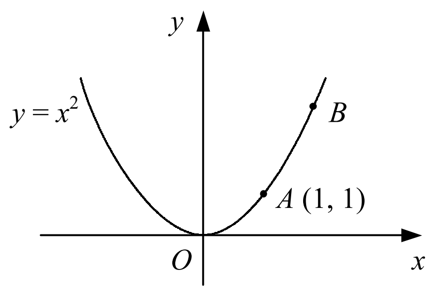
C1 DIFFERENTIATION Worksheet A

You will need to use a calculator for this worksheet

1



The diagram shows the curve $y = x^2$ which passes through the point A (1, 1) and the point B.

a Copy and complete the table to find the gradient of the chord AB when the x-coordinate of B takes each of the given values.

x-coordinate of B	y-coordinate of B	gradient of AB
2	4	$\frac{4-1}{2-1} = 3$
1.1	1.21	
1.01		
1.001		

b Suggest a value for the gradient of the tangent to the curve $y = x^2$ at the point (1, 1).

c Repeat part **a** using 0, 0.9, 0.99 and 0.999 as the x-coordinates of B and comment on your answer to part **b**.

2 Use a similar table of values to that in question **1** to find a value for the gradient of the tangent to the curve $y = x^2$ at the point A when A has the coordinates

a (2, 4) **b** (4, 16) **c** (1.5, 2.25) **d** (–3, 9)

3 **a** Using your answers to questions **1** and **2**, suggest an expression in terms of x for the gradient of the curve $y = x^2$ at the point (x, y).

b Write down the gradient of the curve $y = x^2$ at the points

i (6, 36) **ii** (2.4, 5.76) **iii** (–3.2, 10.24)

4 By considering the gradient of a suitable sequence of chords, find a value for the gradient of each curve at the given point.

a $y = x^4$ at (1, 1) **b** $y = x^2 - 5x + 3$ at (2, –3)

c $y = \sqrt{x}$ at (4, 2) **d** $y = \frac{2}{x}$ at (2, 1)

5 **a** By considering the gradient of a suitable sequence of chords, find a value for the gradient of the curve $y = x^3$ at the points

i (1, 1) **ii** (2, 8) **iii** (3, 27)

b Suggest an expression of the form kx^n for the gradient of the curve $y = x^3$ at the point (x, y).

c Find the gradient of the curve $y = x^3$ at the points

i (4, 64) **ii** (–2, –8) **iii** (1.5, 3.375)

C1 DIFFERENTIATION Worksheet B

1 Differentiate with respect to x

a x^2 **b** x^4 **c** x **d** x^9 **e** x^{-3} **f** x^{-1}

g $4x^2$ **h** $7x$ **i** $2x^5$ **j** 3 **k** $8x^{-2}$ **l** $11x^{-4}$

2 Find $\frac{dy}{dx}$

a $y = x^5 + x^2$ **b** $y = x + x^3$ **c** $y = x^4 + 2$ **d** $y = x^6 - 2x$

e $y = 6x^3 + 5x^{-2}$ **f** $y = x^2 - 4x + 1$ **g** $y = x^{-1} - x^{-5}$ **h** $y = 4x^3 + 3x^{-4}$

3 Differentiate with respect to t

a t^6 **b** $5t^{-3}$ **c** $t^{\frac{1}{2}}$ **d** $t^{\frac{2}{3}}$ **e** $\frac{3}{4}t^2$ **f** $8t^{\frac{1}{4}}$

g $2t^{\frac{7}{2}}$ **h** $t^{-\frac{1}{5}}$ **i** $\frac{1}{2}t^{\frac{6}{5}}$ **j** $t^{-\frac{3}{2}}$ **k** $12t^{-\frac{5}{4}}$ **l** $\frac{1}{6}t^{\frac{4}{3}}$

4 Find $f'(x)$

a $f(x) = 2x + \frac{1}{3}x^6$ **b** $f(x) = x^{\frac{3}{2}} - 5$ **c** $f(x) = x + 4x^{\frac{1}{2}}$ **d** $f(x) = 6x^{\frac{5}{3}} - x^{-4}$

e $f(x) = 7 + x^{-\frac{4}{5}}$ **f** $f(x) = 2x^{\frac{1}{6}} + x^{\frac{3}{4}}$ **g** $f(x) = 3x^{-1} - 5x^{-\frac{3}{2}}$ **h** $f(x) = 2 - 7x^{-1} + x^{-\frac{8}{3}}$

5 Find $\frac{dy}{dx}$

a $y = \sqrt{x}$ **b** $y = 4 - \frac{1}{x}$ **c** $y = 3x^2 + \sqrt[3]{x}$ **d** $y = 9x + \frac{3}{x}$

e $y = \frac{1}{4x} - \frac{1}{x^2}$ **f** $y = \frac{6}{\sqrt[4]{x}}$ **g** $y = \sqrt{x^5}$ **h** $y = 8\sqrt{x} + \frac{4}{3x^2}$

6 Find $\frac{ds}{dt}$

a $s = t(t + 3)$ **b** $s = (t - 2)^2$ **c** $s = 5t(t^3 + 4t)$ **d** $s = t^2(7t - t^{-1})$

e $s = (t + 1)(t + 6)$ **f** $s = (t - 4)(t + 2)$ **g** $s = t(t^4 + 3t^2 + 9)$ **h** $s = t(t - 1)(2t - 3)$

7 Find $\frac{dy}{dx}$

a $y = \sqrt{x}(x - 4)$ **b** $y = \frac{x^3 - 2x}{x}$ **c** $y = \frac{4x^3 + x}{x^2}$ **d** $y = \frac{x + 3}{\sqrt{x}}$

e $y = \frac{4 - x^3}{2x}$ **f** $y = \frac{5 + \sqrt{x}}{x^2}$ **g** $y = \frac{9x - 2}{3x}$ **h** $y = \frac{8x + x^3}{4\sqrt{x}}$

8 In each case, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

a $y = 4x^2 - x + 3$ **b** $y = x^3 + 5x^2 + 2x - 6$ **c** $y = 8 - \frac{2}{x}$

d $y = 2x^4 + 3x^2 - 9$ **e** $y = \frac{3x^6 - 4}{x^2}$ **f** $y = 6x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

C1 DIFFERENTIATION Worksheet C

1 Find the gradient at the point with x-coordinate 3 on each of the following curves.

a $y = x^3$ **b** $y = 4x - x^2$ **c** $y = 2x^2 - 8x + 3$ **d** $y = \frac{3}{x} + 2$

2 Find the gradient of each curve at the given point.

a $y = 3x^2 + x - 5$ $(1, -1)$ **b** $y = x^4 + 2x^3$ $(-2, 0)$

c $y = x(2x - 3)$ $(2, 2)$ **d** $y = x^2 - 2x^{-1}$ $(2, 3)$

e $y = x^2 + 6x + 8$ $(-3, -1)$ **f** $y = 4x + x^{-2}$ $(\frac{1}{2}, 6)$

3 Evaluate $f'(4)$ when

a $f(x) = (x + 1)^2$ **b** $f(x) = x^{\frac{1}{2}}$ **c** $f(x) = x - 4x^{-2}$ **d** $f(x) = 5 - 6x^{\frac{3}{2}}$

4 The curve with equation $y = x^3 - 4x^2 + 3x$ crosses the x-axis at the points A, B and C.

a Find the coordinates of the points A, B and C.

b Find the gradient of the curve at each of the points A, B and C.

5 For the curve with equation $y = 2x^2 - 5x + 1$,

a find $\frac{dy}{dx}$,

b find the value of x for which $\frac{dy}{dx} = 7$.

6 Find the coordinates of the points on the curve with the equation $y = x^3 - 8x$ at which the gradient of the curve is 4.

7 A curve has the equation $y = x^3 + x^2 - 4x + 1$.

a Find the gradient of the curve at the point P $(-1, 5)$.

Given that the gradient at the point Q on the curve is the same as the gradient at the point P,

b find, as exact fractions, the coordinates of the point Q.

8 Find an equation of the tangent to each curve at the given point.

a $y = x^2$ $(2, 4)$ **b** $y = x^2 + 3x + 4$ $(-1, 2)$

c $y = 2x^2 - 6x + 8$ $(1, 4)$ **d** $y = x^3 - 4x^2 + 2$ $(3, -7)$

9 Find an equation of the tangent to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = 3 - x^2$ $(-3, -6)$ **b** $y = \frac{2}{x}$ $(2, 1)$

c $y = 2x^2 + 5x - 1$ $(\frac{1}{2}, 2)$ **d** $y = x - 3\sqrt{x}$ $(4, -2)$

10 Find an equation of the normal to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = x^2 - 4$ $(1, -3)$ **b** $y = 3x^2 + 7x + 7$ $(-2, 5)$

c $y = x^3 - 8x + 4$ $(2, -4)$ **d** $y = x - \frac{6}{x}$ $(3, 1)$

11 Find, in the form $y = mx + c$, an equation of

a the tangent to the curve $y = 3x^2 - 5x + 2$ at the point on the curve with x-coordinate 2,

b the normal to the curve $y = x^3 + 5x^2 - 12$ at the point on the curve with x-coordinate -3.

12 A curve has the equation $y = x^3 + 3x^2 - 16x + 2$.

a Find an equation of the tangent to the curve at the point P (2, −10).

The tangent to the curve at the point Q is parallel to the tangent at the point P.

b Find the coordinates of the point Q.

13 A curve has the equation $y = x^2 - 3x + 4$.

a Find an equation of the normal to the curve at the point A (2, 2).

The normal to the curve at A intersects the curve again at the point B.

b Find the coordinates of the point B.

14 $$f(x) \equiv x^3 + 4x^2 - 18.$$

a Find $f'(x)$.

b Show that the tangent to the curve $y = f(x)$ at the point on the curve with x-coordinate -3 passes through the origin.

15 The curve C has the equation $y = 6 + x - x^2$.

a Find the coordinates of the point P, where C crosses the positive x-axis, and the point Q, where C crosses the y-axis.

b Find an equation of the tangent to C at P.

c Find the coordinates of the point where the tangent to C at P meets the tangent to C at Q.

16 The straight line l is a tangent to the curve $y = x^2 - 5x + 3$ at the point A on the curve.

Given that l is parallel to the line $3x + y = 0$,

a find the coordinates of the point A,

b find the equation of the line l in the form $y = mx + c$.

17 The line with equation $y = 2x + k$ is a normal to the curve with equation $y = \frac{16}{x^2}$.

Find the value of the constant k.

18 A ball is thrown vertically downwards from the top of a cliff. The distance, s metres, of the ball from the top of the cliff after t seconds is given by $s = 3t + 5t^2$.

Find the rate at which the distance the ball has travelled is increasing when

a $t = 0.6$,

b $s = 54$.

19 Water is poured into a vase such that the depth, h cm, of the water in the vase after t seconds is given by $h = kt^{\frac{1}{3}}$, where k is a constant. Given that when $t = 1$, the depth of the water in the vase is increasing at the rate of 3 cm per second,

a find the value of k,

b find the rate at which h is increasing when $t = 8$.

C1 DIFFERENTIATION Worksheet D

1 $f(x) = (x + 1)(x - 2)^2$.

a Sketch the curve $y = f(x)$, showing the coordinates of any points where the curve meets the coordinate axes. **(3)**

b Find $f'(x)$. **(4)**

c Show that the tangent to the curve $y = f(x)$ at the point where $x = 1$ has the equation

$$y = 5 - 3x.$$ **(3)**

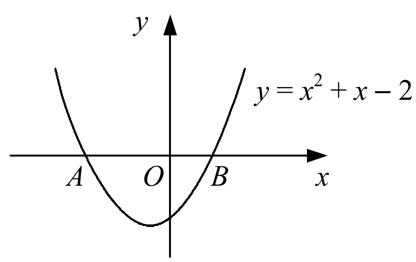
2 The curve *C* has the equation $y = x - 3x^{\frac{1}{2}} + 3$ and passes through the point *P* (4, 1).

a Show that the tangent to *C* at *P* passes through the origin. **(5)**

The normal to *C* at *P* crosses the *y*-axis at the point *Q*.

b Find the area of triangle *OPQ*, where *O* is the origin. **(4)**

3



The diagram shows the curve $y = x^2 + x - 2$. The curve crosses the *x*-axis at the points *A* (*a*, 0) and *B* (*b*, 0) where $a < b$.

a Find the values of *a* and *b*. **(3)**

b Show that the normal to the curve at *A* has the equation

$$x - 3y + 2 = 0.$$ **(5)**

The tangent to the curve at *B* meets the normal to the curve at *A* at the point *C*.

c Find the exact coordinates of *C*. **(4)**

4 Given that $y = \dfrac{x^2 - 6x - 3}{3x^{\frac{1}{2}}}$, show that $\dfrac{dy}{dx}$ can be expressed in the form $\dfrac{(x+a)^2}{bx^{\frac{3}{2}}}$, where *a* and *b* are integers to be found. **(6)**

5 The point *A* lies on the curve $y = \dfrac{12}{x^2}$ and the *x*-coordinate of *A* is 2.

a Find an equation of the tangent to the curve at *A*. Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **(5)**

b Verify that the point where the tangent at *A* intersects the curve again has the coordinates (–1, 12). **(3)**

6 A curve has the equation $y = 2 + 3x + kx^2 - x^3$ where *k* is a constant.

Given that the gradient of the curve is –6 at the point *P* where $x = -1$,

a find the value of *k*. **(4)**

Given also that the tangent to the curve at the point *Q* is parallel to the tangent at *P*,

b find the length *PQ*, giving your answer in the form $k\sqrt{5}$. **(5)**

7 Differentiate $x^2 + \frac{1}{2x}$ with respect to x. **(3)**

8 A curve has the equation $y = 2x^2 - 7x + 1$ and the point A on the curve has x-coordinate 2.

a Find an equation of the tangent to the curve at A. **(4)**

The normal to the curve at the point B is parallel to the tangent at A.

b Find the coordinates of B. **(3)**

9
$$y = x^2 + 3x^{\frac{1}{2}}.$$

a Find $\frac{dy}{dx}$. **(2)**

b Show that $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x = 0$. **(4)**

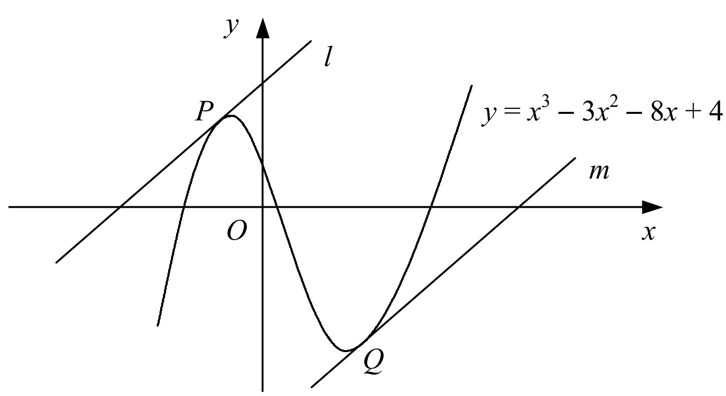
10 A curve has the equation $y = 2 + \frac{4}{x}$.

a Find an equation of the normal to the curve at the point M (4, 3). **(5)**

The normal to the curve at M intersects the curve again at the point N.

b Find the coordinates of the point N. **(5)**

11



The diagram shows the curve with equation $y = x^3 - 3x^2 - 8x + 4$.

The straight line l is the tangent to the curve at the point P (−1, 8).

a Find an equation of line l. **(4)**

The straight line m is parallel to l and is the tangent to the curve at the point Q.

b Find an equation of line m. **(4)**

c Find an equation of the normal to the curve at P. **(2)**

d Hence, or otherwise, show that the distance between lines l and m is $16\sqrt{2}$. **(4)**

12 A curve has the equation $y = \sqrt{x}\,(k - x)$, where k is a constant.

Given that the gradient of the curve is $\sqrt{2}$ at the point P where $x = 2$,

a find the value of k, **(5)**

b show that the normal to the curve at P has the equation

$$x + \sqrt{2}\,y = c,$$

where c is an integer to be found. **(5)**

1 **a**

x-coordinate of B	y-coordinate of B	gradient of AB
2	4	$\frac{4-1}{2-1} = 3$
1.1	1.21	$\frac{1.21-1}{1.1-1} = 2.1$
1.01	1.0201	$\frac{1.0201-1}{1.01-1} = 2.01$
1.001	1.002001	$\frac{1.002001-1}{1.001-1} = 2.001$

b gradient = 2

c

x-coordinate of B	y-coordinate of B	gradient of AB
0	0	$\frac{1-0}{1-0} = 1$
0.9	0.81	$\frac{1-0.81}{1-0.9} = 1.9$
0.99	0.9801	$\frac{1-0.9801}{1-0.99} = 1.99$
0.999	0.998001	$\frac{1-0.998001}{1-0.999} = 1.999$

this table supports the answer to part **b** as the gradient of the chord AB again gets closer to 2 as B gets closer to A

2 possible tables of values are:

a

x-coordinate of B	y-coordinate of B	gradient of AB
3	9	$\frac{9-4}{3-2} = 5$
2.1	4.41	$\frac{4.41-4}{2.1-2} = 4.1$
2.01	4.0401	$\frac{4.0401-4}{2.01-2} = 4.01$
2.001	4.004001	$\frac{4.004001-4}{2.001-2} = 4.001$

$\therefore$ gradient = 4

b

x-coordinate of B	y-coordinate of B	gradient of AB
5	25	$\frac{25-16}{5-4} = 9$
4.1	16.81	$\frac{16.81-16}{4.1-4} = 8.1$
4.01	16.0801	$\frac{16.0801-16}{4.01-4} = 8.01$
4.001	16.008001	$\frac{16.008001-16}{4.001-4} = 8.001$

$\therefore$ gradient = 8

c

x-coordinate of B	y-coordinate of B	gradient of AB
2.5	6.25	$\frac{6.25-2.25}{2.5-1.5} = 4$
1.6	2.56	$\frac{2.56-2.25}{1.6-1.5} = 3.1$
1.51	2.2801	$\frac{2.2801-2.25}{1.51-1.5} = 3.01$
1.501	2.253001	$\frac{2.253001-2.25}{1.501-1.5} = 3.001$

$\therefore$ gradient = 3

d

x-coordinate of B	y-coordinate of B	gradient of AB
−2	4	$\frac{4-9}{-2-(-3)} = -5$
−2.9	8.41	$\frac{8.41-9}{-2.9-(-3)} = -5.9$
−2.99	8.9401	$\frac{8.9401-9}{-2.99-(-3)} = -5.99$
−2.999	8.994001	$\frac{8.994001-9}{-2.999-(-3)} = -5.999$

$\therefore$ gradient = −6

3 **a** gradient = $2x$

b **i** 12 **ii** 4.8 **iii** −6.4

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4 possible answers are:

a let A be (1, 1)

x-coordinate of B	y-coordinate of B	gradient of AB
2	16	$\frac{16-1}{2-1} = 15$
1.1	1.4641	$\frac{1.4641-1}{1.1-1} = 4.641$
1.01	1.04060401	$\frac{1.04060401-1}{1.01-1} = 4.060401$
1.001	1.004006004	$\frac{1.004006004-1}{1.001-1} = 4.006004$

$\therefore$ gradient = 4

b let A be (2, −3)

x-coordinate of B	y-coordinate of B	gradient of AB
3	−3	$\frac{-3-(-3)}{3-2} = 0$
2.1	−3.09	$\frac{-3.09-(-3)}{2.1-2} = -0.9$
2.01	−3.0099	$\frac{-3.0099-(-3)}{2.01-2} = -0.99$
2.001	−3.000999	$\frac{-3.000999-(-3)}{2.001-2} = -0.999$

$\therefore$ gradient = −1

c let A be (4, 2)

x-coordinate of B	y-coordinate of B	gradient of AB
5	2.236067977	$\frac{2.236067977-2}{5-4} = 0.236068$
4.1	2.024845673	$\frac{2.024845673-2}{4.1-4} = 0.248457$
4.01	2.002498439	$\frac{2.002498439-2}{4.01-4} = 0.249844$
4.001	2.000249984	$\frac{2.000249984-2}{4.001-4} = 0.249984$

$\therefore$ gradient = 0.25

d let A be (2, 1)

x-coordinate of B	y-coordinate of B	gradient of AB
3	0.666666667	$\frac{0.666666667-1}{3-2} = -0.333333$
2.1	0.952380952	$\frac{0.952380952-1}{2.1-2} = -0.476190$
2.01	0.995024876	$\frac{0.995024876-1}{2.01-2} = -0.497512$
2.001	0.999500250	$\frac{0.999500250-1}{2.001-2} = -0.499750$

$\therefore$ gradient = −0.5

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5 **a** possible answers are:

i let A be (1, 1)

x-coordinate of B	y-coordinate of B	gradient of AB
2	8	$\frac{8-1}{2-1} = 7$
1.1	1.331	$\frac{1.331-1}{1.1-1} = 3.31$
1.01	1.030301	$\frac{1.030301-1}{1.01-1} = 3.0301$
1.001	1.003003001	$\frac{1.003003001-1}{1.001-1} = 3.003001$

$\therefore$ gradient = 3

ii let A be (2, 8)

x-coordinate of B	y-coordinate of B	gradient of AB
3	27	$\frac{27-8}{3-2} = 19$
2.1	9.261	$\frac{9.261-8}{2.1-2} = 12.61$
2.01	8.120601	$\frac{8.120601-8}{2.01-2} = 12.0601$
2.001	8.012006001	$\frac{8.012006001-8}{2.001-2} = 12.006001$

$\therefore$ gradient = 12

iii let A be (3, 27)

x-coordinate of B	y-coordinate of B	gradient of AB
4	64	$\frac{64-27}{4-3} = 37$
3.1	29.791	$\frac{29.791-27}{3.1-3} = 27.91$
3.01	27.270901	$\frac{27.270901-27}{3.01-3} = 27.0901$
3.001	27.027009	$\frac{27.027009-27}{3.001-3} = 27.009$

$\therefore$ gradient = 27

b gradient = $3x^2$

c **i** 48 **ii** 12 **iii** 6.75

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C1 DIFFERENTIATION — Answers - Worksheet B

1 **a** $2x$ **b** $4x^3$ **c** 1 **d** $9x^8$ **e** $-3x^{-4}$ **f** $-x^{-2}$

g $8x$ **h** 7 **i** $10x^4$ **j** 0 **k** $-16x^{-3}$ **l** $-44x^{-5}$

2 **a** $5x^4 + 2x$ **b** $1 + 3x^2$ **c** $4x^3$ **d** $6x^5 - 2$

e $18x^2 - 10x^{-3}$ **f** $2x - 4$ **g** $-x^{-2} + 5x^{-6}$ **h** $12x^2 - 12x^{-5}$

3 **a** $6t^5$ **b** $-15t^{-4}$ **c** $\frac{1}{2}t^{-\frac{1}{2}}$ **d** $\frac{2}{3}t^{-\frac{1}{3}}$ **e** $\frac{3}{2}t$ **f** $2t^{-\frac{3}{4}}$

g $7t^{\frac{5}{2}}$ **h** $-\frac{1}{5}t^{-\frac{6}{5}}$ **i** $\frac{3}{5}t^{\frac{1}{5}}$ **j** $-\frac{3}{2}t^{-\frac{5}{2}}$ **k** $-15t^{-\frac{9}{4}}$ **l** $\frac{2}{9}t^{\frac{1}{3}}$

4 **a** $2 + 2x^5$ **b** $\frac{3}{2}x^{\frac{1}{2}}$ **c** $1 + 2x^{-\frac{1}{2}}$ **d** $10x^{\frac{2}{3}} + 4x^{-5}$

e $-\frac{4}{5}x^{-\frac{9}{5}}$ **f** $\frac{1}{3}x^{-\frac{5}{6}} + \frac{3}{4}x^{-\frac{1}{4}}$ **g** $-3x^{-2} + \frac{15}{2}x^{-\frac{5}{2}}$ **h** $7x^{-2} - \frac{8}{3}x^{-\frac{11}{3}}$

5 **a** $y = x^{\frac{1}{2}}$, $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ **b** $y = 4 - x^{-1}$, $\frac{dy}{dx} = x^{-2}$ **c** $y = 3x^2 + x^{\frac{1}{3}}$, $\frac{dy}{dx} = 6x + \frac{1}{3}x^{-\frac{2}{3}}$ **d** $y = 9x + 3x^{-1}$, $\frac{dy}{dx} = 9 - 3x^{-2}$

e $y = \frac{1}{4}x^{-1} - x^{-2}$, $\frac{dy}{dx} = -\frac{1}{4}x^{-2} + 2x^{-3}$ **f** $y = 6x^{-\frac{1}{4}}$, $\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{5}{4}}$ **g** $y = x^{\frac{5}{2}}$, $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ **h** $y = 8x^{\frac{1}{2}} + \frac{4}{3}x^{-2}$, $\frac{dy}{dx} = 4x^{-\frac{1}{2}} - \frac{8}{3}x^{-3}$

6 **a** $s = t^2 + 3t$, $\frac{ds}{dt} = 2t + 3$ **b** $s = t^2 - 4t + 4$, $\frac{ds}{dt} = 2t - 4$ **c** $s = 5t^4 + 20t^2$, $\frac{ds}{dt} = 20t^3 + 40t$ **d** $s = 7t^3 - t$, $\frac{ds}{dt} = 21t^2 - 1$

e $s = t^2 + 7t + 6$, $\frac{ds}{dt} = 2t + 7$ **f** $s = t^2 - 2t - 8$, $\frac{ds}{dt} = 2t - 2$ **g** $s = t^5 + 3t^3 + 9t$, $\frac{ds}{dt} = 5t^4 + 9t^2 + 9$ **h** $s = 2t^3 - 5t^2 + 3t$, $\frac{ds}{dt} = 6t^2 - 10t + 3$

7 **a** $y = x^{\frac{3}{2}} - 4x^{\frac{1}{2}}$, $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$ **b** $y = x^2 - 2$, $\frac{dy}{dx} = 2x$ **c** $y = 4x + x^{-1}$, $\frac{dy}{dx} = 4 - x^{-2}$ **d** $y = x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$, $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

e $y = 2x^{-1} - \frac{1}{2}x^2$, $\frac{dy}{dx} = -2x^{-2} - x$ **f** $y = 5x^{-2} + x^{-\frac{3}{2}}$, $\frac{dy}{dx} = -10x^{-3} - \frac{3}{2}x^{-\frac{5}{2}}$ **g** $y = 3 - \frac{2}{3}x^{-1}$, $\frac{dy}{dx} = \frac{2}{3}x^{-2}$ **h** $y = 2x^{\frac{1}{2}} + \frac{1}{4}x^{\frac{5}{2}}$, $\frac{dy}{dx} = x^{-\frac{1}{2}} + \frac{5}{8}x^{\frac{3}{2}}$

8 **a** $\frac{dy}{dx} = 8x - 1$, $\frac{d^2y}{dx^2} = 8$ **b** $\frac{dy}{dx} = 3x^2 + 10x + 2$, $\frac{d^2y}{dx^2} = 6x + 10$ **c** $\frac{dy}{dx} = 2x^{-2}$, $\frac{d^2y}{dx^2} = -4x^{-3}$

d $\frac{dy}{dx} = 8x^3 + 6x$, $\frac{d^2y}{dx^2} = 24x^2 + 6$ **e** $y = 3x^4 - 4x^{-2}$, $\frac{dy}{dx} = 12x^3 + 8x^{-3}$, $\frac{d^2y}{dx^2} = 36x^2 - 24x^{-4}$ **f** $\frac{dy}{dx} = 3x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$, $\frac{d^2y}{dx^2} = -\frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

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C1 DIFFERENTIATION — Answers - Worksheet C

1 **a** $\frac{dy}{dx} = 3x^2$, grad = 27

b $\frac{dy}{dx} = 4 - 2x$, grad = −2

c $\frac{dy}{dx} = 4x - 8$, grad = 4

d $\frac{dy}{dx} = -3x^{-2}$, grad = $-\frac{1}{3}$

2 **a** $\frac{dy}{dx} = 6x + 1$

at (1, −1) grad = 7

b $\frac{dy}{dx} = 4x^3 + 6x^2$

at (−2, 0) grad = −8

c $y = 2x^2 - 3x$, $\frac{dy}{dx} = 4x - 3$

at (2, 2) grad = 5

d $\frac{dy}{dx} = 2x + 2x^{-2}$

at (2, 3) grad = $\frac{9}{2}$

e $\frac{dy}{dx} = 2x + 6$

at (−3, −1) grad = 0

f $\frac{dy}{dx} = 4 - 2x^{-3}$

at $(\frac{1}{2}, 6)$ grad = −12

3 **a** $f(x) = x^2 + 2x + 1$

$f'(x) = 2x + 2$

$f'(4) = 10$

b $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(4) = \frac{1}{4}$

c $f'(x) = 1 + 8x^{-3}$

$f'(4) = \frac{9}{8}$

d $f'(x) = -9x^{\frac{1}{2}}$

$f'(4) = -18$

4 **a** $x(x - 1)(x - 3) = 0$, $x = 0, 1, 3$

∴ (0, 0), (1, 0) and (3, 0)

b $\frac{dy}{dx} = 3x^2 - 8x + 3$

at (0, 0) grad = 3

at (1, 0) grad = −2

at (3, 0) grad = 6

5 **a** $\frac{dy}{dx} = 4x - 5$

b $4x - 5 = 7$

$x = 3$

6 $\frac{dy}{dx} = 3x^2 - 8$

∴ $3x^2 - 8 = 4$

$x^2 = 4$

$x = \pm 2$

∴ (−2, 8) and (2, −8)

7 **a** $\frac{dy}{dx} = 3x^2 + 2x - 4$

grad at $P = -3$

b grad at $Q = -3$

∴ $3x^2 + 2x - 4 = -3$

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$x = -1$ (at P) or $\frac{1}{3}$

∴ $Q\,(\frac{1}{3}, -\frac{5}{27})$

8 **a** $\frac{dy}{dx} = 2x$, grad = 4

∴ $y - 4 = 4(x - 2)$ [$y = 4x - 4$]

b $\frac{dy}{dx} = 2x + 3$, grad = 1

∴ $y - 2 = x + 1$ [$y = x + 3$]

c $\frac{dy}{dx} = 4x - 6$, grad = −2

∴ $y - 4 = -2(x - 1)$ [$y = -2x + 6$]

d $\frac{dy}{dx} = 3x^2 - 8x$, grad = 3

∴ $y + 7 = 3(x - 3)$ [$y = 3x - 16$]

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9 **a** $\frac{dy}{dx} = -2x$, grad = 6

$\therefore\ y + 6 = 6(x + 3)$

$y + 6 = 6x + 18$

$6x - y + 12 = 0$

b $\frac{dy}{dx} = -2x^{-2}$, grad = $-\frac{1}{2}$

$\therefore\ y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$x + 2y - 4 = 0$

c $\frac{dy}{dx} = 4x + 5$, grad = 7

$\therefore\ y - 2 = 7(x - \frac{1}{2})$

$2y - 4 = 14x - 7$

$14x - 2y - 3 = 0$

d $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$, grad = $\frac{1}{4}$

$\therefore\ y + 2 = \frac{1}{4}(x - 4)$

$4y + 8 = x - 4$

$x - 4y - 12 = 0$

10 **a** $\frac{dy}{dx} = 2x$, grad = 2

$\therefore$ grad of normal = $-\frac{1}{2}$

$\therefore\ y + 3 = -\frac{1}{2}(x - 1)$

$2y + 6 = -x + 1$

$x + 2y + 5 = 0$

b $\frac{dy}{dx} = 6x + 7$, grad = -5

$\therefore$ grad of normal = $\frac{1}{5}$

$\therefore\ y - 5 = \frac{1}{5}(x + 2)$

$5y - 25 = x + 2$

$x - 5y + 27 = 0$

c $\frac{dy}{dx} = 3x^2 - 8$, grad = 4

$\therefore$ grad of normal = $-\frac{1}{4}$

$\therefore\ y + 4 = -\frac{1}{4}(x - 2)$

$4y + 16 = -x + 2$

$x + 4y + 14 = 0$

d $\frac{dy}{dx} = 1 + 6x^{-2}$, grad = $\frac{5}{3}$

$\therefore$ grad of normal = $-\frac{3}{5}$

$\therefore\ y - 1 = -\frac{3}{5}(x - 3)$

$5y - 5 = -3x + 9$

$3x + 5y - 14 = 0$

11 **a** $x = 2\ \ \therefore\ y = 4$

$\frac{dy}{dx} = 6x - 5$, grad = 7

$\therefore\ y - 4 = 7(x - 2)$

$y = 7x - 10$

b $x = -3\ \ \therefore\ y = 6$

$\frac{dy}{dx} = 3x^2 + 10x$, grad = -3

$\therefore$ grad of normal = $\frac{1}{3}$

$\therefore\ y - 6 = \frac{1}{3}(x + 3)$

$y = \frac{1}{3}x + 7$

12 **a** $\frac{dy}{dx} = 3x^2 + 6x - 16$, grad = 8

$\therefore\ y + 10 = 8(x - 2)$ $\quad [y = 8x - 26]$

b $3x^2 + 6x - 16 = 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = 2$ (at P) or -4

$\therefore\ Q\ (-4, 50)$

13 **a** $\frac{dy}{dx} = 2x - 3$, grad = 1

$\therefore$ grad of normal = -1

$\therefore\ y - 2 = -(x - 2)$ $\quad [y = 4 - x]$

b $x^2 - 3x + 4 = 4 - x$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 2$ (at A) or 0

$\therefore\ B\ (0, 4)$

14 **a** $f'(x) = 3x^2 + 8x$

b $x = -3\ \ \therefore\ y = -9$

grad = 3

$\therefore\ y + 9 = 3(x + 3)$

$y = 3x$ which passes through (0, 0)

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15 **a** $y = 0 \Rightarrow 6 + x - x^2 = 0$

$(2 + x)(3 - x) = 0$

$x = -2, 3$

+ve x-axis $\therefore P\,(3, 0)$

$x = 0 \Rightarrow y = 6 \therefore Q\,(0, 6)$

b $\frac{dy}{dx} = 1 - 2x$

grad at $P = -5$

$y = -5(x - 3)$ $[\, y = 15 - 5x]$

c grad at $Q = 1$

tangent at Q: $y = x + 6$

$\therefore 15 - 5x = x + 6$

$x = \frac{3}{2}$

$\therefore (\frac{3}{2}, \frac{15}{2})$

16 **a** grad of $l = -3$

for curve, $\frac{dy}{dx} = 2x - 5$

$\therefore$ at A, $2x - 5 = -3$

$x = 1$

$\therefore A\,(1, -1)$

b $y + 1 = -3(x - 1)$

$y = -3x + 2$

17 grad of normal = 2

$\therefore$ grad of curve = $-\frac{1}{2}$

for curve, $\frac{dy}{dx} = -32x^{-3}$

$\therefore -\frac{32}{x^3} = -\frac{1}{2}$

$x^3 = 64$

$x = 4 \therefore (4, 1)$

sub. $1 = 8 + k$

$k = -7$

18 **a** $\frac{ds}{dt} = 3 + 10t$

$t = 0.6 \Rightarrow \frac{ds}{dt} = 9$ metres per second

b $54 = 3t + 5t^2$

$5t^2 + 3t - 54 = 0$

$(5t + 18)(t - 3) = 0$

$t > 0 \therefore t = 3$

$\therefore \frac{ds}{dt} = 33$ metres per second

19 **a** $\frac{dh}{dt} = \frac{1}{3}kt^{-\frac{2}{3}}$

when $t = 1$, $\frac{dh}{dt} = 3$

$\therefore \frac{1}{3}k = 3$

$k = 9$

b $\frac{dh}{dt} = 3 \times 8^{-\frac{2}{3}} = 0.75$ cm per second

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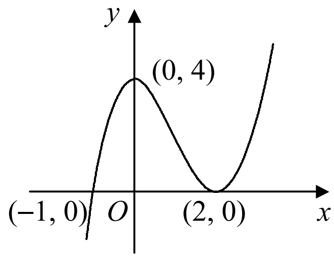
1 **a**

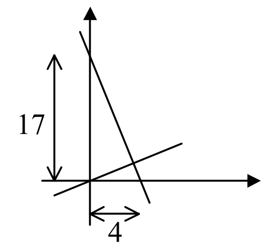


b $f(x) = (x + 1)(x^2 - 4x + 4)$
$= x^3 - 4x^2 + 4x + x^2 - 4x + 4$
$= x^3 - 3x^2 + 4$
$f'(x) = 3x^2 - 6x$

c $x = 1 \therefore y = 2 \times (-1)^2 = 2$
grad $= 3 - 6 = -3$
$\therefore y - 2 = -3(x - 1)$
$y - 2 = -3x + 3$
$y = 5 - 3x$

2 **a** $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$
grad at $P = \frac{1}{4}$
$\therefore y - 1 = \frac{1}{4}(x - 4)$
$y = \frac{1}{4}x$ which passes through (0, 0)

b grad of normal $= -4$
$\therefore y - 1 = -4(x - 4)$ $[y = 17 - 4x]$
at Q, $x = 0 \Rightarrow y = 17$
$\therefore$ area $= \frac{1}{2} \times 17 \times 4 = 34$



3 **a** $x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$x = -2, 1$ $a < b \therefore a = -2, b = 1$

b $\frac{dy}{dx} = 2x + 1$
grad at $A = -3$
$\therefore$ grad of normal $= \frac{1}{3}$
$\therefore y - 0 = \frac{1}{3}(x + 2)$
$3y = x + 2$
$x - 3y + 2 = 0$

c grad at $B = 3$
tangent at B: $y - 0 = 3(x - 1)$
$y = 3x - 3$
at C, $x - 3(3x - 3) + 2 = 0$
$x = \frac{11}{8}$
$\therefore C\left(\frac{11}{8}, \frac{9}{8}\right)$

4 $y = \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$
$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$
$= \frac{x^2 - 2x + 1}{2x^{\frac{3}{2}}}$
$= \frac{(x-1)^2}{2x^{\frac{3}{2}}}$ $[a = -1, b = 2]$

5 **a** $\frac{dy}{dx} = -24x^{-3}$
at A, $y = 3$, grad$= -3$
$\therefore y - 3 = -3(x - 2)$
$3x + y - 9 = 0$

b tangent:
$x = -1 \Rightarrow -3 + y - 9 = 0 \Rightarrow y = 12$
curve:
$x = -1 \Rightarrow y = \frac{12}{1} \Rightarrow y = 12$
$\therefore$ tangent intersects curve at (−1, 12)

6 **a** $\frac{dy}{dx} = 3 + 2kx - 3x^2$
at P, $3 - 2k - 3 = -6$
$k = 3$

b $y = 2 + 3x + 3x^2 - x^3$ $\therefore P(-1, 3)$
at Q, $3 + 6x - 3x^2 = -6$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ (at P) or 3 $\therefore Q(3, 11)$
$PQ = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$

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7 $= \frac{d}{dx}(x^2 + \frac{1}{2}x^{-1})$

$= 2x - \frac{1}{2}x^{-2}$

8 **a** $\frac{dy}{dx} = 4x - 7$

at A, $y = -5$, grad $= 1$

$\therefore\ y + 5 = 1(x - 2)$

$[\ y = x - 7\ ]$

b grad of normal at $B = 1$

$\therefore$ grad of curve at $B = -1$

$\therefore\ 4x - 7 = -1$

$x = \frac{3}{2},\ y = 2(\frac{9}{4}) - 7(\frac{3}{2}) + 1 = -5$

$\therefore\ B\,(\frac{3}{2}, -5)$

9 **a** $\frac{dy}{dx} = 2x + \frac{3}{2}x^{-\frac{1}{2}}$

b $\frac{d^2y}{dx^2} = 2 - \frac{3}{4}x^{-\frac{3}{2}}$

$\therefore\ 2x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x$

$= 2x(2 - \frac{3}{4}x^{-\frac{3}{2}}) + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$

$= 4x - \frac{3}{2}x^{-\frac{1}{2}} + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$

$= 0$

10 **a** $\frac{dy}{dx} = -4x^{-2}$

grad at $M = -\frac{1}{4}$

$\therefore$ grad of normal $= 4$

$\therefore\ y - 3 = 4(x - 4)$ $[\ y = 4x - 13]$

b $4x - 13 = 2 + \frac{4}{x}$

$4x^2 - 15x - 4 = 0$

$(4x + 1)(x - 4) = 0$

$x = 4$ (at M) or $-\frac{1}{4}$

$\therefore\ N\,(-\frac{1}{4}, -14)$

11 **a** $\frac{dy}{dx} = 3x^2 - 6x - 8$

grad at $P = 1$

$\therefore\ y - 8 = 1(x + 1)$ $[\ y = x + 9]$

b at Q, $3x^2 - 6x - 8 = 1$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$ at P $\therefore\ Q\,(3, -20)$

$\therefore\ y + 20 = 1(x - 3)$ $[\ y = x - 23]$

c grad normal $= -1$

$\therefore\ y - 8 = -(x + 1)$ $[\ y = 7 - x]$

d normal at P meets m when

$7 - x = x - 23$

$x = 15$ $\therefore$ $(15, -8)$

dist between lines = dist P to $(15, -8)$

$= \sqrt{16^2 + 16^2} = \sqrt{16^2 \times 2} = 16\sqrt{2}$

12 **a** $y = kx^{\frac{1}{2}} - x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

at P, $\frac{1}{2}k(\frac{1}{\sqrt{2}}) - \frac{3}{2}(\sqrt{2}) = \sqrt{2}$

$k - 6 = 4$

$k = 10$

b $y = \sqrt{x}\,(10 - x)$

at P, $y = \sqrt{2}\,(10 - 2) = 8\sqrt{2}$

grad of normal $= -\frac{1}{\sqrt{2}}$

$\therefore\ y - 8\sqrt{2} = -\frac{1}{\sqrt{2}}(x - 2)$

$\sqrt{2}\,y - 16 = -x + 2$

$x + \sqrt{2}\,y = 18$ $[\ c = 18\ ]$