C2 SEQUENCES AND SERIES Worksheet A

1 For each of the following geometric series, write down the common ratio and find the value of the eighth term.

a $3 + 9 + 27 + 81 + \ldots$ **b** $1024 + 256 + 64 + 16 + \ldots$ **c** $1 - 2 + 4 - 8 + \ldots$

2 For each of the following geometric series, find an expression for the nth term.

a $1 + 5 + 25 + 125 + \ldots$ **b** $3 - 12 + 48 - 192 + \ldots$ **c** $81 + 54 + 36 + 24 + \ldots$

3 Find the sum of the first 12 terms of each of the following geometric series.

a $2 + 4 + 8 + 16 + \ldots$ **b** $640 + 320 + 160 + 80 + \ldots$ **c** $\frac{1}{6} - \frac{1}{2} + 1\frac{1}{2} - 4\frac{1}{2} + \ldots$

4 Given the first term, a, the common ratio, r, and the number of terms, n, find the sum of each of the following geometric series. Give your answers to 3 decimal places where appropriate.

a $a = 4, r = 3, n = 8$ **b** $a = 48, r = \frac{1}{2}, n = 14$ **c** $a = -1, r = -4, n = 12$

d $a = 200, r = 0.7, n = 20$ **e** $a = 120, r = -\frac{3}{4}, n = 15$ **f** $a = -25, r = 1.2, n = 30$

5 Evaluate to an appropriate degree of accuracy

a $\sum_{r=1}^{9} 3^r$ **b** $\sum_{r=1}^{6} 8^{r+1}$ **c** $\sum_{r=1}^{10} (10 \times 2^r)$ **d** $\sum_{r=1}^{8} (0.8)^r$

e $\sum_{r=1}^{10} \left[12 \times (\frac{1}{6})^r\right]$ **f** $\sum_{r=1}^{9} (-4)^r$ **g** $\sum_{r=4}^{20} (\frac{1}{2})^r$ **h** $\sum_{r=3}^{9} \left[2 \times (-3)^r\right]$

6 The second and third terms of a geometric series are 2 and 10 respectively.

a Find the common ratio of the series.

b Find the first term of the series.

c Find the sum of the first eight terms of the series.

7 The first and fourth terms of a geometric series are 2 and 54 respectively.

a Find the common ratio of the series.

b Find the ninth term of the series.

8 The third and fourth terms of a geometric series are 24 and 8 respectively.

a Find the common ratio of the series.

b Find the first term of the series.

c Find, to 3 decimal places, the sum of the first 11 terms of the series.

9 The first and third terms of a geometric series are 6 and 24 respectively.

a Find the two possible values for the common ratio of the series.

Given also that the common ratio of the series is positive,

b find the sum of the first 15 terms of the series.

10 The first and fourth terms of a geometric series are 768 and –96 respectively.

a Find the common ratio of the series.

b Find the tenth term of the series.

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11 The second and fifth terms of a geometric series are 0.5 and 32 respectively.

a Find the first term and common ratio of the series.

b Find the number of terms of the series that are smaller than 10 000.

12 The sum of the first four terms of a geometric series is 130 and its common ratio is $1\frac{1}{2}$.

a Find the first term of the series.

b Find the eighth term of the series.

c Find the least value of n for which the sum of the first n terms of the series is greater than 30 000.

13 All the terms of a geometric series are positive. The sum of the first and second terms of the series is 10.8 and the sum of the third and fourth terms of the series is 43.2

a Find the first term and common ratio of the series.

b Find the sum of the first 16 terms of the series.

14 For each of the following geometric series, either find its sum to infinity or explain why this cannot be found.

a $12 + 6 + 3 + 1.5 + \ldots$ **b** $270 + 90 + 30 + 10 + \ldots$ **c** $25 - 30 + 36 - 43.2 + \ldots$

d $216 + 144 + 96 + 64 + \ldots$ **e** $\frac{8}{25} + \frac{2}{5} + \frac{1}{2} + \frac{5}{8} + \ldots$ **f** $500 - 300 + 180 - 108 + \ldots$

15 Find the sum to infinity of the geometric series with nth term

a $(0.9)^n$ **b** $6 \times (\frac{1}{2})^n$ **c** $(-\frac{3}{4})^{n-1}$ **d** $40 \times (0.8)^n$

16 A geometric series has first term 80 and common ratio 0.2

a Find the sum to infinity of the series.

b Find the difference between the sum to infinity of the series and the sum of the first six terms of the series.

17 A sequence is defined by the recurrence relation

$$u_{n+1} = \tfrac{1}{3} u_n, \quad n > 0, \quad u_1 = 1.$$

a Write down the first four terms of the sequence.

b Evaluate $\sum_{r=1}^{\infty} u_r$.

18 The common ratio of a geometric series is 0.55 and the sum to infinity of the series is 40.

a Find the first term of the series.

b Find the smallest value of n for which the nth term of the series is less than 0.001

19 The sum, S_n, of the first n terms of a geometric series is given by $S_n = 2^n - 1$.

a Find the first term and the fifth term of the series.

b Find an expression for the nth term of the series.

20 The first three terms of a geometric series are $(k + 10)$, k and $(k - 6)$ respectively.

a Find the value of the constant k.

b Find the sum to infinity of the series.

C2 SEQUENCES AND SERIES Worksheet B

1 The third and fourth terms of a geometric series are 27 and $20\frac{1}{4}$ respectively.

a Find the first term of the series.

b Find the sum to infinity of the series.

2 The first three terms of a geometric series are $(k - 8)$, $(k + 4)$ and $(3k + 2)$ respectively, where k is a positive constant.

a Find the value of k.

b Find the sixth term of the series.

c Show that the sum of the first ten terms of the series is 50 857.3 to 1 decimal place.

3 The second and fifth terms of a geometric series are 75 and 129.6 respectively.

a Show that the first term of the series is 62.5

b Find the value of the tenth term of the series to 1 decimal place.

c Find the sum of the first 12 terms of the series to 1 decimal place.

4 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

b A geometric series has first term 2 and common ratio $\sqrt{2}$.

Given that the sum of the first n terms of the series is $126(\sqrt{2} + 1)$, find the value of n.

5 The first term of a geometric series is 18 and the sum to infinity of the series is 15.

a Find the common ratio of the series.

b Find the third term of the series.

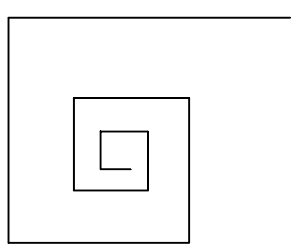
c Find the exact difference between the sum of the first eight terms of the series and the sum to infinity of the series.

6 The sum of the first n terms of a geometric series is given by $5(3^n - 1)$.

a Show that the third term of the series is 90.

b Find an expression for the nth term of the series in the form $k(3^n)$ where k is an exact fraction.

7



A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous one and at right angles to it. The first line is 4 mm long and thereafter each line is 25% longer than the previous one, so that a spiral is formed as shown above.

a Find the length, in mm, of the eighth straight line drawn by the program.

b Find the total length of the spiral, in metres, when 20 straight lines have been drawn.

8 The second and fourth terms of a geometric series are 30 and 2.7 respectively.

Given that the common ratio, r, of the series is positive,

a find the value of r and the first term of the series,

b find the sum to infinity of the series.

9 **a** Evaluate $\sum_{r=3}^{10} 3^r$.

b Show that $\sum_{r=1}^{15} (2^r - 12r) = 64\,094$.

10 A geometric series has common ratio r and the nth term of the series is denoted by u_n.

Given that $u_1 = 64$ and that $u_3 - u_2 = 20$,

a show that $16r^2 - 16r - 5 = 0$,

b find the two possible values of r,

c find the fourth term of the series corresponding to each possible value of r.

d Taking the value of r such that the series converges, find the sum to infinity of the series.

11 A geometric series has first term 4 and common ratio $\frac{1}{2}$.

a Find the eighth term of the series as an exact fraction.

b Find the nth term of the series in the form 2^y where y is a function of n.

c Show that the sum of the first n terms of the series is $8 - 2^{3-n}$.

12 The sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 4 \times 3^n, \quad n \geq 1.$$

a Find u_6 .

b Find the smallest value of t such that the sum of the first t terms of the sequence is greater than 10^{25}.

13 The sum of the first and third terms of a geometric series is 150. The sum of the second and fourth terms of the series is −75.

a Find the first term and common ratio of the series.

b Find the sum to infinity of the series.

14 Three consecutive terms of an arithmetic series are a, b and $(3a + 4)$ respectively.

a Find an expression for b in terms of a.

Given also that a, b and $(6a + 1)$ respectively are consecutive terms of a geometric series and that a and b are integers,

b find the values of a and b.

15 When a ball is dropped onto a horizontal floor it bounces such that it reaches a maximum height of 60% of the height from which it was dropped.

a Find the maximum height the ball reaches after its fourth bounce when it is initially dropped from 3 metres above the floor.

b Show that when the ball is dropped from a height of h metres above the floor it travels a total distance of $4h$ metres before coming to rest.

C2 SEQUENCES AND SERIES — Worksheet C

1 Expand each of the following, simplifying the coefficient in each term.

a $(1 + x)^4$ **b** $(1 - x)^5$ **c** $(1 + 4x)^3$ **d** $(1 - 2y)^3$

e $(1 + \frac{1}{2}x)^4$ **f** $(1 + \frac{1}{3}y)^3$ **g** $(1 + x^2)^5$ **h** $(1 - \frac{3}{2}x)^4$

2 Expand each of the following, simplifying the coefficient in each term.

a $(x + y)^3$ **b** $(a - b)^5$ **c** $(x + 2y)^4$ **d** $(2 + y)^3$

e $(3 - x)^3$ **f** $(5 + 2x)^4$ **g** $(3 - 4y)^5$ **h** $(3 + \frac{1}{2}x)^4$

3 Find the first four terms in the expansion in ascending powers of x of

a $(1 + x)^{10}$ **b** $(1 - x)^6$ **c** $(1 + 2x)^8$ **d** $(1 - \frac{1}{2}x)^7$

e $(1 + x^3)^6$ **f** $(2 + x)^9$ **g** $(3 - x)^7$ **h** $(2 + 5x)^{10}$

4 Find the coefficient indicated in the following expansions.

a $(1 + x)^{20}$, coefficient of x^3 **b** $(1 - x)^{14}$, coefficient of x^4

c $(1 + 4x)^9$, coefficient of x^2 **d** $(1 - 3y)^{14}$, coefficient of y^3

e $(1 - \frac{1}{3}x)^{12}$, coefficient of x^4 **f** $(1 - \frac{1}{2}x)^{16}$, coefficient of x^5

g $(1 + \frac{2}{5}x)^{15}$, coefficient of x^2 **h** $(1 + y^2)^8$, coefficient of y^6

5 Express each of the following in the required form where a and b are integers.

a $(1 + \sqrt{5})^3$ in the form $a + b\sqrt{5}$ **b** $(1 - \sqrt{3})^4$ in the form $a + b\sqrt{3}$

c $(2 + \sqrt{2})^3$ in the form $a + b\sqrt{2}$ **d** $(1 + 2\sqrt{3})^4$ in the form $a + b\sqrt{3}$

6 **a** Expand $(1 + x)^6$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.02^6 **ii** 0.99^6

giving your answers to 4 decimal places.

7 **a** Expand $(1 + 2y)^8$ in ascending powers of y up to and including the term in y^3, simplifying each coefficient.

b By substituting a suitable value of y into your answer for part **a**, obtain an estimate for

i 0.98^8 **ii** 1.01^8

giving your answers to 4 decimal places.

8 Expand and simplify

a $(1 + x)^4 + (1 - x)^4$ **b** $(1 - \frac{1}{3}x)^3 - (1 + \frac{1}{3}x)^3$

9 The coefficient of x^2 in the expansion of $(1 + ax)^4$ in ascending powers of x is 24, where a is a constant and $a < 0$. Find

a the value of a,

b the value of the coefficient of x^3 in the expansion.

C2 SEQUENCES AND SERIES Worksheet D

1 Expand

a $(1 + 3x)^4$ **b** $(2 - x)^5$ **c** $(3 + 10x^2)^3$ **d** $(a + 2b)^5$

e $(x^2 - y)^3$ **f** $(5 + \frac{1}{2}x)^4$ **g** $(x + \frac{1}{x})^4$ **h** $(t - \frac{2}{t^2})^3$

2 Find the first four terms in the expansion in ascending powers of x of

a $(1 + 3x)^6$ **b** $(1 - \frac{1}{4}x)^8$ **c** $(5 - x)^7$ **d** $(3 + 2x^2)^{10}$

3 Find the coefficient indicated in the following expansions

a $(1 + x)^{15}$, coefficient of x^3 **b** $(1 - 2x)^{12}$, coefficient of x^4

c $(3 + x)^7$, coefficient of x^2 **d** $(2 - y)^{10}$, coefficient of y^5

e $(2 + t^3)^8$, coefficient of t^{15} **f** $(1 - \frac{1}{x})^9$, coefficient of x^{-3}

4 **a** Express $(\sqrt{2} - \sqrt{5})^4$ in the form $a + b\sqrt{10}$, where $a, b \in \mathbb{Z}$.

b Express $(\sqrt{2} + \frac{1}{\sqrt{3}})^3$ in the form $a\sqrt{2} + b\sqrt{3}$, where $a, b \in \mathbb{Q}$.

c Express $(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3$ in the form $a\sqrt{5}$, where $a \in \mathbb{Z}$.

5 **a** Expand $(1 + \frac{x}{2})^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.005^{10} **ii** 0.996^{10}

giving your answers to 5 decimal places.

6 **a** Expand $(3 + x)^8$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 3.001^8 **ii** 2.995^8

giving your answers to 7 significant figures.

7 Expand and simplify

a $(1 + 10x)^4 + (1 - 10x)^4$ **b** $(2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$

c $(1 + 4y)(1 + y)^3$ **d** $(1 - x)(1 + \frac{1}{x})^3$

8 Expand each of the following in ascending powers of x up to and including the term in x^3.

a $(1 + x^2)(1 - 3x)^{10}$ **b** $(1 - 2x)(1 + x)^8$

c $(1 + x + x^2)(1 - x)^6$ **d** $(1 + 3x - x^2)(1 + 2x)^7$

9 Find the term independent of y in each of the following expansions.

a $(y + \frac{1}{y})^8$ **b** $(2y - \frac{1}{2y})^{12}$ **c** $(\frac{1}{y} + y^2)^6$ **d** $(3y - \frac{1}{y^2})^9$

10 The coefficient of x^2 in the binomial expansion of $(1 + \frac{2}{5}x)^n$, where n is a positive integer, is 1.6

a Find the value of n.

b Use your value of n to find the coefficient of x^4 in the expansion.

11 Given that $y_1 = (1 - 2x)(1 + x)^{10}$ and $y_2 = ax^2 + bx + c$ and that when x is small, y_2 can be used as an approximation for y_1,

a find the values of the constants a, b and c,

b find the percentage error in using y_2 as an approximation for y_1 when $x = 0.2$

12 In the binomial expansion of $(1 + px)^q$, where p and q are constants and q is a positive integer, the coefficient of x is -12 and the coefficient of x^2 is 60.

Find

a the value of p and the value of q,

b the value of the coefficient of x^3 in the expansion.

13 **a** Expand $(3 - \frac{x}{3})^{12}$ as a binomial series in ascending powers of x up to and including the term in x^3, giving each coefficient as an integer.

b Use your series expansion with a suitable value of x to obtain an estimate for 2.998^{12}, giving your answer to 2 decimal places.

14 **a** Expand $(1 - x)^5$ as a binomial series in ascending powers of x.

b Express $(\sqrt{3} + 1)(\sqrt{3} - 2)$ in the form $A + B\sqrt{3}$, where $A, B \in \mathbb{Z}$.

c Hence express each of the following in the form $C + D\sqrt{3}$, where $C, D \in \mathbb{Z}$.

i $(\sqrt{3} + 1)^5(\sqrt{3} - 2)^5$

ii $(\sqrt{3} + 1)^6(\sqrt{3} - 2)^5$

15 **a** Expand $(1 + \frac{x}{2})^9$ in ascending powers of x up to and including the term in x^4.

Hence, or otherwise, find

b the coefficient of x^3 in the expansion of $(1 + \frac{x}{2})^9 - (1 - \frac{x}{2})^9$,

c the coefficient of x^4 in the expansion of $(1 + 2x)(1 + \frac{x}{2})^9$.

16 The term independent of x in the expansion of $(x^3 + \frac{a}{x^2})^5$ is -80.

Find the value of the constant a.

17 In the binomial expansion of $(1 + \frac{x}{k})^n$, where k is a non-zero constant, n is an integer and $n > 1$, the coefficient of x^2 is three times the coefficient of x^3.

a Show that $k = n - 2$.

Given also that $n = 7$,

b expand $(1 + \frac{x}{k})^n$ in ascending powers of x up to and including the term in x^4, giving each coefficient as a fraction in its simplest form.

C2 SEQUENCES AND SERIES Worksheet E

1 Expand $(1 + 4x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

2 A geometric series has first term 3 and common ratio -2.

a Find the fifth term of the series. **(2)**

b Find the sum of the first ten terms of the series. **(2)**

c Show that the sum of the first eight positive terms of the series is 65 535. **(4)**

3 **a** Expand $(1 + 3x)^7$ in ascending powers of x up to and including the term in x^4, simplifying each coefficient in the expansion. **(4)**

b Use your series with a suitable value of x to estimate the value of 1.03^7 correct to 5 decimal places. **(3)**

4 Evaluate $\sum_{r=3}^{12} 2^r$. **(4)**

5 **a** Expand $(2 + x)^5$, simplifying the coefficient in each term. **(4)**

b Hence, or otherwise, write down the expansion of $(2 - x)^5$. **(1)**

c Show that

$$(2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 = k\sqrt{5},$$

where k is an integer to be found. **(4)**

6 Ginny opens a savings account and decides to pay £200 into the account at the start of each month. At the end of each month, interest of 0.5% is paid into the account.

a Find, to the nearest penny, the interest paid into the account at the end of the third month. **(4)**

b Show that the total interest paid into the account over the first 12 months is £79.45 to the nearest penny. **(5)**

7 Find the first four terms in the expansion of $(1 - 3x)^8$ in ascending powers of x, simplifying each coefficient. **(4)**

8 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$ **(4)**

b Find the exact sum of the first 16 terms of the geometric series with fourth term 3 and fifth term 6. **(5)**

9 **a** Write down the first three terms in the binomial expansion of $(1 + ax)^n$, where n is a positive integer, in ascending powers of x. **(2)**

Given that the coefficient of x^2 is three times the coefficient of x,

b show that $n = \dfrac{6+a}{a}$. **(4)**

Given also that $a = \frac{2}{3}$,

c find the coefficient of x^3 in the expansion. **(3)**

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10 Find the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x, simplifying each coefficient. **(4)**

11 The first term of a geometric series is 162 and the sum to infinity of the series is 486.

a Find the common ratio of the series. **(3)**

b Find the sixth term of the series. **(2)**

c Find, to 3 decimal places, the sum of the first ten terms of the series. **(4)**

12 **a** Expand $(1 + 3x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

b Find the coefficient of x^2 in the expansion of

$$(1 + 4x - x^2)(1 + 3x)^4.$$ **(3)**

13 In a computer game, each player must complete the tasks set at each level within a fixed amount of time in order to progress to the next level.

The time allowed for level 1 is 2 minutes and the time allowed for each of the other levels is 10% less than that allowed in the previous level.

a Find, in seconds, the time allowed for completing level 4. **(2)**

b Find, in minutes and seconds, the maximum total time allowed for completing the first 12 levels of the game. **(4)**

14 Given that

$$(1 + \tfrac{x}{2})^8(1 - x)^6 \equiv 1 + Ax + Bx^2 + \dots,$$

find the values of the constants A and B. **(7)**

15 The terms of a sequence are defined by the recurrence relation

$$u_r = 2u_{r-1}, \quad r > 1, \quad u_1 = 6.$$

a Write down the first four terms of the sequence. **(1)**

b Evaluate $\sum_{r=1}^{10} u_r$. **(3)**

16 **a** Expand $(1 + x)^4$ in ascending powers of x. **(2)**

b Hence, or otherwise, write down the expansion of $(1 - x)^4$ in ascending powers of x. **(1)**

c By using your answers to parts **a** and **b**, or otherwise, solve the equation

$$(1 + x)^4 + (1 - x)^4 = 82,$$

for real values of x. **(5)**

17 The common ratio of a geometric series is 1.5 and the third term of the series is 18.

a Find the first term of the series. **(2)**

b Find the sum of the first six terms of the series. **(2)**

c Find the smallest value of k such that the kth term of the series is greater than 8000. **(4)**

18 The first two terms in the expansion of $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$, in ascending powers of x, are 2 and $90x^2$.

Given that $a < b$, find the values of the constants a and b. **(9)**

C2 SEQUENCES AND SERIES Worksheet F

1 The first and fourth terms of a geometric series are 108 and 32 respectively.

a Find the third term of the series. **(5)**

b Find the sum to infinity of the series. **(2)**

2 Expand $(1 - 2x)^5$ in ascending powers of x, simplifying each coefficient. **(4)**

3 An internet site has 3600 subscribers at the start of a promotional campaign.

In a model of the results of the campaign, it is assumed that the site will gain 200 new subscribers in the first week and that in subsequent weeks the number of new subscribers will be 15% greater each week.

a Show that, according to this model, the site will gain 304 new subscribers in the fourth week of the campaign. **(2)**

b Find the total number of subcribers to the site predicted by the model after ten weeks of the campaign, assuming that no subscriptions are cancelled in this period. **(5)**

4 **a** Find the first three terms in the expansion of $(1 + 4x)^7$ in ascending powers of x. **(3)**

b Hence, find the coefficient of x^2 in the expansion of

$$(1 + 2x)^2(1 + 4x)^7.$$ **(3)**

5 **a** Write down the first four terms in the expansion in ascending powers of x of $(1 + \frac{x}{k})^{2n}$, where k is a non-zero constant, n is an integer and $n > 1$. **(4)**

Given that the coefficient of x^3 is half the coefficient of x^2,

b show that $3k = 4(n - 1)$. **(3)**

Given also that the coefficient of x is 2,

c find the values of n and k. **(3)**

6 The second and third terms of a geometric series are $\sqrt{6}$ and $3\sqrt{2}$ respectively.

a Find, in surd form, the first term and the common ratio of the series. **(4)**

b Show that the sum of the first eight terms of the series is $40\sqrt{2}(\sqrt{3} + 1)$. **(4)**

7 Evaluate $$\sum_{r=1}^{9} (3^r - 1).$$ **(5)**

8 **a** Find the first four terms in the expansion of $(1 + 2x)^9$ in ascending powers of x. **(4)**

b Show that, if terms involving x^4 and higher powers of x may be ignored,

$$(1 + 2x)^9 + (1 - 2x)^9 = 2 + 288x^2.$$ **(3)**

c Hence find the value of

$$1.002^9 + 0.998^9,$$

giving your answer to 7 significant figures. **(2)**

9 Given that

$$(k - x)^9 \equiv a - bx + bx^2 + \ldots,$$

find the values of the positive integers a, b and k. **(7)**

10 Expand $(3 + 2x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

11 The first term of a geometric series is t and the sum to infinity of the series is $3t$.

a Find the common ratio of the series. **(3)**

Given also that the sum of the first four terms of the series is 130,

b find the value of t. **(4)**

12 **a** Expand $(1 - 2x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

b Hence, or otherwise, find the coefficient of y^2 in the expansion of

$$(1 + 4y - 2y^2)^4.$$ **(4)**

13 A company buys a new car for £12 000 at the start of one year. In a model, it is assumed that each year the value of a car decreases by 25% of its value at the start of that year.

a Show that the value of the car after four years is £3800 to 3 significant figures. **(2)**

The company plans to buy one new car for £12 000 at the start of each subsequent year.

b Using the same model, find the total value of all the cars the company will have bought under this plan immediately after the purchase of the eighth car. **(5)**

14 The polynomial p(x) is defined by

$$p(x) = (x + 3)^4 - (x + 1)^4.$$

a Show that $(x + 2)$ is a factor of p(x). **(2)**

b Fully factorise p(x). **(7)**

c Hence show that there is only one real solution to the equation $p(x) = 0$. **(3)**

15 $$f(x) \equiv (1 - x)(1 + 2x)^n, \quad n \in \mathbb{N}.$$

Given that the coefficient of x^2 in the binomial expansion of f(x) is 198, find

a the value of n, **(6)**

b the coefficient of x^3 in the expansion. **(3)**

16 Expand $(\frac{3}{x} - x)^4$ in descending powers of x, simplifying the coefficient in each term. **(4)**

17 The sum, S_n, of the first n terms of a series is given by

$$S_n = 3^n - 1.$$

a Show that the fourth term of the series is 54. **(3)**

b Show that the nth term of the series can be expressed in the form $k(3^n)$ where k is an exact fraction to be found. **(4)**

c Prove that the series is geometric. **(3)**

18 An arithmetic series has first term 3, second term x and fourth term y.

a Find an expression for y in terms of x. **(3)**

Given also that 3, x and y are the first, second and fourth terms respectively of a geometric series,

b show that $x^3 - 27x + 54 = 0$, **(4)**

c by first finding a linear factor of $x^3 - 27x + 54$, find the two possible values of x. **(6)**

C2 SEQUENCES AND SERIES — Answers - Worksheet A

1 **a** $r = 3$
$u_8 = 3 \times 3^7 = 6561$

b $r = \frac{1}{4}$
$u_8 = 1024 \times (\frac{1}{4})^7 = \frac{1}{16}$

c $r = -2$
$u_8 = 1 \times (-2)^7 = -128$

2 **a** $a = 1,\ r = 5$
$u_n = 5^{n-1}$

b $a = 3,\ r = -4$
$u_n = 3 \times (-4)^{n-1}$

c $a = 81,\ r = \frac{2}{3}$
$u_n = 81 \times (\frac{2}{3})^{n-1}$

3 **a** $a = 2,\ r = 2,\ n = 12$
$S_{12} = \frac{2(2^{12}-1)}{2-1} = 8190$

b $a = 640,\ r = \frac{1}{2},\ n = 12$
$S_{12} = \frac{640[1-(\frac{1}{2})^{12}]}{1-\frac{1}{2}} = 1279\frac{11}{16}$

c $a = \frac{1}{6},\ r = -3,\ n = 12$
$S_{12} = \frac{\frac{1}{6}[1-(-3)^{12}]}{1-(-3)} = -22\,143\frac{1}{3}$

4 **a** $S_8 = \frac{4(3^8-1)}{3-1} = 13\,120$

b $S_{14} = \frac{48[1-(\frac{1}{2})^{14}]}{1-\frac{1}{2}} = 95.994$

c $S_{12} = \frac{-[1-(-4)^{12}]}{1-(-4)} = 3\,355\,443$

d $S_{20} = \frac{200[1-(0.7)^{20}]}{1-0.7}$
$= 666.135$

e $S_{15} = \frac{120[1-(-\frac{3}{4})^{15}]}{1-(-\frac{3}{4})}$
$= 69.488$

f $S_{30} = \frac{-25[(1.2)^{30}-1]}{1.2-1}$
$= -29\,547.039$

5 **a** GP: $a = 3$
$r = 3,\ n = 9$
$S_9 = \frac{3(3^9-1)}{3-1}$
$= 29\,523$

b GP: $a = 64$
$r = 8,\ n = 6$
$S_6 = \frac{64(8^6-1)}{8-1}$
$= 2\,396\,736$

c GP: $a = 20$
$r = 2,\ n = 10$
$S_{10} = \frac{20(2^{10}-1)}{2-1}$
$= 20\,460$

d GP: $a = 0.8$
$r = 0.8,\ n = 8$
$S_8 = \frac{0.8[1-(0.8)^8]}{1-0.8}$
$= 3.329$ (3dp)

e GP: $a = 2$
$r = \frac{1}{6},\ n = 10$
$S_{10} = \frac{2[1-(\frac{1}{6})^{10}]}{1-\frac{1}{6}}$
$= 2.400$ (3dp)

f GP: $a = -4$
$r = -4,\ n = 9$
$S_9 = \frac{-4[1-(-4)^9]}{1-(-4)}$
$= -209\,716$

g GP: $a = \frac{1}{16}$
$r = \frac{1}{2},\ n = 17$
$S_{17} = \frac{\frac{1}{16}[1-(\frac{1}{2})^{17}]}{1-\frac{1}{2}}$
$= 0.125$ (3dp)

h GP: $a = -54$
$r = -3,\ n = 7$
$S_7 = \frac{-54[1-(-3)^7]}{1-(-3)}$
$= -29\,538$

6 **a** $r = 10 \div 2 = 5$
b $a \times 5 = 2 \quad \therefore a = 0.4$
c $S_8 = \frac{0.4(5^8-1)}{5-1} = 39\,062.4$

7 **a** $a = 2,\ ar^3 = 54 \quad \therefore r^3 = 54 \div 2 = 27$
$r = \sqrt[3]{27} = 3$
b $u_9 = 2 \times 3^8 = 13\,122$

8 **a** $r = 8 \div 24 = \frac{1}{3}$
b $a \times (\frac{1}{3})^2 = 24 \quad \therefore a = 216$
c $S_{11} = \frac{216[1-(\frac{1}{3})^{11}]}{1-\frac{1}{3}} = 323.998$

9 **a** $a = 6,\ ar^2 = 24 \quad \therefore r^2 = 24 \div 6 = 4$
$r = \pm 2$
b $r = 2,\ S_{15} = \frac{6(2^{15}-1)}{2-1} = 196\,602$

10 **a** $a = 768,\ ar^3 = -96$
$r^3 = -96 \div 768 = -\frac{1}{8}$
$r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
b $u_{10} = 768 \times (-\frac{1}{2})^9 = -1.5$

11 **a** $ar = 0.5,\ ar^4 = 32 \quad \therefore r^3 = 32 \div 0.5 = 64$
$r = \sqrt[3]{64} = 4, \quad a \times 4 = 0.5 \quad \therefore a = 0.125$
b $0.125 \times 4^{n-1} < 10\,000 \quad \therefore 4^{n-1} < 80\,000$
$(n-1) \lg 4 < \lg 80\,000$
$n < \frac{\lg 80000}{\lg 4} + 1$
$n < 9.14 \quad \therefore$ 9 terms

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12 **a** $\dfrac{a[(\frac{3}{2})^4 - 1]}{\frac{3}{2} - 1} = 130$

$a = 130 \div \frac{65}{8} = 16$

b $u_8 = 16 \times (\frac{3}{2})^7 = 273\frac{3}{8}$

c $\dfrac{16[(\frac{3}{2})^n - 1]}{\frac{3}{2} - 1} > 30\,000$

$(\frac{3}{2})^n > 938.5$

$n \lg \frac{3}{2} > \lg 938.5$

$n > \dfrac{\lg 938.5}{\lg 1.5}$

$n > 16.9 \;\therefore$ least $n = 17$

13 **a** $a + ar = a(1 + r) = 10.8$

$ar^2 + ar^3 = ar^2(1 + r) = 43.2$

$\therefore\; r^2 = 43.2 \div 10.8 = 4$

all terms +ve $\therefore\; r$ +ve $\therefore\; r = 2$

sub. $a = 10.8 \div 3 = 3.6$

b $S_{16} = \dfrac{3.6(2^{16} - 1)}{2 - 1} = 235\,926$

14 **a** $a = 12, r = 0.5$

$S_\infty = \dfrac{12}{1 - 0.5} = 24$

b $a = 270, r = \frac{1}{3}$

$S_\infty = \dfrac{270}{1 - \frac{1}{3}} = 405$

c $a = 25, r = -1.2$

no S_∞ as $r < -1 \;\therefore$ diverges

d $a = 216, r = \frac{2}{3}$

$S_\infty = \dfrac{216}{1 - \frac{2}{3}} = 648$

e $a = \frac{8}{25}, r = \frac{5}{4}$

no S_∞ as $r > 1 \;\therefore$ diverges

f $a = 500, r = -0.6$

$S_\infty = \dfrac{500}{1 - (-0.6)} = 312.5$

15 **a** $a = 0.9, r = 0.9$

$S_\infty = \dfrac{0.9}{1 - 0.9} = 9$

b $a = 3, r = \frac{1}{2}$

$S_\infty = \dfrac{3}{1 - \frac{1}{2}} = 6$

c $a = 1, r = -\frac{3}{4}$

$S_\infty = \dfrac{1}{1 - (-\frac{3}{4})} = \frac{4}{7}$

d $a = 32, r = 0.8$

$S_\infty = \dfrac{32}{1 - 0.8} = 160$

16 **a** $S_\infty = \dfrac{80}{1 - 0.2} = 100$

b $S_6 = \dfrac{80[1 - (0.2)^6]}{1 - 0.2} = 99.9936$

$S_\infty - S_6 = 0.0064$

17 **a** $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

b GP: $a = 1, r = \frac{1}{3}$

$S_\infty = \dfrac{1}{1 - \frac{1}{3}} = \frac{3}{2}$

18 **a** $\dfrac{a}{1 - 0.55} = 40$

$a = 0.45 \times 40 = 18$

b $18 \times (0.55)^{n-1} < 0.001$

$(n - 1) \lg 0.55 < \lg 0.0000556$

$n > \dfrac{\lg 0.0000556}{\lg 0.55} + 1$

$n > 17.4 \;\therefore$ smallest $n = 18$

19 **a** $u_1 = S_1 = 2^1 - 1 = 1$

$S_5 = 2^5 - 1 = 31,\; S_4 = 2^4 - 1 = 15$

$u_5 = S_5 - S_4 = 31 - 15 = 16$

b $S_{n-1} = 2^{n-1} - 1$

$u_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1)$

$= 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$

20 **a** $\dfrac{k}{k + 10} = \dfrac{k - 6}{k}$

$k^2 = (k + 10)(k - 6)$

$4k - 60 = 0$

$k = 15$

b $u_1 = 25, u_2 = 15 \;\therefore\; a = 25, r = 0.6$

$S_\infty = \dfrac{25}{1 - 0.6} = 62.5$

C2 SEQUENCES AND SERIES — Answers - Worksheet B

1 **a** $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$

$a \times (\frac{3}{4})^2 = 27$

$a = \frac{16}{9} \times 27 = 48$

b $S_\infty = \frac{48}{1-\frac{3}{4}} = 192$

2 **a** $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$

$(k + 4)^2 = (3k + 2)(k - 8)$

$k^2 - 15k - 16 = 0$

$(k + 1)(k - 16) = 0$

$k > 0 \ \therefore\ k = 16$

b $u_1 = 8, u_2 = 20 \ \therefore\ a = 8, r = \frac{5}{2}$

$u_6 = 8 \times (\frac{5}{2})^5 = 781\frac{1}{4}$

c $S_{10} = \frac{8[(\frac{5}{2})^{10} - 1]}{\frac{5}{2} - 1} = 50\,857.3$

3 **a** $ar = 75, ar^4 = 129.6$

$r^3 = 129.6 \div 75 = 1.728$

$r = \sqrt[3]{1.728} = 1.2$

$a = 75 \div 1.2 = 62.5$

b $u_{10} = 62.5 \times (1.2)^9 = 322.5$

c $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 **a** $S_n = a + ar + ar^2 + \ldots ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots ar^{n-1} + ar^n$

subtracting,

$S_n - rS_n = a - ar^n$

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $\frac{2[1-(\sqrt{2})^n]}{1-\sqrt{2}} = 126(\sqrt{2} + 1)$

$1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$

$1 - (\sqrt{2})^n = 63(1 - 2)$

$(\sqrt{2})^n = 64$

$2^{\frac{1}{2}n} = 2^6$

$n = 12$

5 **a** $\frac{18}{1-r} = 15$

$\therefore\ 1 - r = \frac{18}{15} = 1.2$

$r = -0.2$

b $u_3 = 18 \times (-0.2)^2 = 0.72$

c $S_8 = \frac{18[1-(-0.2)^8]}{1-(-0.2)} = 14.9999616$

$S_\infty - S_8 = 0.000\,0384$

6 **a** $S_3 = 5(3^3 - 1) = 130$

$S_2 = 5(3^2 - 1) = 40$

$u_3 = S_3 - S_2 = 90$

b $S_{n-1} = 5(3^{n-1} - 1)$

$u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$

$= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$

7 **a** $4 \times (1.25)^7 = 19.1$ mm (3sf)

b GP: $a = 4, r = 1.25$

$S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8$ mm

$\therefore$ length = 1.37 m (3sf)

8 **a** $ar = 30, ar^3 = 2.7 \ \therefore\ r^2 = 2.7 \div 30 = 0.09$

$r > 0 \ \therefore\ r = \sqrt{0.09} = 0.3$

$a = 30 \div 0.3 = 100$

b $S_\infty = \frac{100}{1-0.3} = 142.9$ (1dp)

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9 **a** GP: $a = 27, r = 3$

$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$

b $\sum_{r=1}^{15} 2^r$: GP, $a = 2, r = 2$

$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$

$\sum_{r=1}^{15} 12r$: AP, $a = 12, d = 12$

$S_{15} = \frac{15}{2}[24 + (14 \times 12)] = 1440$

$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$

10 **a** $a = 64,\ ar^2 - ar = 20$

$\therefore\ 64r^2 - 64r = 20$

$16r^2 - 16r - 5 = 0$

b $(4r + 1)(4r - 5) = 0$

$r = -\frac{1}{4}$ or $\frac{5}{4}$

c $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times (-\frac{1}{4})^3 = -1$

$r = \frac{5}{4} \Rightarrow u_4 = 64 \times (\frac{5}{4})^3 = 125$

d $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

11 **a** $u_8 = 4 \times (\frac{1}{2})^7 = \frac{1}{32}$

b $u_n = 4 \times (\frac{1}{2})^{n-1}$

$= 2^2 \times 2^{1-n}$

$= 2^{3-n}$

c $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$= 8(1 - 2^{-n})$

$= 8 - (2^3 \times 2^{-n})$

$= 8 - 2^{3-n}$

12 **a** $u_6 = 4 \times 3^6 = 2916$

b GP: $a = 12, r = 3$

$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$

$\therefore\ 6(3^t - 1) > 10^{25}$

$3^t > \frac{10^{25}}{6} + 1$

$t \lg 3 > \lg(\frac{10^{25}}{6} + 1)$

$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$

$t > 50.8\ \therefore$ smallest $t = 51$

13 **a** $a + ar^2 = a(1 + r^2) = 150$

$ar + ar^3 = ar(1 + r^2) = -75$

$\therefore\ r = -75 \div 150 = -\frac{1}{2}$

$a = 150 \div \frac{5}{4} = 120$

b $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

14 **a** $b - a = (3a + 4) - b$

$2b = 4a + 4$

$b = 2a + 2$

b $\frac{2a + 2}{a} = \frac{6a + 1}{2a + 2}$

$(2a + 2)^2 = a(6a + 1)$

$2a^2 - 7a - 4 = 0$

$(2a + 1)(a - 4) = 0$

a integer $\therefore\ a = 4$

sub. $b = 10$

15 **a** after 4th bounce,

reaches $3 \times (0.6)^4 = 0.3888$ m

b total distance

$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \ldots]$

$= h + 2 \times S_\infty$ of GP, $a = 0.6h, r = 0.6$

$= h + \frac{2 \times 0.6h}{1 - 0.6}$

$= h + 3h = 4h$ metres

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1 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$

b $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

c $= 1 + 3(4x) + 3(4x)^2 + (4x)^3$
$= 1 + 12x + 48x^2 + 64x^3$

d $= 1 + 3(-2y) + 3(-2y)^2 + (-2y)^3$
$= 1 - 6y + 12y^2 - 8y^3$

e $= 1 + 4(\tfrac{1}{2}x) + 6(\tfrac{1}{2}x)^2 + 4(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 1 + 2x + \tfrac{3}{2}x^2 + \tfrac{1}{2}x^3 + \tfrac{1}{16}x^4$

f $= 1 + 3(\tfrac{1}{3}y) + 3(\tfrac{1}{3}y)^2 + (\tfrac{1}{3}y)^3$
$= 1 + y + \tfrac{1}{3}y^2 + \tfrac{1}{27}y^3$

g $= 1 + 5(x^2) + 10(x^2)^2 + 10(x^2)^3 + 5(x^2)^4 + (x^2)^5$
$= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}$

h $= 1 + 4(-\tfrac{3}{2}x) + 6(-\tfrac{3}{2}x)^2 + 4(-\tfrac{3}{2}x)^3 + (-\tfrac{3}{2}x)^4$
$= 1 - 6x + \tfrac{27}{2}x^2 - \tfrac{27}{2}x^3 + \tfrac{81}{16}x^4$

2 **a** $= x^3 + 3x^2y + 3xy^2 + y^3$

b $= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

c $= x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$
$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$

d $= 2^3 + 3(2^2)y + 3(2)y^2 + y^3$
$= 8 + 12y + 6y^2 + y^3$

e $= 3^3 + 3(3^2)(-x) + 3(3)(-x)^2 + (-x)^3$
$= 27 - 27x + 9x^2 - x^3$

f $= 5^4 + 4(5^3)(2x) + 6(5^2)(2x)^2 + 4(5)(2x)^3 + (2x)^4$
$= 625 + 1000x + 600x^2 + 160x^3 + 16x^4$

g $= 3^5 + 5(3^4)(-4y) + 10(3^3)(-4y)^2 + 10(3^2)(-4y)^3 + 5(3)(-4y)^4 + (-4y)^5$
$= 243 - 1620y + 4320y^2 - 5760y^3 + 3840y^4 - 1024y^5$

h $= 3^4 + 4(3^3)(\tfrac{1}{2}x) + 6(3^2)(\tfrac{1}{2}x)^2 + 4(3)(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 81 + 54x + \tfrac{27}{2}x^2 + \tfrac{3}{2}x^3 + \tfrac{1}{16}x^4$

3 **a** $= 1 + 10x + \tfrac{10\times9}{2}x^2 + \tfrac{10\times9\times8}{3\times2}x^3 + \ldots$
$= 1 + 10x + 45x^2 + 120x^3 + \ldots$

b $= 1 + 6(-x) + \tfrac{6\times5}{2}(-x)^2 + \tfrac{6\times5\times4}{3\times2}(-x)^3 + \ldots$
$= 1 - 6x + 15x^2 - 20x^3 + \ldots$

c $= 1 + 8(2x) + \tfrac{8\times7}{2}(2x)^2 + \tfrac{8\times7\times6}{3\times2}(2x)^3 + \ldots$
$= 1 + 16x + 112x^2 + 448x^3 + \ldots$

d $= 1 + 7(-\tfrac{1}{2}x) + \tfrac{7\times6}{2}(-\tfrac{1}{2}x)^2 + \tfrac{7\times6\times5}{3\times2}(-\tfrac{1}{2}x)^3 + \ldots$
$= 1 - \tfrac{7}{2}x + \tfrac{21}{4}x^2 - \tfrac{35}{8}x^3 + \ldots$

e $= 1 + 6(x^3) + \tfrac{6\times5}{2}(x^3)^2 + \tfrac{6\times5\times4}{3\times2}(x^3)^3 + \ldots$
$= 1 + 6x^3 + 15x^6 + 20x^9 + \ldots$

f $= 2^9 + 9(2^8)x + \tfrac{9\times8}{2}(2^7)x^2 + \tfrac{9\times8\times7}{3\times2}(2^6)x^3 + \ldots$
$= 512 + 2304x + 4608x^2 + 5376x^3 + \ldots$

g $= 3^7 + 7(3^6)(-x) + \tfrac{7\times6}{2}(3^5)(-x)^2 + \tfrac{7\times6\times5}{3\times2}(3^4)(-x)^3 + \ldots$
$= 2187 - 5103x + 5103x^2 - 2835x^3 + \ldots$

h $= 2^{10} + 10(2^9)(5x) + \tfrac{10\times9}{2}(2^8)(5x)^2 + \tfrac{10\times9\times8}{3\times2}(2^7)(5x)^3 + \ldots$
$= 1024 + 25\,600x + 288\,000x^2 + 1\,920\,000x^3 + \ldots$

4 **a** $= \binom{20}{3} = 1140$

b $= \binom{14}{4} \times (-1)^4 = 1001$

c $= \binom{9}{2} \times 4^2 = 576$

d $= \binom{14}{3} \times (-3)^3 = -9828$

e $= \binom{12}{4} \times (-\tfrac{1}{3})^4 = \tfrac{55}{9}$ or $6\tfrac{1}{9}$

f $= \binom{16}{5} \times (-\tfrac{1}{2})^5 = -136.5$

g $= \binom{15}{2} \times (\tfrac{2}{5})^2 = \tfrac{84}{5}$ or 16.8

h $= \binom{8}{3} = 56$

5 **a** $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3$
$= 1 + 3\sqrt{5} + 15 + 5\sqrt{5}$
$= 16 + 8\sqrt{5}$

b $= 1 + 4(-\sqrt{3}) + 6(-\sqrt{3})^2 + 4(-\sqrt{3})^3 + (-\sqrt{3})^4$
$= 1 - 4\sqrt{3} + 18 - 12\sqrt{3} + 9$
$= 28 - 16\sqrt{3}$

c $= 2^3 + 3(2^2)(\sqrt{2}) + 3(2)(\sqrt{2})^2 + (\sqrt{2})^3$
$= 8 + 12\sqrt{2} + 12 + 2\sqrt{2}$
$= 20 + 14\sqrt{2}$

d $= 1 + 4(2\sqrt{3}) + 6(2\sqrt{3})^2 + 4(2\sqrt{3})^3 + (2\sqrt{3})^4$
$= 1 + 8\sqrt{3} + 72 + 96\sqrt{3} + 144$
$= 217 + 104\sqrt{3}$

6 **a** $= 1 + 6x + \frac{6\times5}{2}x^2 + \frac{6\times5\times4}{3\times2}x^3 + \ldots$
$= 1 + 6x + 15x^2 + 20x^3 + \ldots$

b **i** let $x = 0.02$
$1.02^6 \approx 1 + 6(0.02) + 15(0.02)^2 + 20(0.02)^3$
$= 1 + 0.12 + 0.0060 + 0.000\,160$
$= 1.1262$ (4dp)

ii let $x = -0.01$
$0.99^6 \approx 1 + 6(-0.01) + 15(-0.01)^2 + 20(-0.01)^3$
$= 1 - 0.06 + 0.0015 - 0.000\,020$
$= 0.9415$ (4dp)

7 **a** $= 1 + 8(2y) + \frac{8\times7}{2}(2y)^2 + \frac{8\times7\times6}{3\times2}(2y)^3 + \ldots$
$= 1 + 16y + 112y^2 + 448y^3 + \ldots$

b **i** let $y = -0.01$
$0.98^8 \approx 1 + 16(-0.01) + 112(-0.01)^2 + 448(-0.01)^3$
$= 1 - 0.16 + 0.0112 - 0.000\,448$
$= 0.8508$ (4dp)

ii let $y = 0.005$
$1.01^8 \approx 1 + 16(0.005) + 112(0.005)^2 + 448(0.005)^3$
$= 1 + 0.080 + 0.002\,800 + 0.000\,056\,000$
$= 1.0829$ (4dp)

8 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4 + (1 - 4x + 6x^2 - 4x^3 + x^4)$
$= 2 + 12x^2 + 2x^4$

b $= 1 + 3(-\frac{1}{3}x) + 3(-\frac{1}{3}x)^2 + (-\frac{1}{3}x)^3 - [1 + 3(\frac{1}{3}x) + 3(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3]$
$= 1 - x + \frac{1}{3}x^2 - \frac{1}{27}x^3 - (1 + x + \frac{1}{3}x^2 + \frac{1}{27}x^3)$
$= -2x - \frac{2}{27}x^3$

9 **a** $6(ax)^2 = 24x^2$
$a^2 = 4$
$a < 0 \;\therefore\; a = -2$

b $= 4a^3 = -32$

1 **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$
$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

c $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$
$= 27 + 270x^2 + 900x^4 + 1000x^6$

d $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$
$= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$

e $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$
$= x^6 - 3x^4y + 3x^2y^2 - y^3$

f $= 5^4 + 4(5^3)(\tfrac{1}{2}x) + 6(5^2)(\tfrac{1}{2}x)^2 + 4(5)(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 625 + 250x + \tfrac{75}{2}x^2 + \tfrac{5}{2}x^3 + \tfrac{1}{16}x^4$

g $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$
$= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$

h $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$
$= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$

2 **a** $= 1 + 6(3x) + \tfrac{6\times5}{2}(3x)^2 + \tfrac{6\times5\times4}{3\times2}(3x)^3 + \ldots$
$= 1 + 18x + 135x^2 + 540x^3 + \ldots$

b $= 1 + 8(-\tfrac{1}{4}x) + \tfrac{8\times7}{2}(-\tfrac{1}{4}x)^2 + \tfrac{8\times7\times6}{3\times2}(-\tfrac{1}{4}x)^3 + \ldots$
$= 1 - 2x + \tfrac{7}{4}x^2 - \tfrac{7}{8}x^3 + \ldots$

c $= 5^7 + 7(5^6)(-x) + \tfrac{7\times6}{2}(5^5)(-x)^2 + \tfrac{7\times6\times5}{3\times2}(5^4)(-x)^3 + \ldots$
$= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \ldots$

d $= 3^{10} + 10(3^9)(2x^2) + \tfrac{10\times9}{2}(3^8)(2x^2)^2 + \tfrac{10\times9\times8}{3\times2}(3^7)(2x^2)^3 + \ldots$
$= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \ldots$

3 **a** $= \binom{15}{3} = 455$

b $= \binom{12}{4} \times (-2)^4 = 7920$

c $= \binom{7}{2} \times 3^5 = 5103$

d $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$

e $= \binom{8}{5} \times 2^3 = 448$

f $= \binom{9}{3} \times (-1)^3 = -84$

4 **a** $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$
$= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$
$= 89 - 28\sqrt{10}$

b $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$
$= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \tfrac{1}{9}\sqrt{3}$
$= 3\sqrt{2} + \tfrac{19}{9}\sqrt{3}$

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c $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3]$
$= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}]$
$= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}]$
$= 16\sqrt{5}$

5 **a** $= 1 + 10(\frac{x}{2}) + \frac{10\times9}{2}(\frac{x}{2})^2 + \frac{10\times9\times8}{3\times2}(\frac{x}{2})^3 + \ldots$
$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$

b **i** let $x = 0.01$
$1.005^{10} \approx 1 + 0.05 + 0.001\,125 + 0.000\,015$
$= 1.051\,14$ (5dp)

ii let $x = -0.008$
$0.996^{10} \approx 1 - 0.040 + 0.000\,720 - 0.000\,007\,680$
$= 0.960\,71$ (5dp)

6 **a** $= 3^8 + 8(3^7)x + \frac{8\times7}{2}(3^6)x^2 + \frac{8\times7\times6}{3\times2}(3^5)x^3 + \ldots$
$= 6561 + 17\,496x + 20\,412x^2 + 13\,608x^3 + \ldots$

b **i** let $x = 0.001$
$3.001^8 \approx 6561 + 17.496 + 0.020\,412 + 0.000\,013\,608$
$= 6578.516$ (7sf)

ii let $x = -0.005$
$2.995^8 \approx 6561 - 87.480 + 0.510\,300 - 0.001\,701\,000$
$= 6474.029$ (7sf)

7 **a** $(1 + 10x)^4 = 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4$
$= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4$
$\therefore (1 + 10x)^4 + (1 - 10x)^4$
$= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10\,000x^4)$
$= 2 + 1200x^2 + 20\,000x^4$

b $(2 + \frac{1}{3}x)^3 = 2^3 + 3(2^2)(\frac{1}{3}x) + 3(2)(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3$
$= 8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3$
$\therefore (2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$
$= 8 - 4x + \frac{2}{3}x^2 - \frac{1}{27}x^3 - (8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3)$
$= -8x - \frac{2}{27}x^3$

c $= (1 + 4y)(1 + 3y + 3y^2 + y^3)$
$= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4$
$= 1 + 7y + 15y^2 + 13y^3 + 4y^4$

d $= (1 - x)(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3})$
$= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} - x - 3 - \frac{3}{x} - \frac{1}{x^2}$
$= -x - 2 + \frac{2}{x^2} + \frac{1}{x^3}$

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8 **a** $= (1 + x^2)[1 + 10(-3x) + \frac{10\times9}{2}(-3x)^2 + \frac{10\times9\times8}{3\times2}(-3x)^3 + \ldots]$
$= (1 + x^2)[1 - 30x + 405x^2 - 3240x^3 + \ldots]$
$= 1 - 30x + 405x^2 - 3240x^3 + x^2 - 30x^3 + \ldots$
$= 1 - 30x + 406x^2 - 3270x^3 + \ldots$

b $= (1 - 2x)[1 + 8x + \frac{8\times7}{2}x^2 + \frac{8\times7\times6}{3\times2}x^3 + \ldots]$
$= (1 - 2x)[1 + 8x + 28x^2 + 56x^3 + \ldots]$
$= 1 + 8x + 28x^2 + 56x^3 - 2x - 16x^2 - 56x^3 + \ldots$
$= 1 + 6x + 12x^2 + \ldots$

c $= (1 + x + x^2)[1 + 6(-x) + \frac{6\times5}{2}(-x)^2 + \frac{6\times5\times4}{3\times2}(-x)^3 + \ldots]$
$= (1 + x + x^2)[1 - 6x + 15x^2 - 20x^3 + \ldots]$
$= 1 - 6x + 15x^2 - 20x^3 + x - 6x^2 + 15x^3 + x^2 - 6x^3 + \ldots$
$= 1 - 5x + 10x^2 - 11x^3 + \ldots$

d $= (1 + 3x - x^2)[1 + 7(2x) + \frac{7\times6}{2}(2x)^2 + \frac{7\times6\times5}{3\times2}(2x)^3 + \ldots]$
$= (1 + 3x - x^2)[1 + 14x + 84x^2 + 280x^3 + \ldots]$
$= 1 + 14x + 84x^2 + 280x^3 + 3x + 42x^2 + 252x^3 - x^2 - 14x^3 + \ldots$
$= 1 + 17x + 125x^2 + 518x^3 + \ldots$

9 **a** $= \binom{8}{4} \times y^4 \times (\frac{1}{y})^4 = 70$

b $= \binom{12}{6} \times (2y)^6 \times (-\frac{1}{2y})^6 = 924$

c $= \binom{6}{2} \times (\frac{1}{y})^4 \times (y^2)^2 = 15$

d $= \binom{9}{3} \times (3y)^6 \times (-\frac{1}{y^2})^3 = -61\,236$

10 **a** $\frac{n(n-1)}{2} \times (\frac{2}{5})^2 = 1.6$
$n(n - 1) = \frac{25}{2} \times 1.6 = 20$
$n^2 - n - 20 = 0$
$(n + 4)(n - 5) = 0$
$n > 0 \;\; \therefore \; n = 5$

b $= 5 \times (\frac{2}{5})^4 = \frac{16}{125}$ or 0.128

11 **a** $y_1 = (1 - 2x)[1 + 10x + \frac{10\times9}{2}x^2 + \ldots]$
$= 1 + 10x + 45x^2 - 2x - 20x^2 + \ldots$
$= 1 + 8x + 25x^2 + \ldots$
$\therefore\; a = 25, b = 8, c = 1$

b $x = 0.2$: $y_1 = 0.6 \times (1.2)^{10} = 3.71504$
$y_2 = (25\times0.04) + (8\times0.2) + 1 = 3.6$
$\% \text{ error} = \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\%$ (2sf)

12 **a** $(1 + px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \ldots$
$\therefore\; pq = -12$ and $\frac{1}{2}p^2q(q - 1) = 60$
sub. $p = -\frac{12}{q}$
$\Rightarrow \frac{72}{q}(q - 1) = 60$
$72(q - 1) = 60q$
$q = 6,\; p = -2$

b $= \frac{6\times5\times4}{3\times2} \times (-2)^3 = -160$

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13 **a** $= 3^{12} + 12(3^{11})(-\frac{x}{3}) + \frac{12\times11}{2}(3^{10})(-\frac{x}{3})^2 + \frac{12\times11\times10}{3\times2}(3^9)(-\frac{x}{3})^3 + \ldots$

$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \ldots$

b let $\frac{x}{3} = 0.002 \quad \therefore x = 0.006$

$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$

$= 527\,205.03$ (2dp)

14 **a** $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

b $= 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$

c **i** $= [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$

$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$

$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$

$= 76 - 44\sqrt{3}$

ii $= (\sqrt{3} + 1)(76 - 44\sqrt{3})$

$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$

$= -56 + 32\sqrt{3}$

15 **a** $= 1 + 9(\frac{x}{2}) + \frac{9\times8}{2}(\frac{x}{2})^2 + \frac{9\times8\times7}{3\times2}(\frac{x}{2})^3 + \frac{9\times8\times7\times6}{4\times3\times2}(\frac{x}{2})^4 + \ldots$

$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \ldots$

b $= \frac{21}{2} - (-\frac{21}{2}) = 21$

c $= (1 \times \frac{63}{8}) + (2 \times \frac{21}{2}) = 28\frac{7}{8}$

16 $10(x^3)^2(\frac{a}{x^2})^3 = -80$

$a^3 = -8$

$a = -2$

17 **a** $(1 + \frac{x}{k})^n = 1 + n(\frac{x}{k}) + \frac{n(n-1)}{2}(\frac{x}{k})^2 + \frac{n(n-1)(n-2)}{3\times2}(\frac{x}{k})^3 + \ldots$

$\therefore \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$

$kn(n-1) = n(n-1)(n-2)$

$n(n-1)[k - (n-2)] = 0$

$n > 1 \quad \therefore k - (n-2) = 0$

$k = n - 2$

b $k = 7 - 2 = 5$

$(1 + \frac{x}{5})^7 = 1 + 7(\frac{x}{5}) + \frac{7\times6}{2}(\frac{x}{5})^2 + \frac{7\times6\times5}{3\times2}(\frac{x}{5})^3 + \frac{7\times6\times5\times4}{4\times3\times2}(\frac{x}{5})^4 + \ldots$

$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \ldots$

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C2 SEQUENCES AND SERIES — Answers - Worksheet E

1 $= 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4$

$= 1 + 16x + 96x^2 + 256x^3 + 256x^4$

2 **a** $u_5 = 3 \times (-2)^4 = 48$

b $S_{10} = \frac{3[1-(-2)^{10}]}{1-(-2)} = -1023$

c positive terms form GP:
$a = 3$, $r = (-2)^2 = 4$

$S_8 = \frac{3(4^8-1)}{4-1} = 65\,535$

3 **a** $= 1 + 7(3x) + \frac{7\times6}{2}(3x)^2$

$+ \frac{7\times6\times5}{3\times2}(3x)^3 + \frac{7\times6\times5\times4}{4\times3\times2}(3x)^4 + \ldots$

$= 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \ldots$

b let $x = 0.01$

$1.03^7 \approx 1 + 0.21 + 0.0189$
$+ 0.000\,945 + 0.000\,028\,35$
$= 1.229\,87$ (5dp)

4 GP: $a = 8$, $r = 2$, $n = 10$

$S_{10} = \frac{8(2^{10}-1)}{2-1} = 8184$

5 **a** $= 2^5 + 5(2^4)x + 10(2^3)x^2$
$+ 10(2^2)x^3 + 5(2)x^4 + x^5$
$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$

b $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

c $(2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2$
$+ 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5$

$= 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5}$

$= 682 + 305\sqrt{5}$

$\therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5$

$= (682 + 305\sqrt{5}) - (682 - 305\sqrt{5})$

$= 610\sqrt{5}$, $k = 610$

6 **a** amount in account after 3rd payment in
$= 200 + (1.005 \times 200) + (1.005^2 \times 200)$
$= 603.005$
interest paid at end of 3rd month
$= 0.005 \times 603.005 = £3.02$ (nearest penny)

b amount paid in $= 12 \times 200 = £2400$

amount in account after 12 months

$= 200(1.005 + 1.005^2 + \ldots + 1.005^{12})$

$= 200 \times S_{12}$ [GP: $a = 1.005$, $r = 1.005$]

$= 200 \times \frac{1.005(1.005^{12}-1)}{1.005-1} = 2479.45$

total interest $= 2479.45 - 2400 = £79.45$

7 $= 1 + 8(-3x) + \frac{8\times7}{2}(-3x)^2$

$+ \frac{8\times7\times6}{3\times2}(-3x)^3 + \ldots$

$= 1 - 24x + 252x^2 - 1512x^3 + \ldots$

8 **a** $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$

subtracting, $S_n - rS_n = a - ar^n$
$S_n(1 - r) = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $r = 6 \div 3 = 2$
$a \times 2^3 = 3 \quad \therefore a = \frac{3}{8}$

$S_{16} = \frac{\frac{3}{8}(2^{16}-1)}{2-1} = 24\,575\frac{5}{8}$

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9 **a** $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$

$= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \ldots$

b $\frac{1}{2}a^2n(n-1) = 3an$

$a^2n(n-1) = 6an$

$an[a(n-1) - 6] = 0$

$n \neq 0 \;\therefore\; a(n-1) - 6 = 0$

$an - a = 6$

$n = \frac{6+a}{a}$

c $n = 10 \;\therefore$ coeff. of $x^3 = \frac{10\times9\times8}{3\times2} \times (\frac{2}{3})^3 = 35\frac{5}{9}$

10 $= 2^6 + 6(2^5)(5x) + \frac{6\times5}{2}(2^4)(5x)^2 + \ldots$

$= 64 + 960x + 6000x^2 + \ldots$

11 **a** $\frac{162}{1-r} = 486$

$1 - r = \frac{162}{486} = \frac{1}{3} \quad \therefore r = \frac{2}{3}$

b $u_6 = 162 \times (\frac{2}{3})^5 = \frac{64}{3}$ or $21\frac{1}{3}$

c $S_{10} = \frac{162[1-(\frac{2}{3})^{10}]}{1-\frac{2}{3}} = 477.572$

12 **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$

$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b term in $x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$

coefficient of $x^2 = 54 + 48 - 1 = 101$

13 **a** time $= 120 \times (0.9)^3 = 87.48$ seconds

b GP: $a = 120, r = 0.9, n = 12$

$S_{12} = \frac{120[1-(0.9)^{12}]}{1-0.9}$

$= 861.08$ seconds

$= 14$ mins 21 secs (nearest sec.)

14 $= [1+8(\frac{x}{2})+\frac{8\times7}{2}(\frac{x}{2})^2+\ldots][1+6(-x)+\frac{6\times5}{2}(-x)^2+\ldots]$

$= [1 + 4x + 7x^2 + \ldots][1 - 6x + 15x^2 + \ldots]$

$= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \ldots$

$= 1 - 2x - 2x^2 + \ldots$

$\therefore\; A = -2, B = -2$

15 **a** 6, 12, 24, 48

b GP: $a = 6, r = 2, n = 10$

$S_{10} = \frac{6(2^{10}-1)}{2-1} = 6138$

16 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$

b $= 1 - 4x + 6x^2 - 4x^3 + x^4$

c $(1 + 4x + 6x^2 + 4x^3 + x^4)$

$+ (1 - 4x + 6x^2 - 4x^3 + x^4) = 82$

$2 + 12x^2 + 2x^4 = 82$

$x^4 + 6x^2 - 40 = 0$

$(x^2 + 10)(x^2 - 4) = 0$

$x^2 = -10$ [no real solutions] or $x^2 = 4$

$x = \pm 2$

17 **a** $a \times (1.5)^2 = 18$

$a = 18 \div 2.25 = 8$

b $S_6 = \frac{8[(1.5)^6 - 1]}{1.5-1} = 166.25$

c $8 \times (1.5)^{k-1} > 8000$

$(k-1) \lg 1.5 > \lg 1000$

$k > \frac{\lg 1000}{\lg 1.5} + 1$

$k > 18.04 \;\therefore$ smallest $k = 19$

18 $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$

$= 1 + 10(\frac{ax}{2}) + \frac{10\times9}{2}(\frac{ax}{2})^2 + \ldots$

$+ 1 + 10(bx) + \frac{10\times9}{2}(bx)^2 + \ldots$

$= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \ldots$

$\therefore\; 5a + 10b = 0 \quad \Rightarrow \quad a = -2b$

and $\frac{45}{4}a^2 + 45b^2 = 90 \quad \Rightarrow \quad a^2 + 4b^2 = 8$

sub. $(-2b)^2 + 4b^2 = 8$

$b^2 = 1$

$a < b \;\therefore\; b = 1, a = -2$

C2 SEQUENCES AND SERIES — Answers - Worksheet F

1 **a** $a = 108,\ ar^3 = 32$

$\therefore\ r^3 = 32 \div 108 = \frac{8}{27}$

$r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

$u_3 = 108 \times (\frac{2}{3})^2 = 48$

b $S_\infty = \dfrac{108}{1-\frac{2}{3}} = 324$

2 $= 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

3 **a** new subscribers in 4th week

$= 200 \times (1.15)^3 = 304.175$

$= 304$ (nearest unit)

b new subscribers: GP, $a = 200$, $r = 1.15$

$S_{10} = \dfrac{200[(1.15)^{10}-1]}{1.15-1} = 4060.74$

total no. of subscribers $= 3600 + S_{10}$

$= 7661$ (nearest unit)

4 **a** $= 1 + 7(4x) + \frac{7\times6}{2}(4x)^2 + \ldots$

$= 1 + 28x + 336x^2 + \ldots$

b $(1 + 2x)^2(1 + 4x)^7$

$= (1 + 4x + 4x^2)(1 + 28x + 336x^2 + \ldots)$

term in x^2

$= (1)(336x^2) + (4x)(28x) + (4x^2)(1)$

coefficient of $x^2 = 336 + 112 + 4 = 452$

5 **a** $= 1 + 2n(\frac{x}{k}) + \frac{2n(2n-1)}{2}(\frac{x}{k})^2 + \frac{2n(2n-1)(2n-2)}{3\times2}(\frac{x}{k})^3 + \ldots$

$= 1 + \dfrac{2n}{k}x + \dfrac{n(2n-1)}{k^2}x^2 + \dfrac{2n(n-1)(2n-1)}{3k^3}x^3 + \ldots$

b $\dfrac{2n(n-1)(2n-1)}{3k^3} = \dfrac{1}{2} \times \dfrac{n(2n-1)}{k^2}$

$4n(n-1)(2n-1) = 3kn(2n-1)$

$n(2n-1)[4(n-1) - 3k] = 0$

$n > 1\ \therefore\ 4(n-1) - 3k = 0$

$3k = 4(n-1)$

c $\dfrac{2n}{k} = 2\ \therefore\ n = k$

$\therefore\ 3k = 4k - 4$

$k = 4,\ n = 4$

6 **a** $r = 3\sqrt{2} \div \sqrt{6} = \sqrt{3}$

$a = \sqrt{6} \div \sqrt{3} = \sqrt{2}$

b $S_8 = \dfrac{\sqrt{2}[(\sqrt{3})^8 - 1]}{\sqrt{3}-1}$

$= \dfrac{80\sqrt{2}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{80\sqrt{2}(\sqrt{3}+1)}{3-1}$

$= 40\sqrt{2}(\sqrt{3}+1)$

7 $\sum_{r=1}^{9} 3^r$: GP, $a = 3, r = 3$

$S_9 = \dfrac{3(3^9-1)}{3-1} = 29523$

$\therefore\ \sum_{r=1}^{9}(3^r - 1) = 29523 - 9$

$= 29\,514$

8 **a** $= 1 + 9(2x) + \frac{9\times8}{2}(2x)^2 + \frac{9\times8\times7}{3\times2}(2x)^3 + \ldots$

$= 1 + 18x + 144x^2 + 672x^3 + \ldots$

b $(1 - 2x)^9 = 1 - 18x + 144x^2 - 672x^3 + \ldots$

$\therefore\ (1 + 2x)^9 + (1 - 2x)^9$

$= (1 + 18x + 144x^2 + 672x^3 + \ldots) + (1 - 18x + 144x^2 - 672x^3 + \ldots)$

$= 2 + 288x^2$ (ignoring terms in x^4 and higher)

c let $x = 0.001$

$\therefore\ 1.002^9 + 0.998^9 \approx 2 + 0.000\,288$

$= 2.000\,288$ (7sf)

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9 $(k-x)^9 = k^9 + 9(k^8)(-x) + \frac{9\times 8}{2}(k^7)(-x)^2 + \ldots$
$= k^9 - 9k^8x + 36k^7x^2 + \ldots$
$\therefore\ -b = -9k^8$ and $b = 36k^7$
$9k^8 = 36k^7$
$9k^7(k-4) = 0$
$k \neq 0\ \therefore k = 4$
$a = k^9 = 262\,144$
$b = 9k^8 = 589\,824$

10 $= 3^4 + 4(3)^3(2x) + 6(3)^2(2x)^2 + 4(3)(2x)^3 + (2x)^4$
$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$

11 **a** $\frac{t}{1-r} = 3t$
$1 - r = \frac{t}{3t} = \frac{1}{3}\ \therefore r = \frac{2}{3}$
b $\frac{t[1-(\frac{2}{3})^4]}{1-\frac{2}{3}} = 130$
$t = (\frac{1}{3} \times 80) \div \frac{65}{81} = 54$

12 **a** $= 1 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3 + (-2x)^4$
$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$
b let $x = y^2 - 2y$
$(1 + 4y - 2y^2)^4$
$= 1 - 8(y^2 - 2y) + 24(y^2 - 2y)^2 + \ldots$
term in $y^2 = -8y^2 + 24(-2y)^2$
coefficient of $y^2 = -8 + 96 = 88$

13 **a** $= 12000 \times (0.75)^4$
$= 3796.875$
$= £3800$ (3sf)
b GP: $a = 12000$, $r = 0.75$
$S_8 = \frac{12000[1-(0.75)^8]}{1-0.75}$
$= £43\,200$ (3sf)

14 **a** $p(-2) = 1^4 - (-1)^4 = 1 - 1 = 0$
$\therefore\ (x+2)$ is a factor of $p(x)$
b $p(x) = [x^4 + 4(x^3)(3) + 6(x^2)(3^2) + 4(x)(3^3) + 3^4] - [x^4 + 4x^3 + 6x^2 + 4x + 1]$
$= 8x^3 + 48x^2 + 104x + 80$
$= 8(x^3 + 6x^2 + 13x + 10)$

$$\begin{array}{r|l} & x^2 + 4x + 5 \\ \hline x+2 & x^3 + 6x^2 + 13x + 10 \\ & x^3 + 2x^2 \\ \hline & 4x^2 + 13x \\ & 4x^2 + 8x \\ \hline & 5x + 10 \\ & 5x + 10 \\ \hline \end{array}$$

$p(x) = 8(x+2)(x^2 + 4x + 5)$
c $8(x+2)(x^2 + 4x + 5) = 0$
$x = -2$ or $(x^2 + 4x + 5) = 0$
$b^2 - 4ac = 16 - 20 = -4$
$b^2 - 4ac < 0\ \therefore$ no real sols to $(x^2 + 4x + 5) = 0$
$\therefore$ only one real solution to $p(x) = 0$

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15 **a** $(1 - x)(1 + 2x)^n$

$= (1 - x)[1 + n(2x) + \frac{n(n-1)}{2}(2x)^2 + \ldots]$

$= (1 - x)[1 + 2nx + 2n(n - 1)x^2 + \ldots]$

$\therefore\ 2n(n - 1) - 2n = 198$

$n^2 - 2n - 99 = 0$

$(n + 9)(n - 11) = 0$

$n \geq 0\ \therefore\ n = 11$

b $(1 - x)(1 + 2x)^{11}$

$= (1 - x)[\ldots + \frac{11\times10}{2}(2x)^2 + \frac{11\times10\times9}{3\times2}(2x)^3 + \ldots]$

$= (1 - x)[\ldots + 220x^2 + 1320x^3 + \ldots]$

$\therefore$ coefficient of $x^3 = 1320 - 220 = 1100$

16 $= (\frac{3}{x})^4 + 4(\frac{3}{x})^3(-x) + 6(\frac{3}{x})^2(-x)^2$

$+ 4(\frac{3}{x})(-x)^3 + (-x)^4$

$= x^4 - 12x^2 + 54 - \frac{108}{x^2} + \frac{81}{x^4}$

17 **a** $S_4 = 3^4 - 1 = 80$

$S_3 = 3^3 - 1 = 26$

$u_4 = S_4 - S_3 = 80 - 26 = 54$

b $S_{n-1} = 3^{n-1} - 1$

$u_n = S_n - S_{n-1}$

$= (3^n - 1) - (3^{n-1} - 1)$

$= 3^n - 3^{n-1}$

$= 3^n(1 - \frac{1}{3}) = \frac{2}{3}(3^n)$ $\quad [k = \frac{2}{3}]$

c $u_{n-1} = \frac{2}{3}(3^{n-1})$

$u_n \div u_{n-1} = \frac{2}{3}(3^n) \div \frac{2}{3}(3^{n-1}) = 3$

$u_n \div u_{n-1}$ is constant $\therefore$ geometric

18 **a** $3(x - 3) = y - 3$

$y = 3x - 6$

b $\left(\frac{x}{3}\right)^3 = \frac{y}{3}$

$x^3 = 9y = 9(3x - 6)$

$x^3 - 27x + 54 = 0$

c trying $x = 1, 2$ etc. $\Rightarrow$ $x = 3$ is a solution

$\therefore$ $(x - 3)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & +\ 3x & -\ 18 & \\ \hline x-3 & x^3 & +\ 0x^2 & -\ 27x & +\ 54 \\ & x^3 & -\ 3x^2 & & \\ \hline & & 3x^2 & -\ 27x & \\ & & 3x^2 & -\ 9x & \\ \hline & & & -\ 18x & +\ 54 \\ & & & -\ 18x & +\ 54 \\ \hline \end{array}$$

$(x - 3)(x^2 + 3x - 18) = 0$

$(x - 3)(x + 6)(x - 3) = 0$

$x = -6$ or 3