

Further Statistics 1 Unit Test 1: Discrete probability distributions

- 1 The discrete random variable X has a probability distribution function

$$P(X = x) = \frac{x}{k} \quad x = 1, 2, 3, 4$$

where k is a constant.

- a Show that $k = 10$. (2 marks)
b Show that $E(X) = 3$ and find the value of $E(X^2)$. (4 marks)
c Calculate the variance of X . (1 mark)

The random variable $Y = 3X - 2$

- d Find $E(Y)$ and $\text{Var}(Y)$. (2 marks)

- 2 Caitlin is designing a game of chance for her school fete.

She has a fair, five-sided spinner marked with the numbers 1, 2, 3, 4 and 5.

Players get 20 virtual points to have a go at spinning an even number.

If they are successful, they win their points back plus k times the number spun.

Points won can then be exchanged for small prizes.

Given that Caitlin's expected winnings per game is 3 points, show that $k = 7.5$. (4 marks)

- 3 The discrete random variable X has a probability distribution given in the table below.

Table 1

x	-3	-2	-1	0	1
$P(X = x)$	0.1	0.2	k	0.4	0.15

- a Write down the value of k . (1 mark)
b Find the expectation of X and show that $\text{Var}(X) = 1.51$. (4 marks)

The random variable $Y = 2X + 1$.

- c Find $P(X > 2Y)$. (2 marks)

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4 The discrete random variable X has a probability distribution function

$$P(X = x) = \begin{cases} k(x+1) & x = 1, 2 \\ k(x-1) & x = 4, 5 \end{cases}$$

- a Show that $k = \frac{1}{12}$ **(2 marks)**
- b Find the exact values of $E(X)$ and $\text{Var}(X)$. **(4 marks)**
- c Show that $\text{Var}(2 - 4X) = \frac{344}{9}$ **(2 marks)**

5 The discrete random variable X has a probability distribution given in the table below.

Table 2

x	-1	0	2	4	5
$P(X = x)$	0.2	0.1	a	b	0.3

The random variable Y is defined as $Y = 2X - 3$.

Given that $E(Y) = 2.4$,

- a Find the values of a and b . **(5 marks)**
- b Calculate $E(X^2)$ and show that $\text{Var}(X) = 5.61$. **(3 marks)**
- c Write down the value of $\text{Var}(Y)$. **(1 mark)**
- d Find $P(X - 3 > 2Y)$. **(2 marks)**

6 The discrete random variable X has a probability distribution given by

Table 3

x	-1	0	1	2	3	4
$P(X=x)$	a	b	a	b	a	c

The random variable Y is defined as $Y = 4 - 3X$.

Given that $E(Y) = -1.7$ and the $P(Y < 0) = 0.6$,

a Calculate the values of a , b and c . **(7 marks)**

Given that $\text{Var}(Y) = 27.81$,

b Find the exact value of $\text{Var}(X)$. **(2 marks)**

c Find $P(X < Y)$. **(2 marks)**