Core Pure (A level/Year 2) Unit Test 1: Complex numbers

1 Given that $z_1 = -4 + 4\text{i}$ and $z_2 = 6 - 6\sqrt{3}\text{i}$

a Express z_1 and z_2 in the form $z = r\text{e}^{\text{i}\theta}$ where $r > 0$ and $-\pi \leqslant \theta \leqslant \pi$ **(5 marks)**

b Hence, or otherwise, calculate $z_1 z_2$ and $\frac{z_2}{z_1}$, giving your answers in the form $z = r\text{e}^{\text{i}\theta}$ where $r > 0$ and $-\pi \leqslant \theta \leqslant \pi$ **(5 marks)**

c Display $z_1 z_2$ and $\frac{z_2}{z_1}$ on an Argand diagram. **(2 marks)**

2 Use de Moivre's theorem to fully simplify

a $(\cos 4\theta + \text{i} \sin 4\theta)^3$ **(1 mark)**

b $\dfrac{\cos 7\theta + \text{i} \sin 7\theta}{(\cos 3\theta + \text{i} \sin 3\theta)^2}$ **(2 marks)**

c $(\cos 2\theta + \text{i} \sin 2\theta)^4 \, (\cos 3\theta - \text{i} \sin 3\theta)^3$ **(3 marks)**

3 Express $\cos 7\theta$ in terms of powers of $\cos\theta$, giving your answer in its simplest form. **(6 marks)**

4 Solve, giving your answers, where appropriate, in the form $r(\cos\theta + \text{i}\sin\theta)$ where $-\pi < \theta \leqslant \pi$

a $z^5 = 4\sqrt{2}$ showing the solutions on an Argand diagram. **(6 marks)**

b $\sqrt{2}z^3 - 1 + \text{i} = 0$ showing the solutions on an Argand diagram. **(6 marks)**

5 A complex number z has modulus 1 and argument θ

a Show that $z^n - \dfrac{1}{z^n} = 2\text{i} \sin n\theta$ **(3 marks)**

b Hence show that

$$\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$$ **(6 marks)**

c Hence find the exact value of

$$\int_0^{\frac{\pi}{3}} \sin^5\theta \, \text{d}\theta$$ **(5 marks)**