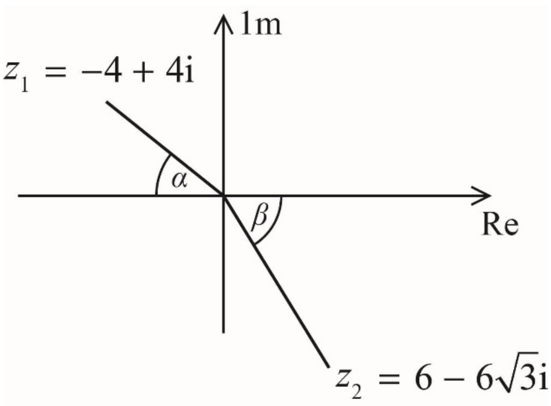
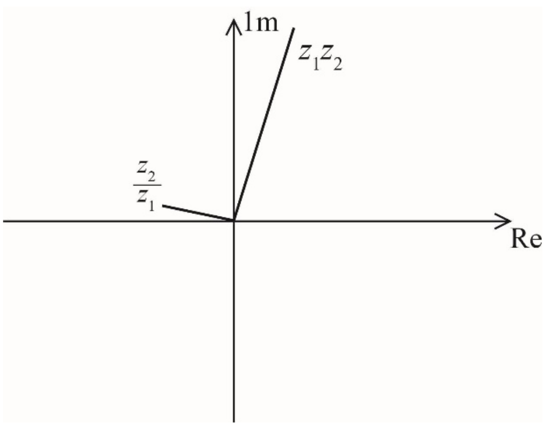
Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
1a	$\lvert z_1\rvert = \sqrt{(-4)^2 + 4^2}$	M1	1.1a	2nd Calculate the modulus/ argument of a complex number
	$\lvert z_1\rvert = 4\sqrt{2}$			
	$\lvert z_2\rvert = \sqrt{6^2 + \left(-6\sqrt{3}\right)^2}$			
	$\lvert z_2\rvert = 12$	A1	1.1b	
	 			
	$\tan\alpha = 1$ $\alpha = \frac{\pi}{4}$	M1	1.1a	
	$\arg z_1 = \frac{3\pi}{4}$			
	$\tan\beta = \sqrt{3}$			
	$\beta = \frac{\pi}{3}$			
	$\arg z_2 = \frac{-\pi}{3}$	A1	1.1b	
	Hence $z_1 = 4\sqrt{2}e^{\frac{3\pi}{4}i}$ and $z_2 = 12e^{-\frac{\pi}{3}i}$	A1	1.1b	
		(5)		

1b	$r = 4\sqrt{2} \times 12 = 48\sqrt{2}$ $\theta = \frac{3\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12}$ Hence $z_1 z_2 = 48\sqrt{2}e^{\frac{5\pi}{12}i}$	**Alt method:** $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$ $\lvert z_1 z_2 \rvert = 48\sqrt{2}$ $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ $\arg(z_1 z_2) = \frac{3\pi}{4} + \frac{-\pi}{3}$ $\arg(z_1 z_2) = \frac{5\pi}{12}$ Hence $z_1 z_2 = 48\sqrt{2}e^{\frac{5\pi}{12}i}$	**M1** **A1**	1.1a 1.1b	4th Know how to multiply/divide complex numbers in exponential form
	$r = \frac{12}{4\sqrt{2}} = \frac{3\sqrt{2}}{2}$ $\theta = -\frac{\pi}{3} - \frac{3\pi}{4} = -\frac{13\pi}{12}$ Hence $\frac{z_2}{z_1} = \frac{3\sqrt{2}}{2}e^{\frac{11\pi}{12}i}$	$\left\lvert \frac{z_2}{z_1} \right\rvert = \frac{\lvert z_2 \rvert}{\lvert z_1 \rvert}$ $\left\lvert \frac{z_2}{z_1} \right\rvert = \frac{3\sqrt{2}}{2}$ $\arg\left(\frac{z_2}{z_1}\right) = \arg(z_2) - \arg(z_1)$ $\arg\left(\frac{z_2}{z_1}\right) = \frac{-\pi}{3} - \frac{3\pi}{4}$ $\arg\left(\frac{z_2}{z_1}\right) = \frac{-13\pi}{12}$ $\arg\left(\frac{z_2}{z_1}\right) = \frac{11\pi}{12}$ Hence $\frac{z_2}{z_1} = \frac{3\sqrt{2}}{2}e^{\frac{11\pi}{12}i}$	**M1** **A1** **A1**	1.1a 1.1b 2.5	
			(5)		

1c		**B1** **B1**	2.5 2.5	2nd Represent complex numbers on an Argand diagram
		(2)		
(12 marks)				

Notes

1a: **M1** for use of Pythagoras

A1 for both values correct

M1 for use of trigonometry to find α and/or β

A1 for correct values of arguments

A1 for both correct final answers in correct form

1b: **M1** for multiplying moduli and adding angles

A1 cao

M1 for dividing moduli and subtracting angles

A1 for correct r and $-\frac{13\pi}{12}$

A1 cao

1c: **B1B1** for each correct

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
2a	$\cos 12\theta + \mathrm{i}\sin 12\theta$	**B1**	1.1b	4th Remember and be able to use de Moivre's theorem
		(1)		
2b	$(\cos 7\theta + \mathrm{i}\sin 7\theta)(\cos(-6\theta) + \mathrm{i}\sin(-6\theta))$ $\cos\theta + \mathrm{i}\sin\theta$	**M1** **A1**	1.1a 1.1b	4th Remember and be able to use de Moivre's theorem
		(2)		
2c	$(\cos 8\theta + \mathrm{i}\sin 8\theta)(\cos(-9\theta) + \mathrm{i}\sin(-9\theta))$ $(\cos(-\theta) + \mathrm{i}\sin(-\theta))$ $\cos\theta - \mathrm{i}\sin\theta$	**M1** **A1** **A1**	1.1a 1.1b 1.1b	4th Remember and be able to use de Moivre's theorem
		(3)		
				(6 marks)

Notes

2a: **B1** accept terms in either order

2b: **M1** for product of two complex numbers with index of –2 or -6θ seen

A1 accept terms in either order

2c: **M1** for product of two complex numbers with 8 and 9 or –9 seen

A1 accept terms in either order

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
3	$(\cos\theta + \mathrm{i}\sin\theta)^7 = \cos 7\theta + \mathrm{i}\sin 7\theta$	M1	1.1a	5th Be able to derive multiple angle formulae and expressions using de Moivre's theorem
	$(\cos\theta + \mathrm{i}\sin\theta)^7 = \cos^7\theta + 7\mathrm{i}\cos^6\theta\sin\theta - 21\cos^5\theta\sin^2\theta$ $-35\mathrm{i}\cos^4\theta\sin^3\theta + 35\cos^3\theta\sin^4\theta$ $+21\mathrm{i}\cos^2\theta\sin^5\theta - 7\cos\theta\sin^6\theta - \mathrm{i}\sin^7\theta$	M1 A1	1.1a 1.1b	
	$\cos 7\theta = \cos^7\theta - 21\cos^5\theta(1-\cos^2\theta)$ $+35\cos^3\theta(1-\cos^2\theta)^2$ $-7\cos(1-\cos^2\theta)^3$	M1 M1	1.1b 1.1b	
	Hence $\cos 7\theta = 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta$	A1	1.1b	
				(6 marks)

Notes

M1 for use of de Moivre's theorem

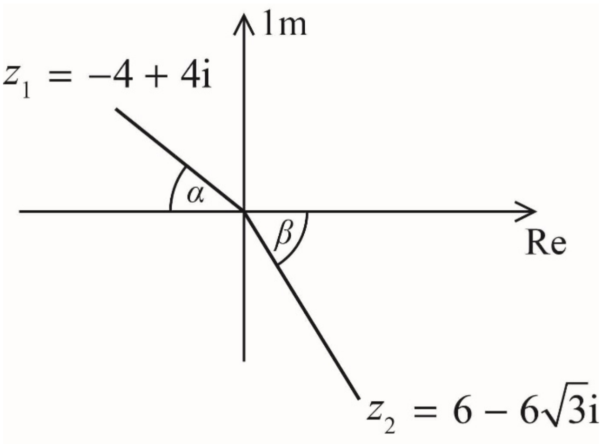
M1 for use of binomial expansion

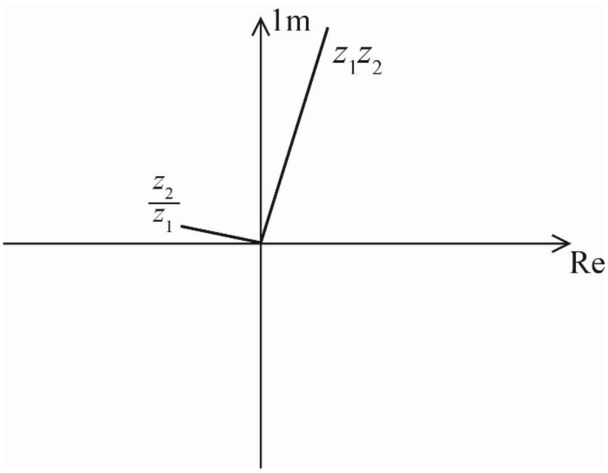
A1 for correct expansion

M1 for choosing real terms

M1 for replacing $\sin^2\theta$

A1 cao

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
4a	$z^5 = \left(\sqrt{2}\right)^5 (\cos 2n\pi + \mathrm{i} \sin 2n\pi);\ n = 0,1,2,3,4$	B1	1.1a	8th
	$z = \sqrt{2}\left(\cos\frac{2n\pi}{5} + \mathrm{i}\sin\frac{2n\pi}{5}\right);\ n = 0,1,2,3,4$	M1	1.1b	Be able to find the nth roots of equations of the form $z = r\mathrm{e}^{\mathrm{i}\theta}$ and know that they form the vertices of a regular n-gon in the Argand diagram
	$z = \sqrt{2},\ \sqrt{2}\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right),$	A1	1.1b	
	$\sqrt{2}\left(\cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5}\right), \sqrt{2}\left(\cos\frac{-4\pi}{5} + \mathrm{i}\sin\frac{-4\pi}{5}\right),$	A1	1.1b	
	$\sqrt{2}\left(\cos\frac{-2\pi}{5} + \mathrm{i}\sin\frac{-2\pi}{5}\right)$	B1	2.5	
		B1	2.5	
		(6)		

4b	$z^3 = \frac{1}{\sqrt{2}}(1-\text{i})$	**B1**	1.1a	8th Be able to find the *n*th roots of equations of the form $z = r\text{e}^{\text{i}\theta}$ and know that they form the vertices of a regular *n*-gon in the Argand diagram
	$z^3 = \left(\cos\frac{-\pi}{4} + \text{i}\sin\frac{-\pi}{4}\right)$	**M1**	1.1b	
	$z^3 = \left(\cos\frac{(8n-1)\pi}{4} + \text{i}\sin\frac{(8n-1)\pi}{4}\right), n = 0,1,2$			
	$z^3 = \left(\cos\frac{(8n-1)\pi}{12} + \text{i}\sin\frac{(8n-1)\pi}{12}\right), n = 0,1,2$	**M1**	1.1b	
	$z = \left(\cos\frac{-\pi}{12} + \text{i}\sin\frac{-\pi}{12}\right), \left(\cos\frac{7\pi}{12} + \text{i}\sin\frac{7\pi}{12}\right),$ $\left(\cos\frac{-3\pi}{4} + \text{i}\sin\frac{-3\pi}{4}\right)$	**A1**	1.1b	
		**B1** **B1**	2.5 2.5	
		(6)		
				(12 marks)

Notes

4a: **B1** for rewriting equation

M1 for use of de Moivre's theorem

A1 for at least four roots found by substituting n

A1 for all roots correct in the interval of the principal argument

B1 for all lines the same length

B1 for correct spacing around the circle

4b: **B1** for rewriting equation

M1 for rewriting in terms of $\cos\theta$ and $\sin\theta$

M1 for use of de Moivre's theorem

A1 for substitutes values for n and use of the interval of the principal argument

B1 for all roots the same length

B1 for correct spacing around the circle

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
5a	$z = \cos\theta + \text{i}\sin\theta$ $z^n = (\cos\theta + \text{i}\sin\theta)^n = \cos n\theta + \text{i}\sin n\theta$ $z^{-n} = (\cos\theta + \text{i}\sin\theta)^{-n} = \cos(-n\theta) + \text{i}\sin(-n\theta)$ $= \cos n\theta - \text{i}\sin n\theta$ Subtracting: $z^n - z^{-n} = z^n - \frac{1}{z^n} = 2\text{i}\sin n\theta$	M1 A1 B1	1.1b 2.1 1.2	4th Remember and be able to use de Moivre's theorem
		(3)		
5b	$\left(z - \frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{-1}{z}\right) + 10z^3\left(\frac{-1}{z}\right)^2 + 10z^2\left(\frac{-1}{z}\right)^3$ $+ 5z\left(\frac{-1}{z}\right)^4 + \left(\frac{-1}{z}\right)^5$ $\left(z - \frac{1}{z}\right)^5 = \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$ $(2\text{i}\sin\theta)^5 = 2\text{i}\sin 5\theta - 5(2\text{i}\sin 3\theta) + 10(2\text{i}\sin\theta)$ $32\text{i}\sin^5\theta = 2\text{i}\sin 5\theta - 10\text{i}\sin 3\theta + 20\text{i}\sin\theta$ $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$	M1 A1 M1 A1 M1 A1	3.1a 1.1b 1.1b 1.1b 1.1b 1.1b	5th Be able to derive multiple angle formulae and expressions using de Moivre's theorem
		(6)		
5c	$\int_0^{\frac{\pi}{3}} \sin^5\theta \, \text{d}\theta = \frac{1}{16}\int_0^{\frac{\pi}{3}} (\sin 5\theta - 5\sin 3\theta + 10\sin\theta)\text{d}\theta$ $I = \frac{1}{16}\left[-\frac{1}{5}\cos 5\theta + \frac{5}{3}\cos 3\theta - 10\cos\theta\right]_0^{\frac{\pi}{3}}$ $I = \frac{1}{16}\left[\left(-\frac{1}{10} - \frac{5}{3} - 5\right) - \left(-\frac{1}{5} + \frac{5}{3} - 10\right)\right]$ $I = \frac{53}{480}$	B1 M1 A1 M1 A1	3.1a 1.1b 1.1b 1.1b 1.1b	5th Be able to derive multiple angle formulae and expressions using de Moivre's theorem
		(5)		
				(14 marks)

Notes

5a: **B1** (may be implied) for stating $z = \cos\theta + \mathrm{i}\sin\theta$

M1 for correct use of de Moivre's theorem

A1 cso – must be clearly seen as A.G

5b: **M1** for use of binomial expansion

A1 all terms correct – may be simplified or unsimplified

M1 for attempting to simplify and collect terms

A1 cao

M1 for substituting expression given in **a** (allow *n*)

A1 cso – must be clearly seen as A.G

5c: **B1** for use of correct expression

M1 for integrating with sin changed to –cos at least once

A1 all terms correct

M1(dep) for substituting upper and lower limits into *their* expression

A1 cao