Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
1a	$\frac{3}{2-x-x^2}=\frac{3}{(1-x)(2+x)}$	B1	5.4	4th
	$=\frac{A}{1-x}+\frac{B}{2+x}$	M1	5.4	Integrate functions using partial fractions with linear denominators
	$A(2+x)+B(1-x)=3$	A1	5.4	
	$A=1, B=1$			
		(3)		
1b	$\int\frac{3}{2-x-x^2}\,\mathrm{d}x=\int\left(\frac{1}{1-x}+\frac{1}{2+x}\right)\mathrm{d}x$	M1	5.4	4th
	$=-\ln\lvert 1-x\rvert+\ln\lvert 2+x\rvert+c$	A1	5.4	Integrate functions using partial fractions with linear denominators
	$=\ln A\left\lvert\frac{2+x}{1-x}\right\rvert$	A1	5.4	
		A1	5.4	
		(4)		
				(7 marks)

Notes

1b **M1** Integrates to obtain logs

A1 One term correct

A1 Both terms correct

A1 Combines terms, including the "$+c$", using laws of logarithms

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
2a	$\int_1^{\infty} \frac{1}{1+x^2}\,dx = \lim_{c\to\infty}\left[\arctan x\right]_1^c$	M1	5.2	5th
	$= \lim_{c\to\infty} \arctan c - \arctan 1$	M1	5.2	Evaluate integrals which extend to infinity
	$= \frac{\pi}{2} - \frac{\pi}{4}$	A1	5.2	
	$= \frac{\pi}{4}$	A1	5.2	
		(4)		
2b	$\int_0^9 \frac{1}{(x-1)^{\frac{2}{3}}}\,dx = \int_1^9 \frac{1}{(x-1)^{\frac{2}{3}}}\,dx + \int_0^1 \frac{1}{(x-1)^{\frac{2}{3}}}\,dx$	M1	5.2	6th
	$= \lim_{c\to 1+}\left[3(x-1)^{\frac{1}{3}}\right]_c^9 + \lim_{c\to 1-}\left[3(x-1)^{\frac{1}{3}}\right]_0^c$	M1	5.2	Integrate functions across limits which include values where the function is undefined
	$= (6-0) + (0-(-3))$	A1	5.2	
	$= 9$	A1	5.2	
		A1	5.2	
		(5)		
				(9 marks)

Notes

2a **M1** Use of limit

M1 Integrates

A1 Evaluates values

A1 cao

2b **M1** Splits integral into two parts

M1 Integrates

A1 Uses limits twice

A1 Evaluates values

A1 cao

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
3a	$\text{Volume} = \pi\int x^2 \frac{dy}{d\theta} d\theta$	M1	5.1	7th Find volumes of revolution when given functions in parametric form
	$\frac{dy}{d\theta} = 8\cos\theta$		5.1	
	$V = \pi\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (4\cos\theta)^2\, 8\cos\theta\, d\theta$	A1	5.1	
	$V = 128\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\theta\, d\theta$	A1		
		(3)		
3b	$V = 128\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\theta - \cos\theta\sin^2\theta)\, d\theta$	M1	5.1	7th Find volumes of revolution when given functions in parametric form
		M1	5.1	
	$= 128\pi\left[\sin\theta - \frac{1}{3}\sin^3\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$	A1	5.1	
	$= 128\pi\left(\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right)$	A1	5.1	
	$= \frac{512\pi}{3}$	A1	5.1	
		(5)		
3c	$\text{Mass} = \frac{512\pi}{3}\times 8.7$	M1	5.1	7th Find volumes of revolution when given functions in parametric form
	$= 4664.64$ g	A1	5.1	
		(2)		
	(10 marks)			

Notes

3a **M1** Use of parametric form of volume

A1 Differential correct

A1 Substitutes into formula correctly (A.G.)

3b **M1** Substitutes for $\cos^2\theta$

M1 Attempts to integrate

A1 Both terms correct

A1 Evalutes

A1 cao

3c **M1** Uses Mass = Density × Volume

A1 cao (3 sf or better)

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
4ai	$\frac{dx}{dy} = 2\sec^2 2y$	M1	5.5	6th
		A1	5.5	Be able to differentiate inverse trigonometric functions using chain, product and quotient rules
	$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$	M1	5.5	
	$\frac{dy}{dx} = \frac{1}{2\sec^2 2y}$	M1	5.5	
	$= \frac{1}{2(1+\tan^2 2y)}$	A1	5.5	
	$= \frac{1}{2(1+x^2)}$			
		(5)		
4aii	$\frac{dy}{dx} = \frac{2}{3(1+4x^2)}$	B1	5.5	6th
		B1	5.5	Be able to differentiate inverse trigonometric functions using chain, product and quotient rules
		(2)		

4b	$f'(x) = 4x - 3 + \frac{2}{(1+4x^2)}$ $f'(x) = 0$, when $(4x-3)(1+4x^2) + 2 = 0$ $16x^3 - 12x^2 + 4x - 1 = 0$ $x = \frac{1}{2}$, $f(x) = \frac{\pi}{4}$ $f''(x) = 4 - \frac{16x}{(1+4x^2)^2}$ $f''\left(\frac{1}{2}\right) = 2$ Therefore minimum	**M1** **M1** **M1** **A1** **A1** **M1** **A1**	5.5 5.5 5.5 5.5 5.5 5.5 5.5	6th Be able to differentiate inverse trigonometric functions using chain, product and quotient rules
		(7)		
				(14 marks)

Notes

4ai **M1** Uses standard result and chain rule

A1 Correct $\frac{dx}{dy}$

M1 Uses reciprocal

M1 Uses $1 + \tan^2 2y$

A1 cao

4aii **B1** Uses chain rule or previous result

B1 cao

4b **M1** Differentiates

M1 Sets $f'(x) = 0$

M1 Obtains equation as polynomial in x

A1 $x = \frac{1}{2}$

A1 $f(x) = \frac{\pi}{4}$

M1 Differentiates again

A1 Shows $f''\left(\frac{1}{2}\right) > 0$ and conclusion

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress Descriptor
5a	$\frac{\mathrm{d}x}{\mathrm{d}u}=\sqrt{2}\cos u$	B1	5.6	6th Choose appropriate trigonometric substitutions to integrate given functions
	$(2-x^2)^{\frac{3}{2}}=2\sqrt{2}\cos^3 u$	B1	5.6	
	$I=\int\frac{4\sqrt{2}\cos u}{2\sqrt{2}\cos^3 u}\mathrm{d}u$	M1	5.6	
	$I=\int 2\sec^2 u\ \mathrm{d}u$ $=2\tan u+c$	A1	5.6	
	$=\frac{2x}{\sqrt{2-x^2}}+c$	A1	5.6	
		(5)		
5b	Mean value			6th Evaluate the mean value of a function
	$=\frac{1}{\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}}\int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{6}}{2}}\mathrm{f}(x)\ \mathrm{d}x$	M1	5.3	
	$=\frac{\sqrt{2}\left(\sqrt{3}+1\right)}{2}\left[\frac{2x}{\sqrt{2-x^2}}\right]_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{6}}{2}}$	A1	5.3	
	$=\frac{\sqrt{2}\left(\sqrt{3}+1\right)}{2}\left(\sqrt{12}-\frac{2}{\sqrt{3}}\right)$ $=\frac{2\left(3\sqrt{2}+\sqrt{6}\right)}{3}$	A1	5.3	
		(3)		
5c	Mean value	M1	5.3	6th Evaluate the mean value of a function
	$=\frac{2\left(3\sqrt{2}+\sqrt{6}\right)}{3}+\frac{\sqrt{3}}{3}$ $=\frac{2\sqrt{2}+\sqrt{3}+\sqrt{3}}{3}$	A1	5.3	
		(2)		
				(10 marks)

Notes

5a **B1** Differentiates

B1 Substitutes for x

M1 Simplifies expression and integrates

A1 Answer in terms of u

A1 Answer in terms of x

5b **M1** Correct use of formula

A1 Substitutes values into limits

A1 Simplifies correctly

5c **M1** Adds $\frac{\sqrt{3}}{3}$

A1 cao

Alternative for **5b**

Changes limits to u $\frac{\sqrt{2}}{2} \Rightarrow \frac{\pi}{6}, \frac{\sqrt{6}}{2} \Rightarrow \frac{\pi}{3}$