1 An economist predicts that t years after a company's appointment of a new managing director, the number of subscribers to the company's social media website S, in millions, will be given by

$$S = 5\cosh 2t - 3\sinh 2t$$

a Use calculus to show clearly that the minimum number of subscribers predicted by the economist is exactly 4 million. **(6 marks)**

b Justify, by further differentiation, that this value of S is a minimum. **(2 marks)**

2 **a** Using the definitions for $\cosh x$ and $\sinh x$ in terms of exponentials, show that

$$\cosh^2 x - \sinh^2 x = 1$$

(3 marks)

b Find the exact value of x for which

$$2\tanh x - \cosh x = 0$$

giving your answer as a natural logarithm. **(5 marks)**

3 The curves C_1 and C_2 have equations $y = 12\sinh\frac{x}{5}$ and $y = 6e^{-\frac{x}{5}} - 1$ respectively.

a Sketch the graphs of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the exact coordinates of any points where the curves cross the axes. **(5 marks)**

b Use algebra to show that the x-coordinate of the point of intersection of the curves C_1 and C_2 is $5\ln k$ where k is a rational number to be found. **(5 marks)**

4 **a** A function $y = \text{artanh } x$ has domain $\{x \in \square : x \in [0, \frac{12}{13}]\}$ **(2 marks)**

and range $\{ y \in \mathbb{R} : y \in [0, \ln k] \}$

Use the logarithmic form of artanh x to show that $k = 5$

b Differentiate $x \text{ artanh } 3x$ with respect to x **(3 marks)**

c Hence, or otherwise, use algebraic integration to find the exact value of

$$\int_0^{\frac{1}{9}} \text{artanh } 3x \, \mathrm{d}x$$

giving your answer in the form $\frac{1}{18}\ln k$ where k is a rational number to be found. **(6 marks)**

5 **a** Use the substitution $x = 3 \cosh u$ to prove that

$$\int \sqrt{x^2 - 9} \, \mathrm{d}x = \frac{1}{2}x\sqrt{x^2 - 9} - \frac{9}{2} \text{arcosh} \frac{x}{3} + c$$

(8 marks)

b A veterinary science student is investigating how cats respond to being called and their subsequent movement. The student conducts experiments where stationary cats are individually called by their owners. As soon as a cat begins to move, its velocity in m s^{-1} is modelled by the equation

$$v = \sqrt{t^2 - 9}$$

where t is the time in seconds after the cat is called.

The cat in a first experiment, Harry, begins to move 2 seconds after being called.

i Explain clearly why the model is unsuitable for Harry's movement. **(1 mark)**

The cat in a second experiment, Holly, begins to move 3 seconds after being called. The owner stands 15 m away from Holly's initial position.

Assuming that Holly moves in a straight line towards her owner,

ii use part **a** to find, in its simplest form, the exact distance in metres that the model predicts Holly will be from her owner 5 seconds after being called. **(4 marks)**