1 **a** Prove that

$$\sum_{r=1}^{n} \frac{3}{r(r+1)} = \frac{an}{n+1}, \ n \in \square$$

where a is a constant to be found. **(5 marks)**

b Find the value of $\sum_{r=1}^{50} \frac{3}{r(r+1)}$, giving your answer as an exact fraction. **(1 mark)**

c Find an expression in its simplest form for

$$\sum_{r=n}^{2n} \frac{3}{r(r+1)}$$

(4 marks)

2 **a** Simplify

$$r^2(r+1)^2 - (r-1)^2 r^2$$

(2 marks)

b Use the method of differences to show that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ **(3 marks)**

3 **a** Express in partial fractions

$$\frac{2}{(r+2)(r+3)(r+4)}$$

(3 marks)

b Show that

$$\sum_{r=1}^{n} \frac{2}{(r+2)(r+3)(r+4)} = \frac{n(n+b)}{c(n+3)(n+4)}$$

where a, b and c are constants to be found. **(5 marks)**

4 $\mathrm{f}(x) = \sec x$

a Show that $\mathrm{f}''(x) = 2\sec^3 x - \sec x$ **(4 marks)**

b Obtain the first three non-zero terms of the series expansion of $\sec x$, in ascending powers of x. **(5 marks)**

5 **a** Use standard results to show that the first four terms of the series expansion of e^{2-x} in ascending powers of x can be expressed as

$$e^2\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}\right)$$ **(3 marks)**

b Use standard results to obtain the first four non-zero terms of the series expansion of

$$\sin(3x^2)$$ **(3 marks)**

6 **a** Show that the series expansion of $\ln\left(\frac{1+3x}{1-2x}\right)$ in ascending powers of x, up to and including the term in x^4, is $5x-\frac{5x^2}{2}+\frac{35x^3}{3}-\frac{65x^4}{4}$ **(5 marks)**

b State the range of values of x for which the answer to part **a** is valid. **(1 mark)**

c By choosing a suitable value for x, use the expansion from part **a** to obtain an estimate for the value of $\ln\frac{1}{2}$

Give your answer to 3 decimal places. **(4 marks)**

d Write down the first four terms of the series expansion for $\ln\sqrt{\frac{1+3x}{1-2x}}$ **(2 marks)**