Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
1	$\frac{dy}{dx} + 2y \cot x = 2\cos x$	**B1**	9.1	5th Find particular solutions to first order differential equations using an integrating factor
	I.F. $e^{\int 2\cot x\,dx} = \sin^2 x$	**M1**	9.1	
	$\frac{d}{dx}(\sin^2 x\, y) = 2\sin^2 x \cos x$	**A1**	9.1	
	$\sin^2 x\, y = \frac{2}{3}\sin^3 x + c$	**M1**	9.1	
	$\frac{1}{4} = \frac{1}{12} + c$	**A1**	9.1	
	$c = \frac{1}{6}$	**A1**	9.2	
	$y = \frac{2}{3}\sin x + \frac{1}{6}\operatorname{cosec}^2 x$	**A1**	9.2	
				(7 marks)

Notes

B1 Rearranges equation

M1 Use of integrating factor

A1 Correct IF

M1 Multiplies by IF

A1 Integrates

A1 Evaluates c

A1 cao

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
2	$\lambda^2 - 6\lambda + 9 = 0$	M1	9.4	5th
	$(\lambda - 3)^2 = 0$	A1	9.4	Solve second order homogeneous differential equations using the auxiliary equation
	$\lambda = 3$	A1	9.6	
	$y = (Ax + B)\mathrm{e}^{3x}$	B1	9.6	
				(4 marks)

Notes

M1 Use of auxilary equation

A1 Solves correctly

A1 Correct power

B1 (Ax + B)

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
3	C.F.			6th
	$\lambda^2 - 4\lambda + 3 = 0$	M1	9.6	Solve second order non-homogeneous differential equations using a particular integral
	$(\lambda - 3)(\lambda - 1) = 0$			
	$\lambda = 3,\ \lambda = 1$	A1	9.6	
	$\text{C.F.} = Ae^{3x} + Be^{x}$	A1	9.6	
	P.I. Try			
	$y = kxe^{3x}$	M1	9.6	
	$\frac{dy}{dx} = ke^{3x} + 3kxe^{3x}$			
	$\frac{dy}{dx} = ke^{3x}(3x+1)$	A1	9.6	
	$\frac{d^2y}{dx^2} = 3ke^{3x}(3x+1) + 3ke^{3x}$	A1	9.6	
	$3ke^{3x}(3x+1) + 3ke^{3x} - 4ke^{3x}(3x+1) + 3kxe^{3x} = 4e^{3x}$			
	$2ke^{3x} = 4e^{3x}$			
	$k = 2$	A1	9.6	
	General solution			
	$y = 2xe^{3x} + Ae^{3x} + Be^{x}$	M1	9.6	
	$\frac{dy}{dx} = 6xe^{3x} + 2e^{3x} + 3Ae^{3x} + Be^{x}$			
	Substituting	A1	9.6	
	$A + B = 0$			
	$3A + B + 2 = 0$	A1	9.6	
	$A = -1,\ B = 1$			
	Solution is			
	$y = (2x-1)e^{3x} + e^{x}$	A1	9.6	
				(11 marks)

Notes
M1 Use of auxilary equation
A1 Solves correctly
A1 C.F. correct
M1 Correct form for P.I.
A1 First differential correct
A1 Second differential correct
A1 Substitutes and solves for k
M1 Attempts general solution
A1 Differentiates
A1 Forms simultaneous equations
A1 cao

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
4ai	$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.1\cos 5t + 0.05\sin 5t$	**M1**	9.6	6th Solve second order non-homogeneous differential equations using a particular integral
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -0.5\sin 5t + 0.25\cos 5t$	**A1**	9.6	
	Substituting $-0.5\sin 5t + 0.25\cos 5t + 10(0.1\cos 5t + 0.05\sin 5t) + 125(0.02\sin 5t - 0.01\cos 5t)$ $= 2.5\sin 5t$	**A1**	9.6	
		(3)		
4aii	$\lambda^2 + 10\lambda + 125 = 0$	**M1**	9.6	6th Solve second order non-homogeneous differential equations using a particular integral
	$\lambda = \frac{-10 \pm \sqrt{10^2 - 4\times 125}}{2}$	**A1**	9.6	
	$\lambda = -5 \pm 10i$	**A1**	9.6	
	C.F. $x = \mathrm{e}^{-5t}(A\cos 10t + B\sin 10t)$	**A1**	9.3	
	General solution $x = \mathrm{e}^{-5t}(A\cos 10t + B\sin 10t) + 0.02\sin 5t - 0.01\cos 5t$	**B1**	9.3	
		(5)		

4b	$\frac{dx}{dt} = -5e^{-5t}(A\cos 10t + B\sin 10t)$ $+10e^{-5t}(-A\sin 10t + B\cos 10t)$ $+0.1\cos 5t + 0.05\sin 5t$	**M1**	9.3	6th Solve second order non-homogeneous differential equations using a particular integral
	$0 = A - 0.01$	**M1**	9.3	
	$A = 0.01$	**A1**	9.3	
	$0 = -5A + 10B + 0.1$ $B = -0.005$	**A1**	9.3	
	$x = e^{-5t}(0.01\cos 10t - 0.005\sin 10t) + 0.02\sin 5t$ $-0.01\cos 5t$			
		(4)		
4c	$t \to \infty,\ e^{-5t} \to 0$ $x = 0.02\sin 5t - 0.01\cos 5t$ So the mass will oscillate (with amplitude $0.01\sqrt{5}$) it tends to SMH	**B1**	9.3	8th Model damped oscillations with a forced vibration using second order differential equations and interpret their solutions
		(1)		
				(13 marks)

Notes

4ai **M1** Attempts to differentiate

A1 First and second differentials correct

A1 cao (A.G.)

4aii **M1** Auxiliary equation used

A1 Use of quadratic formula

A1 cao

A1 C.F. correct

B1 General solution correct

4b **M1** Differentiates

M1 Substitutes values

A1 Solves for A and B

A1 cao

4c **B1** Shows that the mass tends to simple harmonic motion or an oscillation with constant amplitude and period.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
5a	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 3\frac{\mathrm{d}x}{\mathrm{d}t} - 5\frac{\mathrm{d}y}{\mathrm{d}t}$	**M1**	9.9	8th Solve pairs of coupled first order differential equations and interpret the models in context
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 3\frac{\mathrm{d}x}{\mathrm{d}t} - 5(y - 1.6x)$	**A1**	9.9	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 3\frac{\mathrm{d}x}{\mathrm{d}t} - \left(3x - \frac{\mathrm{d}x}{\mathrm{d}t}\right) + 8x$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 4\frac{\mathrm{d}x}{\mathrm{d}t} - 5x = 0$	**A1***	9.9	
		(3)		
5b	$\lambda^2 - 4\lambda - 5 = 0$	**M1**	9.4	8th Solve pairs of coupled first order differential equations and interpret the models in context
	$(\lambda - 5)(\lambda + 1) = 0$ $\lambda = 5, \quad \lambda = -1$	**A1**	9.4	
	$x = Ae^{5t} + Be^{-t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 5Ae^{5t} - Be^{-t}$	**M1**	9.6	
	Initially $\frac{\mathrm{d}x}{\mathrm{d}t} = 3$	**A1**	9.6	
	Substituting $A + B = 21$ $5A - B = 3$ $A = 4,\ B = 17$ $x = 4e^{5t} + 17e^{-t}$	**A1**	9.6	
		(5)		

5c	$y=\frac{3}{5}x-\frac{1}{5}\frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	9.1	8th Solve pairs of coupled first order differential equations and interpret the models in context
	$\frac{\mathrm{d}x}{\mathrm{d}t}=20\mathrm{e}^{4t}-17\mathrm{e}^{-t}$	**M1**	9.1	
	$y=\frac{3}{5}(4\mathrm{e}^{4t}+17\mathrm{e}^{-t})-\frac{1}{5}(20\mathrm{e}^{4t}-17\mathrm{e}^{-t})$	**A1**	9.1	
	$y=13.6\mathrm{e}^{-t}-1.6\mathrm{e}^{5t}$	**A1**	9.1	
		(4)		
5d	$13.6\mathrm{e}^{-t}-1.6\mathrm{e}^{5t}=0$	**M1**	9.9	8th Solve pairs of coupled first order differential equations and interpret the models in context
	$\mathrm{e}^{6t}=\frac{13.6}{1.6}$			
	$t=\frac{1}{6}\ln\frac{13.6}{1.6}$	**A1**	9.9	
	$t=0.35667\ldots$			
	Therefore 35.67 years or 35 years 8 months	**B1**	9.9	
	$x=356\ 999$			
		(3)		
				(15 marks)

Notes

5a **M1** Differentiates

A1 Substitutes

A1* Correct simplification (A.G.)

5b **M1** Attempts to solve auxiliary equation

A1 General solution for *x*

M1 Differentiates

A1 Correct initial condition

A1 Solves for A and B

5c **M1** Rearranges the given expression.

M1 Differentiates their expression for *x* and attempts to substitute

A1 Correct unsimplified substitution

A1 Correct solution for *y*

5d **M1** Attempts to solve equation

A1cao for *t*

B1 cao for *x*