

Further Statistics 1 Unit Test 2: Poisson and binomial distributions

- 1 The random variable X is the number of bacterial colonies on a Petri dish.
- a State two conditions under which a Poisson distribution is a suitable model for X . **(2 marks)**

The number of bacterial colonies follows a Poisson distribution with an average of 2 per cm^2 .

Find the probability that

- b there will be no bacterial colonies in a given 2cm^2 section of the dish **(3 marks)**
- c there will be at least four bacterial colonies in a given 3cm^2 section of the dish. **(3 marks)**
- 2 The table shows the numbers of telesales calls received by a private number over a period of 150 days.

Number of telesales calls	0	1	2	3	4
Number of days	51	54	36	6	3

- a Find the mean number of telesales calls received per day. **(1 mark)**
- b Use the mean from part a to estimate the expected frequencies for 0, 1, 2, 3 and 4 telesales calls to the private number modelled using a Poisson distribution. **(4 marks)**
- c By considering the observed and expected frequencies, state, with a reason, whether a Poisson distribution is a good model for this situation. **(2 marks)**
- 3 The monthly demand for a certain magazine at a small newsagent's shop has a Poisson distribution with mean 3.

The newsagent always orders four copies of the magazine for sale each month; any demand for the magazine in excess of four is not met.

- a Calculate the probability that the newsagent will not be able to meet the demand in a given month. **(3 marks)**
- b Write down the expectation and variance of the number of magazines sold in one month. **(1 mark)**
- c Determine the smallest number of copies of the magazine that the newsagent should order each month in order to meet the demand with a probability of at least 0.95. **(3 marks)**

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- 4 A particular genetic mutation occurs in 1.5% of a population.
- a Find the mean and the variance of the number of people with the genetic mutation in a random sample of 150 people. **(2 marks)**
- A genetic engineer tests a random sample of 150 people.
- She decides to conduct research into a possible cure for the mutation if more than five people in the sample have the mutation.
- b Explain why your answer to part a justifies the use of a Poisson approximation in this situation. **(1 mark)**
- c Using a suitable Poisson approximation, find the probability that the genetic engineer will conduct research into a possible cure. **(3 marks)**
- 5 The probability of a particular low energy lightbulb failing after fewer than 200 hours use is 0.003.
- A random sample of 1200 lightbulbs is taken.
- a Suggest a suitable approximating distribution that can be used to model the random variable X , the number of lightbulbs that fail after fewer than 200 hours. Give reasons for your choice. **(2 marks)**
- b Using your approximation in part a, find the probability that three or fewer lightbulbs fail in a single random sample of 1200 lightbulbs. **(2 marks)**
- Johan takes 10 independent random samples of 1200 lightbulbs.
- c Find the mean and standard deviation of the random variable Y , the number of samples that contain three or fewer lightbulbs that fail. **(4 marks)**
- 6 In a fibre optic communication system, transmission errors occur at a constant mean rate of 1.5 every 10 seconds.
- The presence of one error does not influence the presence of another.
- A network engineer believes that he has designed a new system that reduces the average number of errors and wants to test his belief.
- He records the number of errors in a randomly selected minute.
- a Write down the null and alternative hypotheses for the engineer's test. **(1 mark)**
- b Find, at the 5% level of significance, the critical region for the engineer's test. **(3 marks)**
- c State the actual significance level of the engineer's test. **(1 mark)**
- The engineer finds that there are four errors in his randomly selected minute.
- d Comment on this observation in light of your answer to part b. **(2 marks)**

- 7 On a particular stretch of road, the average number of cars passing a recording point is 3.4 in a 30 second interval.

The average number of vans passing the recording point in the same interval is 1.3.

- a Find the probability that there will be at least nine cars or vans passing the recording point in a randomly selected 30 second interval. **(3 marks)**

A traffic management company changes the routing of traffic through that part of the town and claims that the average number of cars and vans passing the recording point has changed.

They carry out a survey and find that nine cars or vans passed the recording point in a randomly selected 30 second interval.

- b Stating your hypotheses clearly, test, at the 10% level of significance, the management company's claim. **(4 marks)**