metres in 4 minutes or less, decide whether the functions $f(t)$ and $F(t)$ would be appropriate models to use for the time, in minutes, taken by a professional track athlete to run 1000 metres.

Give a reason for your answer. **(2 marks)**

4 The time, x minutes, that patients at a clinic wait to be attended to is modelled by the cumulative distribution function F(x) given below.

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^2}{13500}(45 - x) & 0 \leqslant x \leqslant 30 \\ 1 & x > 30 \end{cases}$$

a Show that the probability density function, f(x), for waiting times in the domain $0 \leqslant x \leqslant 30$ can be written in the form $ax(bx + c)$, where a, b and c are constants to be found. **(2 marks)**

b Show, by an appropriate method, that the mean, median and mode of the waiting times are equal. **(2 marks)**

c Show that $P(x < 20) \approx 0.74$ and interpret this result in the context of this question. **(2 marks)**

d Evaluate $P(x \geqslant 12)$ **(2 marks)**

e Find the value of the constant k, given that $E(kx - 20) = E(2x)$ **(2 marks)**

f Give a reason why the cumulative distribution function used to model the waiting times may not be accurate in real life. **(1 mark)**

5 The probability density function of the random variable X is defined as

$$\mathrm{f}(x) = \begin{cases} kx & 1 \leqslant x \leqslant 9 \\ 9k(10 - x) & 9 \leqslant x \leqslant 10 \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

a Show that $k = \frac{2}{89}$ **(3 marks)**

b Explain how you know that the mode of X is equal to 9 **(2 marks)**

c Determine, with a reason, whether the median of X is less than or greater than 6.5 **(2 marks)**

It is claimed by a student that the distribution of X is positively skewed. The student decides to find a suitable summary value to test this claim.

d By finding the summary value, determine if the student's claim is true, giving a reason for your answer. **(4 marks)**

6 The continuous random variable Q is uniformly distributed over the interval $\left[-\frac{3}{4}, \frac{23}{12}\right]$.

a Sketch the probability density function, $\mathrm{f}(q)$, of Q. **(2 marks)**

b Find $\mathrm{P}(\overline{q} \leqslant Q < (\overline{q})^{-1})$ **(2 marks)**

c Determine, showing your reasoning, whether the lower quartile of Q is less than, equal to, or greater than zero. **(2 marks)**

d Find the value of k such that $\mathrm{P}\left(Q > \frac{1}{4}[k - Q]\right) = \frac{5}{32}$ **(2 marks)**