Paper: P3 **Date: June 1996**

The points $A(24, 6, 0)$, $B(30, 12, 12)$ and $C(18, 6, 36)$ are referred to cartesian axes, origin O.

(a) Find a vector equation for the line passing through the points A and B.

The points P lies on the line passing through A and B.

(b) Show that $\overrightarrow{CP}$ can be expressed as

$$(6 + t)\mathbf{i} + t\mathbf{j} + (2t - 36)\mathbf{k}, \text{ where } t \text{ is a parameter.}$$

(c) Given that $\overrightarrow{CP}$ is perpendicular to $\overrightarrow{AB}$, find the coordinates of P.

(d) Hence, or otherwise, find the area of the triangle ABC, giving your answer to 3 significant figures.

[16]

Question 2

Paper: P3 **Date: January 1997**

The lines l_1, l_2 and l_3 are given by

$$l_1 : \mathbf{r} = 10\mathbf{i} + \mathbf{j} + 9\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$$
$$l_2 : x = \frac{y+9}{2} = \frac{z-13}{-3},$$
$$l_3 : \mathbf{r} = -3\mathbf{i} - 5\mathbf{j} - 4\mathbf{k} + \lambda(4\mathbf{i} + 3\mathbf{j} + \mathbf{k}),$$

where μ and λ are parameters.

(a) Show that the point $A(4, -1, 1)$ lies on both l_1 and l_2.

(b) Rewrite the equation for l_2 in the form $\mathbf{r} = \mathbf{a} + v\mathbf{b}$, where v is a parameter.

(c) Show that l_2 and l_3 intersect and find the coordinates of B, the point of intersection.

The lines l_1 and l_3 intersect at the point $C(1, -2, -3)$.

(d) Show that $AC = BC$.

(e) Find the size of angle ACB, giving your answer in degrees to the nearest degree.

(f) Write down the coordinates of the point D on AB such that CD is perpendicular to AB.

[18]

Question 3

Paper: P3 **Date: June 1997**

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + \lambda(-2\mathbf{i} + 4\mathbf{j} + \mathbf{k}),$$
$$l_2 : \mathbf{r} = (-6\mathbf{i} - 3\mathbf{j} + \mathbf{k}) + \mu(5\mathbf{i} + \mathbf{j} - 2\mathbf{k}),$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(b) Find, to the nearest 0.1°, the acute angle between l_1 and l_2.

[10]

Question 4

Paper: P3 **Date: January 1998**

Referred to a fixed origin O, the points P, Q and R have position vectors $(2\mathbf{i} + \mathbf{j} + \mathbf{k})$, $(5\mathbf{j} + 3\mathbf{k})$ and $(5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively.

(a) Find in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line PQ.

(b) Show that the point S with position vector $(4\mathbf{i} - 3\mathbf{j} - \mathbf{k})$ lies on PQ.

(c) Show that the lines PQ and RS are perpendicular.

(d) Find the size of $\angle PQR$, giving your answer to 0.1°.

[13]

Question 5

Paper: P3 Date: June 1998

With respect to an origin O, the position vectors of the points L and M are $\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ and $2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ respectively.

(a) Write down the vector $\overrightarrow{LM}$.

(b) Show that $|\overrightarrow{OL}| = |\overrightarrow{LM}|$.

(c) Find $\angle OLM$, giving your answer to the nearest tenth of a degree.

[7]

Question 6

Paper: P3 Date: January 1999

The line l passes through the points with position vectors $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 6\mathbf{j}$ relative to an origin O.

(a) Find an equation for l in vector form.

The line m has equation $\mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$.

(b) Find the acute angle between l and m, giving your answer to the nearest degree.

[7]

Question 7

Paper: P3 **Date: June 1999**

Referred to a fixed origin, the line l_1 has equation $\mathbf{r} = 12\mathbf{i} + 5\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and the line l_2 has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} + \mu(3\mathbf{i} - \mathbf{k})$.

(a) Show that the lines l_1 and l_2 intersect and find the position vector of A, their point of intersection.

(b) Find, to the nearest degree, the acute angle between the lines l_1 and l_2.

The point B with position vector $16\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$ lies on the line l_1. The point C with position vector $22\mathbf{i} + \mathbf{j} - 9\mathbf{k}$ and the point D both lie on the line l_2.

(c) Given that BD is perpendicular to AC, find the position vector of D.

(d) Hence, or otherwise, prove that ΔABC is isosceles.

[17]

Question 8

Paper: P3 **Date: January 2000**

(a) Find a vector equation of the straight line l through the points with position vectors $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} + 2\mathbf{k}$, relative to the origin O.

The line m has cartesian equations $7 - x = \frac{y+6}{2} = z + 4$.

(b) Write down a vector equation of m.

(c) Show that the lines l and m meet and find the position vector of their point of intersection.

The distinct points P and Q lie on l and are each a distance $\sqrt{17}$ units from O.

(d) Find the position vectors of P and Q.

[16]

Question 9

Paper: P3 **Date: June 2000**

The straight line l has equation

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k}),$$

and the point P has coordinates $(2, 3, -1)$.

(a) Show that the coordinates of the foot of the perpendicular from P to l are $(\frac{2}{3}, 3\frac{2}{3}, -\frac{1}{3})$.

A circle centre P cuts the line l at the points A and B. Given that the coordinates of A are $(1, 4, 0)$,

(b) find the coordinates of B.

[10]

Question 10

Paper: P3 **Date: June 2001**

Two submarines are travelling in straight lines through the ocean. Relative to a fixed origin, the vector equations of the two lines, l_1 and l_2, along which they travel are

$$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$
$$\text{and } \mathbf{r} = 9\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \mu(4\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ and μ are scalars.

(a) Show that the submarines are moving in perpendicular directions.

(b) Given that l_1 and l_2 intersect at the point A, find the position vector of A.

The point B has position vector $10\mathbf{j} - 11\mathbf{k}$.

(c) Show that only one of the submarines passes through the point B.

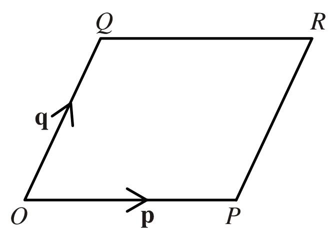
(d) Given that 1 unit on each coordinate axis represents 100 m, find, in km, the distance AB.

[12]

Question 11

Paper: P3 **Date: June (old) 2001**

Figure 1



(i) Figure 1 shows a parallelogram OPRQ, with $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

(a) Express $\overrightarrow{OR}$ and $\overrightarrow{OR}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

Given that $\mathbf{p} = 5\mathbf{i} + 2\mathbf{j} - 11\mathbf{k}$ and $\mathbf{q} = \mathbf{i} + 6\mathbf{j} - 13\mathbf{k}$,

(b) find the acute angle between the diagonals OR and PQ of the parallelogram, giving your answer to the nearest degree.

(ii) **(a)** Given two vectors $\mathbf{x}$ and $\mathbf{y}$, such that the scalar product $(\mathbf{x} + \mathbf{y}) \cdot (\mathbf{x} - \mathbf{y})$ is zero, show that $|\mathbf{x}| = |\mathbf{y}|$.

(b) Given that $\mathbf{x}$ and $\mathbf{y}$ represent adjacent sides of a parallelogram, give a geometrical interpretation to part (ii)(a).

[14]

Question 12

Paper: P3　　**Date: January (old) 2001**

The lines with vector equations

$$\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 9\mathbf{i} + 7\mathbf{j} + 9\mathbf{k} + \mu(-\mathbf{i} + \mathbf{j} + \mathbf{k})$$

meet at the point P.

(a) Find the coordinates of the point P.

(b) Find $\cos \alpha$ where α is the acute angle between the two lines.

(c) Express the exact value for the area of the triangle with vertices at the points (1, 3, 5), (9, 7, 9) and P in the form $N\sqrt{2}$, giving the value of the integer N.

[13]

Total = 153