1 Show in each case that there is a root of the equation $f(x) = 0$ in the given interval.

a $f(x) = x^3 + 3x - 7$ (1, 2) **b** $f(x) = 5\cos x - 3x$ (0.5, 1)

c $f(x) = 2e^x + x + 5$ (−6, −5) **d** $f(x) = x^4 - 5x^2 + 1$ (2.1, 2.2)

e $f(x) = \ln(4x - 1) + x^2$ (0.4, 0.5) **f** $f(x) = e^{-x} - 9\cos 4x$ (10, 11)

2 Given that $|N| \le 5$, find in each case the integer N such that there is a root of the equation $f(x) = 0$ in the interval $(N, N + 1)$.

a $f(x) = x^3 - 3\sqrt{x} - 4$ **b** $f(x) = x\ln x - \frac{12}{x}$ **c** $f(x) = 2x^5 + 4x + 15$

d $f(x) = e^{x-1} + 4x - 2$ **e** $f(x) = e^x - 3\sin x$ **f** $f(x) = \tan(0.1x) + x - 6$

3 Show in each case that there is a root of the given equation in the given interval.

a $x^3 = 12 - \frac{x}{4}$ [2, 3] **b** $12e^x = 9 - 4x$ [−1, 0]

c $10\ln 3x = 5 - 7x^2$ [0.47, 0.48] **d** $\sin 4x = 7e^x$ [−6.5, −6]

e $4^x = 3x + 10$ [−4, −3] **f** $\tan(\frac{1}{2}x) = 2x - 1$ [2.6, 2.7]

4 In each case there is a root of the equation $f(x) = 0$ in the given interval.
Find the integer, a, such that this root lies in the interval $(\frac{a}{10}, \frac{a+1}{10})$.

a $f(x) = x^4 + \frac{3}{x} - 5$ (1, 2) **b** $f(x) = x - \ln(6 + x^2)$ (2, 3)

c $f(x) = 5x^3 - 3x^2 + 11$ (−2, −1) **d** $f(x) = \frac{8}{x} - \cos x$ (11, 12)

e $f(x) = \operatorname{cosec} x + \sqrt{x}$ (5, 6) **f** $f(x) = x^2 - 7e^{2x+5}$ (−3, −2)

5 **a** On the same set of axes, sketch the graphs of $y = x^3$ and $y = 4 - x$.

b Hence, show that the equation $x^3 + x - 4 = 0$ has exactly one real root.

c Show that this root lies in the interval (1, 1.5).

6 $$f : x \to x\ln x - 1, \quad x \in \mathbb{R}, \quad x > 0.$$

a On the same set of axes, sketch the curves $y = \ln x$ and $y = \frac{1}{x}$.

b Hence show that the equation $f(x) = 0$ has exactly one real root.

The real root of $f(x) = 0$ is α.

c Find the integer n such that $n < \alpha < n + 1$.

7 **a** On the same set of axes, sketch the curves $y = e^x$ and $y = 5 - x^2$.

b Hence show that the equation $e^x + x^2 - 5 = 0$ has exactly one negative and one positive real root.

c Show that the negative root lies in the interval (−3, −2).

The positive root, α, is such that $\frac{n}{10} < \alpha < \frac{n+1}{10}$, where n is an integer.

d Find the value of n.

C3 NUMERICAL METHODS Worksheet B

1 For each equation, show that it can be rearranged into the given iterative form. Use this and the given value of x_0 to find x_1, x_2 and x_3. Give your value of x_3 correct to 4 decimal places.

a $9 + 4x - 2x^3 = 0$ $\quad x_{n+1} = \sqrt[3]{2x_n + 4.5}$ $\quad x_0 = 2$

b $e^x - 8x + 5 = 0$ $\quad x_{n+1} = \ln(8x_n - 5)$ $\quad x_0 = 3$

c $\tan x - 5x + 13 = 0$ $\quad x_{n+1} = \arctan(5x_n - 13)$ $\quad x_0 = -1.2$

d $\ln x + \sqrt{x} + 1.4 = 0$ $\quad x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$ $\quad x_0 = 0.16$

2 For each equation, show that it can be rearranged into the given iterative form and state the values of the constants a and b. Use this and the given value of x_0 to find x_1, x_2 and x_3. Give your value of x_3 correct to 3 decimal places.

a $e^{2x-1} - 6x = 0$ $\quad x_{n+1} = a(\ln bx_n + 1)$ $\quad x_0 = 1.7$

b $\frac{2}{x} + \cos x - 3 = 0$ $\quad x_{n+1} = \frac{a}{b - \cos x_n}$ $\quad x_0 = 0.8$

c $2x^3 - 6x - 11 = 0$ $\quad x_{n+1} = \sqrt{a + \frac{b}{x_n}}$ $\quad x_0 = 2$

d $15\ln(x + 3) - 4x = 0$ $\quad x_{n+1} = e^{ax_n} + b$ $\quad x_0 = -2.5$

3 In each case, use the given iteration formula and value of x_0 to find a root of the equation $f(x) = 0$ to the stated degree of accuracy. Justify the accuracy of your answers.

a $f(x) = 10^x + 3x - 4$ $\quad x_{n+1} = \log_{10}(4 - 3x_n)$ $\quad x_0 = 0.44$ $\quad$ 3 decimal places

b $f(x) = x^2 + \frac{1}{x-5}$ $\quad x_{n+1} = \sqrt{\frac{x_n^3 + 1}{5}}$ $\quad x_0 = 0.5$ $\quad$ 2 significant figures

c $f(x) = 30 - 5x + \sin 2x$ $\quad x_{n+1} = 6 + 0.2\sin 2x_n$ $\quad x_0 = 6$ $\quad$ 3 significant figures

d $f(x) = e^{4-x} - \ln x$ $\quad x_{n+1} = 4 - \ln(\ln x_n)$ $\quad x_0 = 3.7$ $\quad$ 3 decimal places

4 $$f(x) = x^5 - 10x^3 + 4.$$

The equation $f(x) = 0$ has a root in the interval $-4 < x < -3$.

a Use the iteration formula $x_{n+1} = \sqrt[5]{10x_n^3 - 4}$ and the starting value $x_0 = -3.2$ to find the value of this root correct to 2 decimal places.

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = \sqrt[3]{\frac{a}{b - x_n^2}}$.

b Find the values of the constants a and b in this formula.

The equation $f(x) = 0$ has another root in the interval $0 < x < 1$.

c Using the iteration formula with your values from part **b** and the starting value $x_0 = 1$, find the value of this root correct to 3 decimal places.

5 $$f : x \rightarrow \arcsin 2x - 0.5x - 0.7, \ x \in \mathbb{R}, \ |x| \leq 0.5$$

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = a\sin(bx_n + c)$.

a Find the values of the constants a, b and c in this formula.

The equation $f(x) = 0$ has a solution in the interval (0.3, 0.4).

b Using the iterative formula with your values from part **a** and the starting value $x_0 = 0.4$, find this solution correct to 3 decimal places.

1 **a** Show that the equation $x^3 - 7x - 11 = 0$ has a real root in the interval (3, 4).

b Using the iterative formula $x_{n+1} = \sqrt{7 + \frac{11}{x_n}}$, with $x_0 = 3.2$, find x_1, x_2 and x_3, giving the value of x_3 correct to 2 decimal places.

2 $$f(x) \equiv 4\operatorname{cosec} x - 5 + 2x.$$

a Find the values of f(4) and f(5).

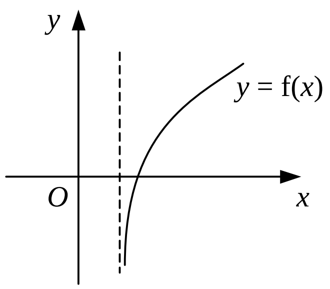
b Hence show that the equation $f(x) = 0$ has a root in the interval (4, 5).

The iterative formula $x_{n+1} = a + \frac{b}{\sin x_n}$, where a and b are constants, is used to find this root.

c Find the values of a and b.

d Starting with $x_0 = 4.5$, use the iterative formula with your values of a and b to find 3 further approximations of the root, giving your final answer correct to 3 decimal places.

3



The diagram shows the curve with equation $y = f(x)$ where

$$f : x \to 2x + \ln(3x - 1),\ x \in \mathbb{R},\ x > \tfrac{1}{3}.$$

Given that $f(\alpha) = 0$,

a show that $0.4 < \alpha < 0.5$,

b use the iterative formula $x_{n+1} = \frac{1}{3}(1 + e^{-2x_n})$, with $x_0 = 0.45$, to find the value of α correct to 3 decimal places.

4 **a** On the same set of axes, sketch the curves $y = \cos x$ and $y = x^2$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

b Show that the equation $\cos x - x^2 = 0$ has exactly one positive and one negative real root.

c Show that the positive real root lies in the interval [0.8, 0.9].

d Use the iteration formula $x_{n+1} = \sqrt{\cos x_n}$ and the starting value $x_0 = 0.8$ to find the positive root correct to 2 decimal places.

5 $$f(x) \equiv e^{5-2x} - x^5.$$

Show that the equation $f(x) = 0$

a has a root in the interval (1.4, 1.5),

b can be written as $x = e^{1-kx}$, stating the value of k.

c Using the iteration formula $x_{n+1} = e^{1-kx_n}$, with $x_0 = 1.5$ and the value of k found in part **b**, find x_1, x_2 and x_3. Give the value of x_3 correct to 3 decimal places.

6 $f : x \to 2^x + x^3 - 5,\ x \in \mathbb{R}.$

a Show that there is a solution of the equation $f(x) = 0$ in the interval $1.3 < x < 1.4$

b Using the iterative formula $x_{n+1} = \sqrt[3]{5 - 2^{x_n}}$, with $x_0 = 1.4$, find x_1, x_2, x_3 and x_4.

c Hence write down an approximation for this solution of the equation $f(x) = 0$ to an appropriate degree of accuracy.

Another attempt is made to find the solution using the iterative formula $x_{n+1} = \dfrac{\ln(5 - x_n^3)}{\ln 2}$.

d Describe the outcome of this attempt.

7 $f(x) = 2x^3 + 4x - 9.$

a Find $f'(x)$.

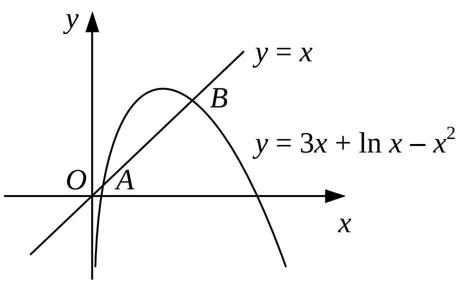
b Hence show that the equation $f(x) = 0$ has exactly one real root.

c Show that this root lies in the interval $(1.2, 1.3)$.

d Use the iterative formula $x_{n+1} = \sqrt[3]{4.5 - 2x_n}$, with $x_0 = 1.2$, to find the root of $f(x) = 0$ correct to 2 decimal places.

e Justify the accuracy of your answer.

8



The diagram shows part of the curve with equation $y = 3x + \ln x - x^2$ and the line $y = x$.
Given that the curve and line intersect at the points A and B, show that

a the x-coordinates of A and B are the solutions of the equation $x = e^{x^2 - 2x}$,

b the x-coordinate of A lies in the interval $(0.4, 0.5)$,

c the x-coordinate of B lies in the interval $(2.3, 2.4)$.

d Use the iteration formula $x_{n+1} = e^{x_n^2 - 2x_n}$, with $x_0 = 0.5$, to find the x-coordinate of A correct to 2 decimal places.

e Justify the accuracy of your answer to part **d**.

9 **a** On the same set of axes, sketch the graphs of $y = x^4$ and $y = 5x + 2$.

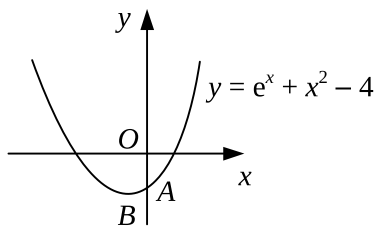
b Show that the equation $x^4 - 5x - 2 = 0$ has exactly one positive and one negative real root.

c Use the iteration formula $x_{n+1} = \sqrt[4]{5x_n + 2}$, with $x_0 = 1.8$, to find x_1, x_2, x_3 and x_4, giving the value of x_4 correct to 3 decimal places.

d Show that the equation $x^4 - 5x - 2 = 0$ can be written in the form $x = \dfrac{a}{x^3 + b}$, stating the values of a and b.

e Use the iteration formula $x_{n+1} = \dfrac{a}{x_n^3 + b}$, with $x_0 = -0.4$ and your values of a and b, to find the negative real root of the equation correct to 4 decimal places.

1



The diagram shows the curve $y = e^x + x^2 - 4$. The curve intersects the y-axis at the point A and has a stationary point at B.

a Find $\frac{dy}{dx}$. **(1)**

b Find an equation for the tangent to the curve at A. **(2)**

c Show that the x-coordinate of B lies in the interval [–0.4, –0.3]. **(3)**

d Using the iteration formula $x_{n+1} = \frac{1}{3}(x_n - e^{x_n})$, with $x_0 = -0.3$, find the x-coordinate of B correct to 3 decimal places. **(4)**

2 The function f is defined by

$$f(x) \equiv \sin(x - 6) - \ln(x^2 + 1), \quad x \in \mathbb{R},$$

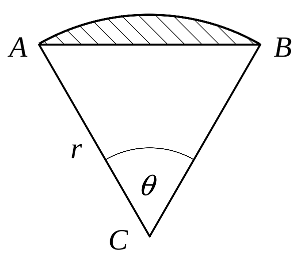
where x is measured in radians.

The equation $f(x) = 0$ has a root in the interval $k < x < k + 1$, where k is a positive integer.

a Find the value of k. **(3)**

b Use the iteration formula $x_{n+1} = \sqrt{e^{\sin(x_n - 6)} - 1}$, with $x_0 = k$, to find three further approximations for this root, giving your answers to 4 decimal places. **(3)**

3



The diagram shows a sector ABC of a circle, centre C, radius r. Angle ACB is θ radians.

Given that the ratio of the area of the shaded segment to the area of triangle ABC is 1 : 4,

a show that $4\theta - 5\sin\theta = 0$, **(4)**

b use the iterative formula $\theta_{n+1} = \frac{5}{4}\sin\theta_n$, with $\theta_0 = 1.1$, to find the value of θ correct to 2 decimal places. **(4)**

4

$$f : x \to e^{x^2} - x - 3, \quad x \in \mathbb{R}.$$

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = \sqrt{\ln(ax_n + b)}$.

a Find the values of the constants a and b in this formula. **(3)**

The equation $f(x) = 0$ has a solution in the interval (1, 2).

b Using the iterative formula with your values from part **a** and a suitable starting value, find this solution correct to 3 decimal places. **(4)**

5 $$f : x \to x^2 - 9,\ x \in \mathbb{R},\ x \geq 0,$$
$$g : x \to x^3,\ x \in \mathbb{R}.$$

a Find $f^{-1}(x)$ and state its domain and range. **(4)**

b On the same set of axes, sketch the curves $y = f(x)$ and $y = f^{-1}(x)$. **(2)**

c Show that the equation $f^{-1}(x) + g(x) = 0$ has a root in the interval $[-2, -1]$. **(3)**

d Use the iterative formula $x_{n+1} = -(x_n + 9)^{\frac{1}{6}}$, with $x_0 = -1$, to find this root correct to 3 decimal places. **(4)**

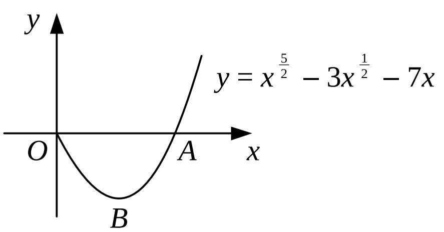
6 **a** On the same diagram, sketch the curves $y = \frac{1}{x}$ and $y = |-x^2 - 3x|$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

The curves intersect at the point P.

b Show that the x-coordinate of P can be found by solving the equation $x^3 + 3x^2 - 1 = 0$. **(3)**

c Use the iteration formula $x_{n+1} = \frac{1}{\sqrt{x_n + 3}}$, with $x_0 = 0$, to find the x-coordinate of P correct to 3 decimal places. **(4)**

7



The diagram shows the curve $y = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x$, $x \geq 0$, which crosses the x-axis at the point A, where $x = \alpha$, and has a stationary point at B, where $x = \beta$.

Show that

a $4 < \alpha < 5$, **(2)**

b $2 < \beta < 3$, **(4)**

c $x = \beta$ is a solution to the equation $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$. **(3)**

d Use the iterative formula $x_{n+1} = \sqrt{0.6 + 2.8x_n^{\frac{1}{2}}}$, with $x_0 = 2.1$, to find β correct to 4 significant figures. **(4)**

8 The curve with equation $y = 3x - \ln x$ passes through the point P (1, 3).

a Find an equation for the normal to the curve at P. **(4)**

The normal to the curve at P intersects the curve again at the point Q.

b Show that the x-coordinate of Q satisfies the equation

$$2 \ln x - 7x + 7 = 0.$$ **(1)**

The x-coordinate of Q is to be found using an iteration of the form $x_{n+1} = e^{k(x_n - 1)}$.

c Find the value of the constant k. **(2)**

d Using $x_0 = 0.5$, find the x-coordinate of Q correct to 3 decimal places. **(4)**

e Justify the accuracy of your answer to part **d**. **(2)**

1 **a** $f(1) = -3 \quad f(2) = 7$
sign change, $f(x)$ continuous $\therefore$ root

b $f(0.5) = 2.89 \quad f(1) = -0.298$
sign change, $f(x)$ continuous $\therefore$ root

c $f(-6) = -0.995 \quad f(-5) = 0.0135$
sign change, $f(x)$ continuous $\therefore$ root

d $f(2.1) = -1.60 \quad f(2.2) = 0.226$
sign change, $f(x)$ continuous $\therefore$ root

e $f(0.4) = -0.351 \quad f(0.5) = 0.25$
sign change, $f(x)$ continuous $\therefore$ root

f $f(10) = 6.00 \quad f(11) = -9.00$
sign change, $f(x)$ continuous $\therefore$ root

2 **a** $f(0) = -4$
$f(3) = 17.8$
$f(1) = -6$
$f(2) = -0.243$
$\therefore N = 2$

b $f(1) = -12$
$f(5) = 5.65$
$f(3) = -0.704$
$f(4) = 2.55$
$\therefore N = 3$

c $f(0) = 15$
$f(-2) = -57$
$f(-1) = 9$
$\therefore N = -2$

d $f(0) = -1.63$
$f(1) = 3$
$\therefore N = 0$

e $f(0) = 1$
$f(-5) = -2.87$
$f(-4) = -2.25$
$f(-3) = 0.473$
$\therefore N = -4$

f $f(0) = -6$
$f(4) = -1.58$
$f(5) = -0.454$
$f(6) = 0.684$
$\therefore N = 5$

3 **a** let $f(x) = x^3 - 12 + \frac{x}{4}$
$f(2) = -3.5 \quad f(3) = 15.75$
sign change, $f(x)$ continuous $\therefore$ root

b let $f(x) = 12e^x - 9 + 4x$
$f(-1) = -8.59 \quad f(0) = 3$
sign change, $f(x)$ continuous $\therefore$ root

c let $f(x) = 10 \ln 3x - 5 + 7x^2$
$f(0.47) = -0.0178 \quad f(0.48) = 0.259$
sign change, $f(x)$ continuous $\therefore$ root

d let $f(x) = \sin 4x - 7e^x$
$f(-6.5) = -0.773 \quad f(-6) = 0.888$
sign change, $f(x)$ continuous $\therefore$ root

e let $f(x) = 4^x - 3x - 10$
$f(-4) = 2.00 \quad f(-3) = -0.984$
sign change, $f(x)$ continuous $\therefore$ root

f let $f(x) = \tan(\frac{1}{2}x) - 2x + 1$
$f(2.6) = -0.598 \quad f(2.7) = 0.0552$
sign change, $f(x)$ continuous $\therefore$ root

4 **a** $f(1) = -1$
$f(2) = 12.5$
$f(1.1) = -0.809$
$f(1.2) = -0.426$
$f(1.3) = 0.164$
$\therefore a = 12$

b $f(2) = -0.303$
$f(3) = 0.292$
$f(2.5) = -0.00553$
$f(2.6) = 0.0537$
$\therefore a = 25$

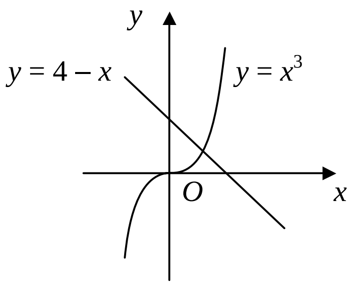
c $f(-2) = -41$
$f(-1) = 3$
$f(-1.1) = 0.715$
$f(-1.2) = -1.96$
$\therefore a = -12$

d $f(11) = 0.723$
$f(12) = -0.177$
$f(11.7) = 0.0362$
$f(11.8) = -0.0425$
$\therefore a = 117$

e $f(5) = 1.19$
$f(6) = -1.13$
$f(5.5) = 0.928$
$f(5.8) = 0.256$
$f(5.9) = -0.246$
$\therefore a = 58$

f $f(-3) = 6.42$
$f(-2) = -15.0$
$f(-2.7) = 2.60$
$f(-2.6) = 1.03$
$f(-2.5) = -0.75$
$\therefore a = -26$

5 **a**



b $x^3 + x - 4 = 0 \Rightarrow x^3 = 4 - x$

the graphs $y = x^3$ and $y = 4 - x$

intersect at exactly one point

$\therefore$ one real root

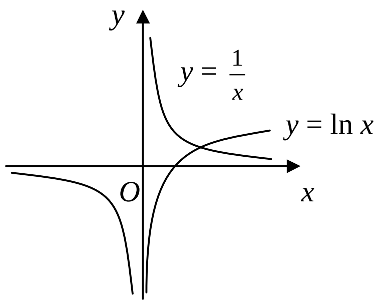
c let $f(x) = x^3 + x - 4$

$f(1) = -2$

$f(1.5) = 0.875$

sign change, $f(x)$ continuous $\therefore$ root

6 **a**



b $x \ln x - 1 = 0 \Rightarrow x \ln x = 1 \Rightarrow \ln x = \frac{1}{x}$

the graphs $y = \ln x$ and $y = \frac{1}{x}$

intersect at exactly one point

$\therefore$ one real root

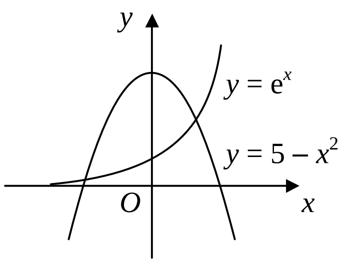
c $f(1) = -1$

$f(2) = 0.386$

$\therefore 1 < \alpha < 2$

$\therefore n = 1$

7 **a**



b $e^x + x^2 - 5 = 0 \Rightarrow e^x = 5 - x^2$

the graphs $y = e^x$ and $y = 5 - x^2$

intersect at two points,

one for $x < 0$ and one for $x > 0$

$\therefore$ one negative and one positive real root

c let $f(x) = e^x + x^2 - 5$

$f(-3) = 4.05$

$f(-2) = -0.865$

sign change, $f(x)$ continuous $\therefore$ root

d $f(1) = -1.28$

$f(2) = 6.39$

$f(1.2) = -0.240$

$f(1.3) = 0.359$

$\therefore 1.2 < \alpha < 1.3$

$\therefore n = 12$

1 **a** $9 + 4x - 2x^3 = 0$
$x^3 = 2x + 4.5$
$x = \sqrt[3]{2x+4.5}$
$\therefore x_{n+1} = \sqrt[3]{2x_n + 4.5}$
$x_1 = 2.040828$
$x_2 = 2.047342$
$x_3 = 2.048377 = 2.0484$ (4dp)

b $e^x - 8x + 5 = 0$
$e^x = 8x - 5$
$x = \ln (8x - 5)$
$\therefore x_{n+1} = \ln (8x_n - 5)$
$x_1 = 2.944439$
$x_2 = 2.920767$
$x_3 = 2.910508 = 2.9105$ (4dp)

c $\tan x - 5x + 13 = 0$
$\tan x = 5x - 13$
$x = \arctan (5x - 13)$
$\therefore x_{n+1} = \arctan (5x_n - 13)$
$x_1 = -1.518213$
$x_2 = -1.522270$
$x_3 = -1.522317 = -1.5223$ (4dp)

d $\ln x + \sqrt{x} + 1.4 = 0$
$\ln x = -(\sqrt{x} + 1.4)$
$x = e^{-(\sqrt{x}+1.4)}$
$\therefore x_{n+1} = e^{-(\sqrt{x_n}+1.4)}$
$x_1 = 0.165299$
$x_2 = 0.164216$
$x_3 = 0.164436 = 0.1644$ (4dp)

2 **a** $e^{2x-1} - 6x = 0$
$e^{2x-1} = 6x$
$2x - 1 = \ln 6x$
$x = \frac{1}{2}(\ln 6x + 1)$
$\therefore x_{n+1} = \frac{1}{2}(\ln 6x_n + 1)$, $a = \frac{1}{2}$, $b = 6$
$x_1 = 1.661194$
$x_2 = 1.649648$
$x_3 = 1.646161 = 1.646$ (3dp)

b $\frac{2}{x} + \cos x - 3 = 0$
$\frac{2}{x} = 3 - \cos x$
$2 = x(3 - \cos x)$
$x = \frac{2}{3 - \cos x}$
$\therefore x_{n+1} = \frac{2}{3 - \cos x_n}$, $a = 2$, $b = 3$
$x_1 = 0.868322$
$x_2 = 0.849657$
$x_3 = 0.854789 = 0.855$ (3dp)

c $2x^3 - 6x - 11 = 0$
$2x^3 = 6x + 11$
$x^2 = 3 + \frac{11}{2x}$
$x = \pm\sqrt{3 + \frac{11}{2x}}$
$\therefore x_{n+1} = \sqrt{3 + \frac{5.5}{x_n}}$, $a = 3$, $b = 5.5$
$x_1 = 2.397916$
$x_2 = 2.300795$
$x_3 = 2.321740 = 2.322$ (3dp)

d $15 \ln (x + 3) - 4x = 0$
$\ln (x + 3) = \frac{4}{15}x$
$x + 3 = e^{\frac{4}{15}x}$
$x = e^{\frac{4}{15}x} - 3$
$\therefore x_{n+1} = e^{\frac{4}{15}x_n} - 3$, $a = \frac{4}{15}$, $b = -3$
$x_1 = -2.486583$
$x_2 = -2.484743$
$x_3 = -2.484490 = -2.484$ (3dp)

3 **a** $x_1 = 0.428135$
$x_2 = 0.433865$
$x_3 = 0.431107$
$x_4 = 0.432437$
$x_5 = 0.431796$
$\therefore$ root = 0.432 (3dp)
f(0.4315) = –0.00465
f(0.4325) = 0.00457
sign change, f(x) continuous $\therefore$ root

b $x_1 = 0.474342$
$x_2 = 0.470474$
$x_3 = 0.469923$
$\therefore$ root = 0.47 (2sf)
f(0.465) = –0.00428
f(0.475) = 0.00463
sign change, f(x) continuous $\therefore$ root

c $x_1 = 5.892685$
$x_2 = 5.859202$
$x_3 = 5.850013$
$x_4 = 5.847607$
$x_5 = 5.846985$
$x_6 = 5.846825$
$\therefore$ root = 5.85 (3sf)
f(5.845) = 0.00658
f(5.855) = –0.0305
sign change, f(x) continuous $\therefore$ root

d $x_1 = 3.731246$
$x_2 = 3.724839$
$x_3 = 3.726145$
$x_4 = 3.725879$
$\therefore$ root = 3.726 (3dp)
f(3.7255) = 0.000672
f(3.7265) = –0.000912
sign change, f(x) continuous $\therefore$ root

4 **a** $x_1 = -3.192595$
$x_2 = -3.188214$
$x_3 = -3.185620$
$x_4 = -3.184084$
$x_5 = -3.183174$
$x_6 = -3.182635$
$\therefore$ root = –3.18 (2dp)

b $x^5 - 10x^3 + 4 = 0$
$4 = 10x^3 - x^5 = x^3(10 - x^2)$
$x^3 = \frac{4}{10 - x^2}$
$x = \sqrt[3]{\frac{4}{10 - x^2}}$ $\therefore a = 4, b = 10$

c $x_1 = 0.763143$
$x_2 = 0.751692$
$x_3 = 0.751231$
$x_4 = 0.751212$
$\therefore$ root = 0.751 (3dp)

5 **a** $\arcsin 2x - 0.5x - 0.7 = 0$
$\arcsin 2x = 0.5x + 0.7$
$2x = \sin(0.5x + 0.7)$
$x = 0.5\sin(0.5x + 0.7)$ $\therefore a = 0.5, b = 0.5, c = 0.7$

b $x_1 = 0.391663$
$x_2 = 0.390365$
$x_3 = 0.390162$
$x_4 = 0.390130$
$\therefore$ solution = 0.390 (3dp)

1 **a** let $f(x) = x^3 - 7x - 11$
$f(3) = -5$
$f(4) = 25$
sign change, $f(x)$ continuous $\therefore$ root

b $x_1 = 3.230712$
$x_2 = 3.225651$
$x_3 = 3.226479 = 3.23$ (2dp)

2 **a** $f(4) = -2.29$ (3sf)
$f(5) = 0.829$ (3sf)

b sign change, $f(x)$ continuous $\therefore$ root

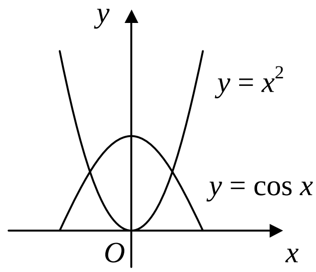
c $4\operatorname{cosec} x - 5 + 2x = 0$
$2x = 5 - 4\operatorname{cosec} x$
$x = 2.5 - \frac{2}{\sin x}$, $a = 2.5$, $b = -2$

d $x_1 = 4.545973$
$x_2 = 4.528018$
$x_3 = 4.534481 = 4.534$ (3dp)

3 **a** $f(0.4) = -0.809$
$f(0.5) = 0.307$
sign change, $f(x)$ continuous $\therefore$ root
$\therefore$ $0.4 < \alpha < 0.5$

b $x_1 = 0.468857$
$x_2 = 0.463841$
$x_3 = 0.465157$
$x_4 = 0.464810$
$\therefore$ $\alpha = 0.465$ (3dp)

4 **a**



b $\cos x - x^2 = 0 \;\Rightarrow\; \cos x = x^2$
the graphs $y = \cos x$ and $y = x^2$ intersect at 2 points, one for $x < 0$ and one for $x > 0$
$\therefore$ one negative and one positive real root

c let $f(x) = \cos x - x^2$
$f(0.8) = 0.0567$
$f(0.9) = -0.188$
sign change, $f(x)$ continuous $\therefore$ root

d $x_1 = 0.834690$
$x_2 = 0.819395$
$x_3 = 0.826235$
$x_4 = 0.823195$
$x_5 = 0.824550$
$\therefore$ root = 0.82 (2dp)

5 **a** $f(1.4) = 3.65$
$f(1.5) = -0.205$
sign change, $f(x)$ continuous $\therefore$ root

b $e^{5-2x} - x^5 = 0 \;\Rightarrow\; x^5 = e^{5-2x}$
$\Rightarrow\; x = (e^{5-2x})^{\frac{1}{5}}$
$\Rightarrow\; x = e^{1-\frac{2}{5}x}$, $k = \frac{2}{5}$

c $x_1 = 1.491825$
$x_2 = 1.496711$
$x_3 = 1.493789 = 1.494$ (3dp)

6 **a** $f(1.3) = -0.341$
$f(1.4) = 0.383$
sign change, $f(x)$ continuous $\therefore$ root

b $x_1 = 1.331571$
$x_2 = 1.354168$
$x_3 = 1.346907$
$x_4 = 1.349261$

c 1.35 (3sf)

d diverges leading to ln of a –ve which is not real

7 **a** $f'(x) = 6x^2 + 4$

b for all real x, $x^2 \geq 0$

$\Rightarrow 6x^2 + 4 > 0$

$\therefore$ f(x) increasing for all x

$\therefore$ y = f(x) only crosses x-axis once so exactly 1 real root

c f(1.2) = –0.744

f(1.3) = 0.594

sign change, f(x) continuous $\therefore$ root

d $x_1 = 1.280579$

$x_2 = 1.246945$

$x_3 = 1.261203$

$x_4 = 1.255199$

$\therefore$ root = 1.26 (2dp)

e f(1.255) = –0.0267

f(1.265) = 0.109

sign change, f(x) continuous $\therefore$ root

8 **a** $3x + \ln x - x^2 = x \Rightarrow \ln x = x^2 - 2x$

$\Rightarrow x = e^{x^2 - 2x}$

b let $f(x) = 2x + \ln x - x^2$

f(0.4) = –0.276

f(0.5) = 0.0569

sign change, f(x) continuous $\therefore$ root

c f(2.3) = 0.143

f(2.4) = –0.0845

sign change, f(x) continuous $\therefore$ root

d $x_1 = 0.472367$

$x_2 = 0.485973$

$x_3 = 0.479134$

$x_4 = 0.482537$

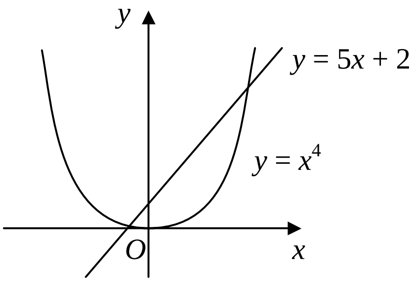
$\therefore$ x-coord of A = 0.48 (2dp)

e f(0.475) = –0.0201

f(0.485) = 0.0112

sign change, f(x) continuous $\therefore$ root

9 **a**



b $x^4 - 5x - 2 = 0 \Rightarrow x^4 = 5x + 2$

the graphs $y = x^4$ and $y = 5x + 2$ intersect at 2 points, one for $x < 0$ and one for $x > 0$

$\therefore$ one negative and one positive real root

c $x_1 = 1.821160$

$x_2 = 1.825524$

$x_3 = 1.826420$

$x_4 = 1.826603 = 1.827$ (3dp)

d $x^4 - 5x - 2 = 0 \Rightarrow x^4 - 5x = 2$

$\Rightarrow x(x^3 - 5) = 2$

$\Rightarrow x = \frac{2}{x^3 - 5}$, $a = 2$, $b = -5$

e $x_1 = -0.394945$

$x_2 = -0.395132$

$x_3 = -0.395125$

$\therefore$ root = –0.3951 (4dp)

1 **a** $\frac{dy}{dx} = e^x + 2x$

b at A, $x = 0$ $\therefore y = -3$, grad = 1
$\therefore y = x - 3$

c SP: $e^x + 2x = 0$
let $f(x) = e^x + 2x$
$f(-0.4) = -0.130$
$f(-0.3) = 0.141$
sign change, f(x) continuous $\therefore$ root
$\therefore$ x-coord of B in interval $[-0.4, -0.3]$

d $x_1 = -0.34694$
$x_2 = -0.35126$
$x_3 = -0.35169$
$x_4 = -0.35173$
$\therefore$ x-coord of $B = -0.352$ (3dp)

2 **a** $f(0) = 0.279$
$f(5) = -4.10$
$f(1) = 0.266$
$f(3) = -2.44$
$f(2) = -0.853$
$\therefore k = 1$

b $x_0 = 1$
$x_1 = 1.2684$
$x_2 = 1.3106$
$x_3 = 1.3106$

3 **a** area of segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$
$= \frac{1}{2}r^2(\theta - \sin\theta)$
$\therefore \frac{1}{2}r^2\sin\theta = 4 \times \frac{1}{2}r^2(\theta - \sin\theta)$
$\sin\theta = 4(\theta - \sin\theta)$
$\sin\theta = 4\theta - 4\sin\theta$
$4\theta - 5\sin\theta = 0$

b $\theta_1 = 1.11401$
$\theta_2 = 1.12184$
$\theta_3 = 1.12613$
$\theta_4 = 1.12844$
$\theta_5 = 1.12968$
$\therefore \theta = 1.13$ (2dp)

4 **a** $e^{x^2} - x - 3 = 0$
$e^{x^2} = x + 3$
$x^2 = \ln(x + 3)$
$x = \sqrt{\ln(x+3)}$ $\therefore a = 1, b = 3$

b e.g. $x_0 = 1.5$
$x_1 = 1.226408$
$x_2 = 1.200563$
$x_3 = 1.198006$
$x_4 = 1.197752$
$x_5 = 1.197727$
$\therefore$ solution = 1.198 (3dp)

5 **a** $y = x^2 - 9$

swap $x = y^2 - 9$

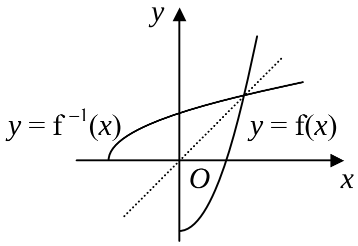
$y = \pm\sqrt{x+9}$

(domain $\Rightarrow$ +)

$f^{-1}(x) = \sqrt{x+9}$, $x \in \mathbb{R}$, $x \geq -9$

range: $f^{-1}(x) \geq 0$

b



c let $h(x) = f^{-1}(x) + g(x) = \sqrt{x+9} + x^3$

$h(-2) = -5.35$

$h(-1) = 1.83$

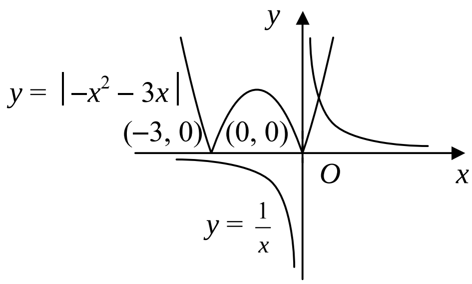
sign change, $h(x)$ continuous $\therefore$ root

d $x_1 = -1.41421$, $x_2 = -1.40174$,

$x_3 = -1.40212$, $x_4 = -1.40211$

$\therefore$ root $= -1.402$ (3dp)

6 **a**



b $-(-x^2 - 3x) = \frac{1}{x}$

$x^2 + 3x = \frac{1}{x}$

$x^3 + 3x^2 = 1$

$x^3 + 3x^2 - 1 = 0$

c $x_1 = 0.57735$

$x_2 = 0.52871$

$x_3 = 0.53234$

$x_4 = 0.53207$

$\therefore$ x-coord of $P = 0.532$ (3dp)

7 **a** at A, $x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x = 0$

let $f(x) = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x$

$f(4) = -2$, $f(5) = 14.2$

sign change, $f(x)$ continuous $\therefore$ root

$\therefore$ $4 < \alpha < 5$

b $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$

at B, $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$

let $g(x) = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$

$g(2) = -0.990$, $g(3) = 5.12$

sign change, $g(x)$ continuous $\therefore$ root

$\therefore$ $2 < \beta < 3$

c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$

$5x^2 - 3 - 14x^{\frac{1}{2}} = 0$

$x^2 = 0.6 + 2.8x^{\frac{1}{2}}$

$x > 0$ $\therefore$ $x = \beta$ is a soln to $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$

d $x_1 = 2.158144$

$x_2 = 2.171031$

$x_3 = 2.173853$

$x_4 = 2.174470$

$x_5 = 2.174604$

$\therefore$ $\beta = 2.175$ (4sf)

8 **a** $\frac{dy}{dx} = 3 - \frac{1}{x}$

grad = 2

$\therefore$ grad of normal $= -\frac{1}{2}$

$\therefore$ $y - 3 = -\frac{1}{2}(x - 1)$

$[\, y = \frac{7}{2} - \frac{1}{2}x \,]$

b $3x - \ln x = \frac{7}{2} - \frac{1}{2}x$

$6x - 2\ln x = 7 - x$

$2\ln x - 7x + 7 = 0$

c $2\ln x = 7x - 7$

$\ln x = \frac{7}{2}(x - 1)$

$x = e^{\frac{7}{2}(x-1)}$ $\therefore$ $k = \frac{7}{2}$

d $x_1 = 0.173774$

$x_2 = 0.055477$

$x_3 = 0.036669$

$x_4 = 0.034333$

$x_5 = 0.034053$

$\therefore$ x-coord of $Q = 0.034$ (3dp)

e let $f(x) = 2\ln x - 7x + 7$

$f(0.0335) = -0.027$

$f(0.0345) = 0.025$

sign change, $f(x)$ continuous $\therefore$ root