

Further Statistics 2 Unit Test 4: Combinations of random variables

1 Two independent random variables are X and Y . It is given that $E(X) = 2E(Y) = a$ and that $\text{Var}(X) = 3\text{Var}(Y) = b$. Find

a i $E(2X - 4Y - 1)$ (2 mark)

ii $\text{Var}(3X - 6Y + 1)$ (2 marks)

b The variance of nY exceeds the variance of the sum of n random observations of Y by $10b$

Find the value of n (3 marks)

2 A particular type of rock has been found to contain three valuable minerals. The masses, in milligrams, of minerals found in 1 tonne of rock are modelled by the following independent normally distributed random variables: $A \sim N(140, 370)$, $B \sim N(225, 480)$ and $C \sim N(310, 154)$.

The market values of these minerals, in dollars (\$) per gram, are given in the following table.

| Mineral mass modelled by | A | B | C |
|--------------------------|-----|------|-----|
| Value (\$ per gram) | 600 | 1000 | 400 |

a Show that the expected total value of the minerals in 1 tonne of rock is \$433. (1 mark)

b Find the probability that the minerals in a randomly chosen tonne of rock have a total value of more than \$500 (3 marks)

To extract and prepare the minerals from each tonne of rock for sale will cost no more than \$365

c Use an appropriately sketched and labelled curve along with your answer to part b to assess the probability that the process of extracting the minerals from the rock will be profitable. (3 marks)

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- 3 The times taken, in minutes, for each of two brands of medication, A and B, to have a beneficial effect on a patient are modelled by the following normally distributed random variables:

$$A \sim N(16.6, 15.5) \text{ for brand A, and } B \sim N(13.4, 13.4) \text{ for brand B.}$$

- a Find the probability that brand B has a beneficial effect in a shorter time than brand A. **(3 marks)**

At 10 a.m., one randomly selected patient takes brand A medication and another randomly selected patient takes brand B medication. A student claims that $P(A + B < 30) = 0.5$ and that there is, therefore, a 50% chance that both patients feel a beneficial effect before 10.30 a.m.

- b Explain why the student's first claim is true and, justifying your answer, determine whether there is any truth in the student's second claim. **(3 marks)**
- c The times taken by another brand of medication can be modelled by a normal distribution with standard deviation 5 minutes. Explain why the mean time taken by this brand of medication is more likely to be 18 minutes than 8 minutes. **(1 mark)**

- 4 The number of vehicles per minute passing a fixed point on a road is denoted by the random variable V

- a In order for V to be modelled by a Poisson distribution, explain why, when a truck transporting three new cars passes the fixed point, this observation must be recorded as one vehicle rather than four. **(1 mark)**

It is given that $V \sim \text{Po}(2.6)$ and that the speeds of the vehicles, in kmh^{-1} , are normally distributed with mean 64 and variance 28

- b Giving your answer correct to 3 significant figures, calculate an estimate of the probability that, in a randomly chosen 2 minute period, exactly 5 vehicles pass the fixed point and that exactly two of them are travelling at speeds greater than 66 kmh^{-1} **(7 marks)**

- 5 At a college, the height of a female student, F metres, is normally distributed with mean 1.63 and variance 0.002. The height of a male student, M metres, is normally distributed with mean 1.79 and variance 0.003

- a State, in terms of F and M , the random variable that can be used to represent the mean height of one randomly selected female student and one randomly selected male student. **(1 mark)**
- b Find the expectation of the random variable in part a. **(1 mark)**
- c Show that the variance of the random variable in part a is equal to 0.00125 **(1 mark)**
- d Find the probability that the mean height of two randomly selected females and two randomly selected males is at least 175cm. **(4 marks)**
- e The variance of the sum of the heights of a group of n female students and $n + 1$ male students is 0.058 m^2 . Find the total number of students in the group. **(3 marks)**

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- 6 a** The random variables B and G are such that $B \sim N(12.4, \sigma^2)$ and $G \sim N(6.8, \frac{2}{3}\sigma^2)$
- i** Explain how you know that $P(B + G < 21) > 0.5$ **(2 marks)**
- ii** Given that $P(B + G < 21) = 0.75$, find the value of σ **(3 marks)**
- iii** B and G respectively represent the ages of a large group of boys and a large group of girls. Find the variance of the sum of the ages of a randomly selected group of 6 boys and 6 girls, giving your answer to one decimal place. **(3 marks)**
- b** The random variables X and Y each have a normal distribution. It is known that the distribution of $3X$ is identical to the distribution of the differences between two randomly observed values of Y
- i** State the mean of X **(1 mark)**
- ii** Using correct mathematical notation, express the standard deviation of X in terms of the standard deviation of Y **(2 marks)**