C2 EXPONENTIALS AND LOGARITHMS *Worksheet A*

1 Express each of the following in the form $\log_a b = c$.

a $10^3 = 1000$ **b** $3^4 = 81$ **c** $256 = 2^8$ **d** $7^0 = 1$

e $3^{-3} = \frac{1}{27}$ **f** $32^{-\frac{1}{5}} = \frac{1}{2}$ **g** $19^1 = 19$ **h** $216 = 36^{\frac{3}{2}}$

2 Express each of the following using index notation.

a $\log_5 125 = 3$ **b** $\log_2 16 = 4$ **c** $5 = \log_{10} 100\,000$ **d** $\log_{23} 1 = 0$

e $\frac{1}{2} = \log_9 3$ **f** $\lg 0.01 = -2$ **g** $\log_2 \frac{1}{8} = -3$ **h** $\log_6 6 = 1$

3 Without using a calculator, find the exact value of

a $\log_7 49$ **b** $\log_4 64$ **c** $\log_2 128$ **d** $\log_3 27$

e $\log_5 625$ **f** $\log_8 8$ **g** $\log_7 1$ **h** $\log_{15} \frac{1}{15}$

i $\log_3 \frac{1}{9}$ **j** $\lg 0.001$ **k** $\log_{16} 2$ **l** $\log_4 8$

m $\log_9 243$ **n** $\log_{100} 0.001$ **o** $\log_{25} 125$ **p** $\log_{27} \frac{1}{9}$

4 Without using a calculator, find the exact value of x in each case.

a $\log_5 25 = x$ **b** $\log_2 x = 6$ **c** $\log_x 64 = 3$ **d** $\lg x = -3$

e $\log_x 16 = \frac{2}{3}$ **f** $\log_5 1 = x$ **g** $\log_x 9 = 1$ **h** $\lg 10^{12} = x$

i $2 \log_x 7 = 1$ **j** $\log_4 x = 1.5$ **k** $\log_x 0.1 = -\frac{1}{3}$ **l** $3 \log_8 x + 1 = 0$

5 Express in the form $\log_a n$

a $\log_a 4 + \log_a 7$ **b** $\log_a 10 - \log_a 5$ **c** $2 \log_a 6$

d $\log_a 9 - \log_a \frac{1}{3}$ **e** $\frac{1}{2} \log_a 25 + 2 \log_a 3$ **f** $\log_a 48 - 3 \log_a 2 - \frac{1}{2} \log_a 9$

6 Express in the form $p \log_q x$

a $\log_q x^5$ **b** $\frac{1}{2} \log_q x^{15}$ **c** $\log_q \frac{1}{x}$ **d** $\log_q \sqrt[3]{x}$

e $4 \log_q \frac{1}{\sqrt{x}}$ **f** $\log_q x^2 + \log_q x^5$ **g** $\log_q \frac{1}{x^2} + \mathrm{loq}_q \frac{1}{x^3}$ **h** $3 \log_q x^2 - \frac{1}{2} \log_q x^4$

7 Express in the form $\lg n$

a $\lg 5 + \lg 4$ **b** $\lg 12 - \lg 6$ **c** $3 \lg 2$ **d** $4 \lg 3 - \lg 9$

e $\frac{1}{2} \lg 16 - \frac{1}{5} \lg 32$ **f** $1 + \lg 11$ **g** $\lg \frac{1}{50} + 2$ **h** $3 - \lg 40$

8 Without using a calculator, evaluate

a $\log_3 54 - \log_3 2$ **b** $\log_5 20 + \log_5 1.25$ **c** $\log_2 16 + \log_3 27$

d $\log_6 24 + \log_6 9$ **e** $\log_3 12 - \log_3 4$ **f** $\log_4 18 - \log_4 9$

g $\log_9 4 + \log_9 0.25$ **h** $2 \lg 2 + \lg 25$ **i** $\frac{1}{3} \log_3 8 - \log_3 18$

j $\frac{1}{3} \log_4 64 + 2 \log_5 25$ **k** $\frac{1}{2} \log_5 (1\frac{9}{16}) + 2 \log_5 10$ **l** $\log_3 5 - 2 \log_3 6 - \log_3 (3\frac{3}{4})$

C2 EXPONENTIALS AND LOGARITHMS — Worksheet B

1 Express in the form $p \log_{10} a + q \log_{10} b$

a $\log_{10} ab$ **b** $\log_{10} ab^7$ **c** $\log_{10} \frac{a^3}{b}$ **d** $\log_{10} a\sqrt{b}$

e $\log_{10} (ab)^2$ **f** $\log_{10} \frac{1}{ab}$ **g** $\log_{10} \sqrt{a^3 b^5}$ **h** $3 \log_{10} \frac{a^2}{\sqrt[3]{b}}$

2 Given that $y = \log_q 8$, express each of the following in terms of y.

a $\log_q 64$ **b** $\log_q 2$ **c** $\log_q \frac{16}{q}$ **d** $\log_q 4q^3$

3 Given that $a = \lg 2$ and $b = \lg 3$, express each of the following in terms of a and b.

a $\lg 18$ **b** $\lg 96$ **c** $\lg \frac{9}{16}$ **d** $\lg 6 - \lg 8$

e $\lg \sqrt{6}$ **f** $\frac{3}{2} \lg 16 + \frac{1}{2} \lg 81$ **g** $4 \lg 3 - 3 \lg 6$ **h** $\lg 60 + \lg 20 - 2$

4 Without using a calculator, evaluate

a $\frac{1}{3} \log_5 1000 - \frac{1}{2} \log_5 4$ **b** $2 \log_{12} 4 + \frac{1}{2} \log_{12} 81$ **c** $\log_4 12 + \log_4 \frac{2}{3}$

d $\frac{\log_7 81}{\log_7 3}$ **e** $3 \log_{27} 12 - 2 \log_{27} 72$ **f** $\frac{\log_{11} 25}{\log_{11} \frac{1}{5}}$

5 Solve each equation, giving your answers correct to 3 significant figures.

a $\log_3 x = 1.8$ **b** $\log_5 x = -0.3$ **c** $\log_8 (x - 3) = 2.1$

d $\log_4 (\frac{1}{2}x + 1) = 3.2$ **e** $15 - \log_2 3y = 9.7$ **f** $\log_6 (1 - 5t) + 4.2 = 3.6$

6 Express in the form $\log_2 [f(x)]$

a $5 \log_2 x$ **b** $\log_2 x + \log_2 (x + 4)$ **c** $2 \log_2 x + \frac{1}{5} \log_2 x^5$

d $3 \log_2 (x - 2) - 4 \log_2 x$ **e** $\log_2 (x^2 - 1) - \log_2 (x + 1)$ **f** $\log_2 x - \frac{1}{2} \log_2 x^4 + \frac{1}{3} \log_2 x^2$

7 Solve each of the following equations.

a $\log_3 x + \log_3 5 = \log_3 (2x + 3)$ **b** $\log_9 x + \log_9 10 = \frac{3}{2}$

c $\log_4 x - \log_4 (x - 1) = \log_4 3 + \frac{1}{2}$ **d** $\log_5 5x - \log_5 (x + 2) = \log_5 (x + 6) - \log_5 x$

e $2 \log_6 x = \log_6 (2x - 5) + \log_6 5$ **f** $\log_7 4x = \log_7 \frac{1}{x-6} + 1$

8 Solve each pair of simultaneous equations.

a $\log_x y = 2$
$xy = 27$

b $\log_5 x - 2 \log_5 y = \log_5 2$
$x + y^2 = 12$

c $\log_2 x = 3 - 2 \log_2 y$
$\log_y 32 = -\frac{5}{2}$

d $\log_y x = \frac{3}{2}$
$x^{\frac{1}{3}} + 3y^{\frac{1}{2}} = 20$

e $\log_a x + \log_a 3 = \frac{1}{2} \log_a y$
$3x + y = 20$

f $\log_{10} y + 2 \log_{10} x = 3$
$\log_2 y - \log_2 x = 3$

C2 EXPONENTIALS AND LOGARITHMS Worksheet C

1 Find, to 3 significant figures, the value of

a $\log_{10} 60$ **b** $\log_{10} 6$ **c** $\log_{10} 253$ **d** $\log_{10} 0.4$

2 Solve each equation, giving your answers to 2 decimal places.

a $10^x = 14$ **b** $2(10^x) - 8 = 0$ **c** $10^{3x} = 49$

d $10^{x-4} = 23$ **e** $10^{2x+1} = 130$ **f** $100^x - 5 = 0$

3 Show that $\log_a b = \dfrac{\log_c b}{\log_c a}$, where a, b and c are positive constants.

4 Find, to 3 significant figures, the value of

a $\log_2 7$ **b** $\log_{20} 172$ **c** $\log_5 49$ **d** $\log_9 4$

5 Solve each equation, giving your answers to 3 significant figures.

a $3^x = 12$ **b** $2^x = 0.7$ **c** $8^{-y} = 3$ **d** $4^{\frac{1}{2}x} - 0.3 = 0$

e $5^{t+3} = 24$ **f** $16 - 3^{4+x} = 0$ **g** $7^{2x+4} = 12$ **h** $5(2^{3x+1}) = 62$

i $4^{2-3x} = 32.7$ **j** $5^x = 6^{x-1}$ **k** $7^{y+2} = 9^{y+1}$ **l** $4^{5-x} = 11^{2x-1}$

m $4^{\frac{1}{2}x+3} - 5^{1-2x} = 0$ **n** $2^{3y-2} = 3^{2y+5}$ **o** $7^{2x+5} = 7(11^{3x-4})$ **p** $3^{2x} = 3^{x-1} \times 2^{4+x}$

6 Solve the following equations, giving your answers to 2 decimal places where appropriate.

a $2^{2x} + 2^x - 6 = 0$ **b** $3^{2x} - 5(3^x) + 4 = 0$ **c** $5^{2x} + 12 = 8(5^x)$

d $2(4^x) + 3(4^{-x}) = 7$ **e** $2^{2y+1} + 7(2^y) - 15 = 0$ **f** $3^{2x+1} - 17(3^x) + 10 = 0$

g $25^t + 5^{t+1} - 24 = 0$ **h** $3^{2x+1} + 15 = 2(3^{x+2})$ **i** $3(16^x) - 4^{x+2} + 5 = 0$

7 Sketch each pair of curves on the same diagram, showing the coordinates of any points of intersection with the coordinate axes.

a $y = 2^x$, $y = 5^x$ **b** $y = 3^x$, $y = (\frac{1}{3})^x$ **c** $y = 4^x$, $y = 4^x - 1$ **d** $y = 2^x$, $y = 2^{x+3}$

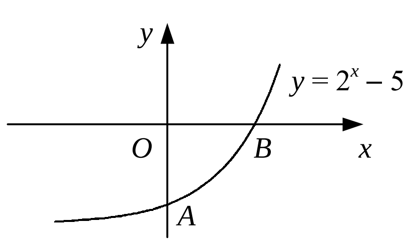
8 A curve has the equation $y = 2 + a^x$ where a is a constant and $a > 1$.

a Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

Given also that the curve passes through the point (3, 29),

b find the value of a.

9



The diagram shows the curve with equation $y = 2^x - 5$ which intersects the coordinate axes at the points A and B. Find the length AB correct to 3 significant figures.

C2 EXPONENTIALS AND LOGARITHMS Worksheet D

1 Given that $a = \log_{10} 2$ and $b = \log_{10} 3$, find expressions in terms of a and b for

a $\log_{10} 1.5$, **(2)**

b $\log_{10} 24$, **(2)**

c $\log_{10} 150$. **(3)**

2 Find, to an appropriate degree of accuracy, the values of x for which

a $4 \log_3 x - 5 = 0$, **(2)**

b $\log_3 x^3 - 5 \log_3 x = 4$. **(3)**

3 **a** Given that $p = \log_2 q$, find expressions in terms of p for

i $\log_2 \sqrt{q}$,

ii $\log_2 8q$. **(4)**

b Solve the equation

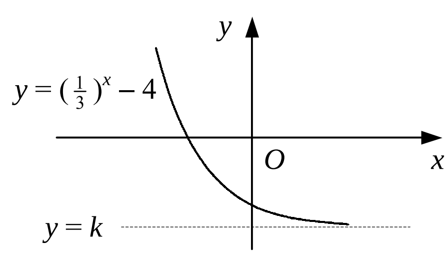
$$\log_2 8q - \log_2 \sqrt{q} = \log_3 9.$$ **(3)**

4 An initial investment of £1000 is placed into a savings account that offers 2.2% interest every 3 months. The amount of money in the account, £P, at the end of t years is given by

$$P = 1000 \times 1.022^{4t}$$

Find, to the nearest year, how long it will take for the investment to double in value. **(4)**

5



The diagram shows the curve with equation $y = (\frac{1}{3})^x - 4$.

a Write down the coordinates of the point where the curve crosses the y-axis. **(1)**

The curve has an asymptote with equation $y = k$.

b Write down the value of the constant k. **(1)**

c Find the x-coordinate of the point where the curve crosses the x-axis. **(3)**

6 **a** Solve the equation

$$\log_3 (x + 1) - \log_3 (x - 2) = 1.$$ **(3)**

b Find, in terms of logarithms to the base 10, the exact value of x such that

$$3^{2x+1} = 2^{x-4}.$$ **(3)**

7 **a** Given that $t = 2^x$, write down expressions in terms of t for

i 2^{x-1},

ii 2^{2x+1}. **(3)**

b Hence solve the equation

$$2^{2x+1} - 14(2^{x-1}) + 6 = 0.$$ **(5)**

8 Find the values of x for which

a $\log_2 (3x + 5) + \log_5 125 = 7$, **(3)**

b $\log_2 (x + 1) = 5 - \log_2 (3x - 1)$. **(5)**

9 Given that $\log_a (x + 4) = \log_a \frac{x}{4} + \log_a 5$,

and that $\log_a (y + 2) = \log_a 12 - \log_a (y + 1)$,

where $y > 0$, find

a the value of x, **(3)**

b the value of y, **(4)**

c the value of the logarithm of x to the base y. **(2)**

10 A colony of fast-breeding fish is introduced into a large, newly-built pond. The number of fish in the pond, n, after t weeks is modelled by

$$n = \frac{18000}{1 + 8c^{-t}}.$$

a Find the initial number of fish in the pond. **(2)**

Given that there are 3600 fish in the pond after 3 weeks, use this model to

b show that $c = \sqrt[3]{2}$, **(3)**

c find the time taken for the initial population of fish to double in size, giving your answer to the nearest day. **(4)**

11 **a** Given that $y = \log_8 x$, find expressions in terms of y for

i $\log_8 x^2$,

ii $\log_2 x$. **(4)**

b Hence, or otherwise, find the value of x such that

$$3 \log_8 x^2 + \log_2 x = 6.$$ **(3)**

12 Solve the simultaneous equations

$$\log_2 y = \log_2 (3 - 2x) + 1$$

$$\log_4 x + \log_4 y = \tfrac{1}{2}$$ **(8)**

13 **a** Sketch on the same diagram the curves $y = 2^x + 1$ and $y = (\tfrac{1}{2})^x$, showing the coordinates of any points where each curve meets the coordinate axes. **(4)**

Given that the curves $y = 2^x + 1$ and $y = (\tfrac{1}{2})^x$ intersect at the point A,

b show that the x-coordinate of A is a solution of the equation

$$2^{2x} + 2^x - 1 = 0,$$ **(2)**

c hence, show that the y-coordinate of A is $\tfrac{1}{2}(\sqrt{5} + 1)$. **(4)**

14 **a** Show that $x = 1$ is a solution of the equation

$$2^{3x} - 4(2^{2x}) + 2^x + 6 = 0. \qquad \text{(I)}$$ **(1)**

b Show that using the substitution $u = 2^x$, equation (I) can be written as

$$u^3 - 4u^2 + u + 6 = 0.$$ **(2)**

c Hence find the other real solution of equation (I) correct to 3 significant figures. **(7)**

1 **a** $\log_{10} 1000 = 3$ **b** $\log_3 81 = 4$ **c** $\log_2 256 = 8$ **d** $\log_7 1 = 0$

e $\log_3 \frac{1}{27} = -3$ **f** $\log_{32} \frac{1}{2} = -\frac{1}{5}$ **g** $\log_{19} 19 = 1$ **h** $\log_{36} 216 = \frac{3}{2}$

2 **a** $5^3 = 125$ **b** $2^4 = 16$ **c** $10^5 = 100\,000$ **d** $23^0 = 1$

e $9^{\frac{1}{2}} = 3$ **f** $10^{-2} = 0.01$ **g** $2^{-3} = \frac{1}{8}$ **h** $6^1 = 6$

3 **a** $= \log_7 7^2 = 2$ **b** $= \log_4 4^3 = 3$ **c** $= \log_2 2^7 = 7$ **d** $= \log_3 3^3 = 3$

e $= \log_5 5^4 = 4$ **f** $= \log_8 8^1 = 1$ **g** $= \log_7 7^0 = 0$ **h** $= \log_{15} 15^{-1} = -1$

i $= \log_3 3^{-2} = -2$ **j** $= \lg 10^{-3} = -3$ **k** $= \log_{16} 16^{\frac{1}{4}} = \frac{1}{4}$ **l** $= \log_4 4^{\frac{3}{2}} = \frac{3}{2}$

m $= \log_9 9^{\frac{5}{2}} = \frac{5}{2}$ **n** $= \log_{100} 100^{-\frac{3}{2}} = -\frac{3}{2}$ **o** $= \log_{25} 25^{\frac{3}{2}} = \frac{3}{2}$ **p** $= \log_{27} 27^{-\frac{2}{3}} = -\frac{2}{3}$

4 **a** $5^x = 25$, $x = 2$ **b** $2^6 = x$, $x = 64$ **c** $x^3 = 64$, $x = 4$ **d** $10^{-3} = x$, $x = \frac{1}{1000}$

e $x^{\frac{2}{3}} = 16$, $x = 64$ **f** $5^x = 1$, $x = 0$ **g** $x^1 = 9$, $x = 9$ **h** $10^x = 10^{12}$, $x = 12$

i $\log_x 7 = \frac{1}{2}$, $x^{\frac{1}{2}} = 7$, $x = 49$ **j** $4^{1.5} = x$, $x = 8$ **k** $x^{-\frac{1}{3}} = 0.1$, $x = 1000$ **l** $\log_8 x = -\frac{1}{3}$, $8^{-\frac{1}{3}} = x$, $x = \frac{1}{2}$

5 **a** $= \log_a (4 \times 7) = \log_a 28$ **b** $= \log_a (10 \div 5) = \log_a 2$ **c** $= \log_a 6^2 = \log_a 36$

d $= \log_a (9 \div \frac{1}{3}) = \log_a 27$

e $= \log_a 25^{\frac{1}{2}} + \log_a 3^2 = \log_a 5 + \log_a 9 = \log_a (5 \times 9) = \log_a 45$

f $= \log_a 48 - \log_a 2^3 - \log_a 9^{\frac{1}{2}} = \log_a 48 - \log_a 8 - \log_a 3 = \log_a [48 \div (8 \times 3)] = \log_a 2$

6 **a** $= 5 \log_q x$ **b** $= \frac{15}{2} \log_q x$ **c** $= \log_q x^{-1} = -\log_q x$ **d** $= \log_q x^{\frac{1}{3}} = \frac{1}{3} \log_q x$

e $= 4 \log_q x^{-\frac{1}{2}} = -2 \log_q x$ **f** $= 2 \log_q x + 5 \log_q x = 7 \log_q x$ **g** $= \log_q x^{-2} + \log_q x^{-3} = -2 \log_q x - 3 \log_q x = -5 \log_q x$ **h** $= 6 \log_q x - 2 \log_q x = 4 \log_q x$

7

a $= \lg (5 \times 4)$
$= \lg 20$

b $= \lg (12 \div 6)$
$= \lg 2$

c $= \lg 2^3$
$= \lg 8$

d $= \lg 3^4 - \lg 9$
$= \lg 81 - \lg 9$
$= \lg (81 \div 9)$
$= \lg 9$

e $= \lg 16^{\frac{1}{2}} - \lg 32^{\frac{1}{5}}$
$= \lg 4 - \lg 2$
$= \lg (4 \div 2)$
$= \lg 2$

f $= \lg 10 + \lg 11$
$= \lg (10 \times 11)$
$= \lg 110$

g $= \lg \frac{1}{50} + \lg 10^2$
$= \lg \frac{1}{50} + \lg 100$
$= \lg (\frac{1}{50} \times 100)$
$= \lg 2$

h $= \lg 10^3 - \lg 40$
$= \lg 1000 - \lg 40$
$= \lg (1000 \div 40)$
$= \lg 25$

8

a $= \log_3 (54 \div 2)$
$= \log_3 27$
$= \log_3 3^3$
$= 3$

b $= \log_5 (20 \times 1.25)$
$= \log_5 25$
$= \log_5 5^2$
$= 2$

c $= \log_2 2^4 + \log_3 3^3$
$= 4 + 3$
$= 7$

d $= \log_6 (24 \times 9)$
$= \log_6 216$
$= \log_6 6^3$
$= 3$

e $= \log_3 (12 \div 4)$
$= \log_3 3$
$= 1$

f $= \log_4 (18 \div 9)$
$= \log_4 2$
$= \log_4 4^{\frac{1}{2}}$
$= \frac{1}{2}$

g $= \log_9 (4 \times 0.25)$
$= \log_9 1$
$= 0$

h $= \lg 2^2 + \lg 25$
$= \lg 4 + \lg 25$
$= \lg (4 \times 25)$
$= \lg 100$
$= \lg 10^2$
$= 2$

i $= \log_3 8^{\frac{1}{3}} - \log_3 18$
$= \log_3 2 - \log_3 18$
$= \log_3 (2 \div 18)$
$= \log_3 \frac{1}{9}$
$= \log_3 3^{-2}$
$= -2$

j $= \log_4 64^{\frac{1}{3}} + (2 \times \log_5 5^2)$
$= \log_4 4 + (2 \times 2)$
$= 1 + 4$
$= 5$

k $= \frac{1}{2} \log_5 \frac{25}{16} + \log_5 10^2$
$= \log_5 (\frac{25}{16})^{\frac{1}{2}} + \log_5 100$
$= \log_5 \frac{5}{4} + \log_5 100$
$= \log_5 (\frac{5}{4} \times 100)$
$= \log_5 125$
$= \log_5 5^3$
$= 3$

l $= \log_3 5 - \log_3 6^2 - \log_3 \frac{15}{4}$
$= \log_3 [5 \div (36 \times \frac{15}{4})]$
$= \log_3 \frac{1}{27}$
$= \log_3 3^{-3}$
$= -3$

1 **a** $= \log_{10} a + \log_{10} b$

b $= \log_{10} a + \log_{10} b^7$
$= \log_{10} a + 7\log_{10} b$

c $= \log_{10} a^3 - \log_{10} b$
$= 3\log_{10} a - \log_{10} b$

d $= \log_{10} a + \log_{10} b^{\frac{1}{2}}$
$= \log_{10} a + \frac{1}{2}\log_{10} b$

e $= 2\log_{10} ab$
$= 2\log_{10} a + 2\log_{10} b$

f $= -\log_{10} ab$
$= -\log_{10} a - \log_{10} b$

g $= \log_{10} a^{\frac{3}{2}} + \log_{10} b^{\frac{5}{2}}$
$= \frac{3}{2}\log_{10} a + \frac{5}{2}\log_{10} b$

h $= 3(\log_{10} a^2 - \log_{10} b^{\frac{1}{3}})$
$= 6\log_{10} a - \log_{10} b$

2 **a** $= \log_q 8^2$
$= 2y$

b $= \log_q 8^{\frac{1}{3}}$
$= \frac{1}{3}y$

c $= \log_q 16 - \log_q q$
$= \log_q 8^{\frac{4}{3}} - 1$
$= \frac{4}{3}y - 1$

d $= \log_q 4 + \log_q q^3$
$= \log_q 8^{\frac{2}{3}} + 3$
$= \frac{2}{3}y + 3$

3 **a** $= \lg(2 \times 3^2)$
$= \lg 2 + 2\lg 3$
$= a + 2b$

b $= \lg(2^5 \times 3)$
$= 5\lg 2 + \lg 3$
$= 5a + b$

c $= \lg 9 - \lg 16$
$= \lg 3^2 - \lg 2^4$
$= 2\lg 3 - 4\lg 2$
$= 2b - 4a$

d $= \lg(2 \times 3) - \lg 2^3$
$= \lg 2 + \lg 3 - 3\lg 2$
$= \lg 3 - 2\lg 2$
$= b - 2a$

e $= \frac{1}{2}\lg 6$
$= \frac{1}{2}(\lg 2 + \lg 3)$
$= \frac{1}{2}(a + b)$

f $= \frac{3}{2}\lg 2^4 + \frac{1}{2}\lg 3^4$
$= 6\lg 2 + 2\lg 3$
$= 6a + 2b$

g $= 4\lg 3 - 3(\lg 2 + \lg 3)$
$= \lg 3 - 3\lg 2$
$= b - 3a$

h $= \lg(6\times10) + \lg(2\times10) - 2$
$= \lg 6 + 1 + \lg 2 + 1 - 2$
$= \lg 2 + \lg 3 + \lg 2$
$= 2a + b$

4 **a** $= \log_5 10 - \log_5 2$
$= \log_5 5$
$= 1$

b $= \log_{12} 16 + \log_{12} 9$
$= \log_{12} 144$
$= 2$

c $= \log_4 8$
$= \log_4 4^{\frac{3}{2}}$
$= \frac{3}{2}$

d $= \dfrac{\log_7 3^4}{\log_7 3}$
$= \dfrac{4\log_7 3}{\log_7 3}$
$= 4$

e $= \log_{27} \dfrac{12^3}{72^2}$
$= \log_{27} \dfrac{12\times12\times12}{6\times12\times6\times12}$
$= \log_{27} \frac{1}{3} = -\frac{1}{3}$

f $= \dfrac{\log_{11} 5^2}{-\log_{11} 5}$
$= \dfrac{2\log_{11} 5}{-\log_{11} 5}$
$= -2$

5 **a** $x = 3^{1.8}$
$x = 7.22$

b $x = 5^{-0.3}$
$x = 0.617$

c $x - 3 = 8^{2.1}$
$x = 3 + 8^{2.1}$
$x = 81.8$

d $\frac{1}{2}x + 1 = 4^{3.2}$
$x = 2(4^{3.2} - 1)$
$x = 167$

e $\log_2 3y = 5.3$
$3y = 2^{5.3}$
$y = \frac{1}{3} \times 2^{5.3}$
$y = 13.1$

f $\log_6 (1 - 5t) = -0.6$
$1 - 5t = 6^{-0.6}$
$t = \frac{1}{5}(1 - 6^{-0.6})$
$t = 0.132$

6 **a** $= \log_2 x^5$

b $= \log_2 (x^2 + 4x)$

c $= \log_2 x^2 + \log_2 x$
$= \log_2 x^3$

d $= \log_2 (x-2)^3 - \log_2 x^4$
$= \log_2 \dfrac{(x-2)^3}{x^4}$

e $= \log_2 \dfrac{x^2-1}{x+1}$
$= \log_2 \dfrac{(x+1)(x-1)}{x+1}$
$= \log_2 (x-1)$

f $= \log_2 x - 2\log_2 x + \frac{2}{3}\log_2 x$
$= -\frac{1}{3}\log_2 x$
$= \log_2 x^{-\frac{1}{3}}$

7 **a** $\log_3 5x = \log_3 (2x + 3)$

$5x = 2x + 3$
$x = 1$

b $\log_9 10x = \frac{3}{2}$

$10x = 9^{\frac{3}{2}} = 27$
$x = 2.7$

c $\log_4 \frac{x}{x-1} = \log_4 3 + \log_4 2 = \log_4 6$

$\frac{x}{x-1} = 6$
$x = 6x - 6$
$x = \frac{6}{5}$

d $\log_5 \frac{5x}{x+2} = \log_5 \frac{x+6}{x}$

$\frac{5x}{x+2} = \frac{x+6}{x}$
$5x^2 = (x + 2)(x + 6) = x^2 + 8x + 12$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1, 3$
$\log_5 x$ not real for $x = -1 \quad \therefore x = 3$

e $\log_6 x^2 = \log_6 5(2x - 5)$

$x^2 = 5(2x - 5)$
$x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x = 5$

f $\log_7 4x - \log_7 \frac{1}{x-6} = 1$

$\log_7 4x(x - 6) = 1$
$4x(x - 6) = 7$
$4x^2 - 24x - 7 = 0$
$x = \frac{24 \pm \sqrt{576+112}}{8} = 3 \pm \frac{1}{2}\sqrt{43}$
$\log_7 4x$ not real for $x = 3 - \frac{1}{2}\sqrt{43}$
$\therefore \ x = 3 + \frac{1}{2}\sqrt{43} \quad [= 6.28 \text{ (3sf)}]$

8 **a** $\log_x y = 2 \Rightarrow y = x^2$

sub. $x^3 = 27$
$x = 3$
$\therefore \ x = 3, y = 9$

b $\log_5 x - 2\log_5 y = \log_5 2 \Rightarrow \frac{x}{y^2} = 2$
$\Rightarrow x = 2y^2$

sub. $3y^2 = 12$
$y^2 = 4$
for real $\log_5 y$, $y > 0 \quad \therefore y = 2$
$\therefore \ x = 8, y = 2$

c $\log_y 32 = -\frac{5}{2} \Rightarrow y^{-\frac{5}{2}} = 32$
$\Rightarrow y = 32^{-\frac{2}{5}} = \frac{1}{4}$

sub. $\log_2 x = 3 - 2\log_2 \frac{1}{4}$
$\log_2 x = 3 - (-4) = 7$
$x = 2^7 = 128$
$\therefore \ x = 128, y = \frac{1}{4}$

d $\log_y x = \frac{3}{2} \Rightarrow y^{\frac{3}{2}} = x$
$\Rightarrow y^{\frac{1}{2}} = x^{\frac{1}{3}}$

sub. $4x^{\frac{1}{3}} = 20$
$x^{\frac{1}{3}} = 5$
$x = 5^3 = 125$
$\therefore \ x = 125, y = 25$

e $\log_a x + \log_a 3 = \frac{1}{2}\log_a y \Rightarrow 3x = y^{\frac{1}{2}}$
$\Rightarrow y = 9x^2$

sub. $3x + 9x^2 = 20$
$9x^2 + 3x - 20 = 0$
$(3x + 5)(3x - 4) = 0$
for real $\log_a x$, $x > 0 \quad \therefore x = \frac{4}{3}$
$\therefore \ x = \frac{4}{3}, y = 16$

f $\log_{10} y + 2\log_{10} x = 3 \Rightarrow x^2 y = 10^3$
$\log_2 y - \log_2 x = 3 \Rightarrow \frac{y}{x} = 2^3$
$\Rightarrow y = 8x$

sub. $8x^3 = 1000$
$x^3 = 125$
$x = 5$
$\therefore \ x = 5, y = 40$

1 **a** 1.78 **b** 0.778 **c** 2.40 **d** −0.398

2
a $x = \lg 14 = 1.15$

b $10^x = 4$
$x = \lg 4 = 0.60$

c $3x = \lg 49$
$x = \frac{1}{3}\lg 49 = 0.56$

d $x - 4 = \lg 23$
$x = 4 + \lg 23 = 5.36$

e $2x + 1 = \lg 130$
$x = \frac{1}{2}(\lg 130 - 1) = 0.56$

f $(10^2)^x = 10^{2x} = 5$
$2x = \lg 5$
$x = \frac{1}{2}\lg 5 = 0.35$

3 let $y = \log_a b \Rightarrow a^y = b$
$y \log_c a = \log_c b$
$y = \frac{\log_c b}{\log_c a}$
$\therefore \log_a b = \frac{\log_c b}{\log_c a}$

4 **a** $= \frac{\lg 7}{\lg 2} = 2.81$ **b** $= \frac{\lg 172}{\lg 20} = 1.72$ **c** $= \frac{\lg 49}{\lg 5} = 2.42$ **d** $= \frac{\lg 4}{\lg 9} = 0.631$

5
a $x \lg 3 = \lg 12$
$x = \frac{\lg 12}{\lg 3}$
$x = 2.26$

b $x \lg 2 = \lg 0.7$
$x = \frac{\lg 0.7}{\lg 2}$
$x = -0.515$

c $-y \lg 8 = \lg 3$
$y = -\frac{\lg 3}{\lg 8}$
$y = -0.528$

d $\frac{1}{2}x \lg 4 = \lg 0.3$
$x = \frac{2\lg 0.3}{\lg 4}$
$x = -1.74$

e $(t + 3)\lg 5 = \lg 24$
$t = \frac{\lg 24}{\lg 5} - 3$
$t = -1.03$

f $(4 + x)\lg 3 = \lg 16$
$x = \frac{\lg 16}{\lg 3} - 4$
$x = -1.48$

g $(2x + 4)\lg 7 = \lg 12$
$x = \frac{1}{2}(\frac{\lg 12}{\lg 7} - 4)$
$x = -1.36$

h $2^{3x+1} = 12.4$
$(3x + 1)\lg 2 = \lg 12.4$
$x = \frac{1}{3}(\frac{\lg 12.4}{\lg 2} - 1)$
$x = 0.877$

i $(2 - 3x)\lg 4 = \lg 32.7$
$x = \frac{1}{3}(2 - \frac{\lg 32.7}{\lg 4})$
$x = -0.172$

j $x \lg 5 = (x - 1)\lg 6$
$x(\lg 6 - \lg 5) = \lg 6$
$x = \frac{\lg 6}{\lg 6 - \lg 5} = 9.83$

k $(y + 2)\lg 7 = (y + 1)\lg 9$
$y(\lg 9 - \lg 7) = 2\lg 7 - \lg 9$
$y = \frac{2\lg 7 - \lg 9}{\lg 9 - \lg 7} = 6.74$

l $(5 - x)\lg 4 = (2x - 1)\lg 11$
$x(2\lg 11 + \lg 4) = 5\lg 4 + \lg 11$
$x = \frac{5\lg 4 + \lg 11}{2\lg 11 + \lg 4} = 1.51$

m $(\frac{1}{2}x + 3)\lg 4 = (1 - 2x)\lg 5$
$x(\frac{1}{2}\lg 4 + 2\lg 5) = \lg 5 - 3\lg 4$
$x = \frac{\lg 5 - 3\lg 4}{\frac{1}{2}\lg 4 + 2\lg 5} = -0.652$

n $(3y - 2)\lg 2 = (2y + 5)\lg 3$
$y(3\lg 2 - 2\lg 3) = 5\lg 3 + 2\lg 2$
$y = \frac{5\lg 3 + 2\lg 2}{3\lg 2 - 2\lg 3} = -58.4$

o $7^{2x+4} = 11^{3x-4}$
$(2x + 4)\lg 7 = (3x - 4)\lg 11$
$x(3\lg 11 - 2\lg 7) = 4\lg 7 + 4\lg 11$
$x = \frac{4\lg 7 + 4\lg 11}{3\lg 11 - 2\lg 7} = 5.26$

p $3^{x+1} = 2^{4+x}$
$(x + 1)\lg 3 = (4 + x)\lg 2$
$x(\lg 3 - \lg 2) = 4\lg 2 - \lg 3$
$x = \frac{4\lg 2 - \lg 3}{\lg 3 - \lg 2} = 4.13$

6 **a** $(2^x + 3)(2^x - 2) = 0$
$2^x = -3$ [no sols], 2
$x = 1$

b $(3^x - 1)(3^x - 4) = 0$
$3^x = 1, 4$
$x = 0, \frac{\lg 4}{\lg 3} = 0, 1.26$

c $5^{2x} - 8(5^x) + 12 = 0$
$(5^x - 2)(5^x - 6) = 0$
$5^x = 2, 6$
$x = \frac{\lg 2}{\lg 5}, \frac{\lg 6}{\lg 5} = 0.43, 1.11$

d $2(4^{2x}) - 7(4^x) + 3 = 0$
$(2(4^x) - 1)(4^x - 3) = 0$
$4^x = \frac{1}{2}, 3$
$x = -\frac{1}{2}, \frac{\lg 3}{\lg 4} = -\frac{1}{2}, 0.79$

e $2(2^{2y}) + 7(2^y) - 15 = 0$
$(2(2^y) - 3)(2^y + 5) = 0$
$2^y = -5$ [no sols], $\frac{3}{2}$
$y = \frac{\lg \frac{3}{2}}{\lg 2} = 0.58$

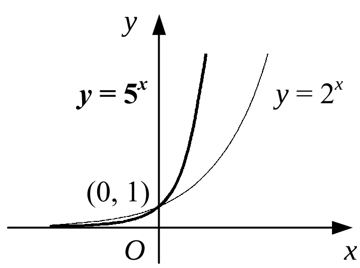
f $3(3^{2x}) - 17(3^x) + 10 = 0$
$(3(3^x) - 2)(3^x - 5) = 0$
$3^x = \frac{2}{3}, 5$
$x = \frac{\lg \frac{2}{3}}{\lg 3}, \frac{\lg 5}{\lg 3} = -0.37, 1.46$

g $5^{2t} + 5(5^t) - 24 = 0$
$(5^t + 8)(5^t - 3) = 0$
$5^t = -8$ [no sols], 3
$t = \frac{\lg 3}{\lg 5} = 0.68$

h $3(3^{2x}) - 18(3^x) + 15 = 0$
$3(3^x - 1)(3^x - 5) = 0$
$3^x = 1, 5$
$x = 0, \frac{\lg 5}{\lg 3} = 0, 1.46$

i $3(4^{2x}) - 16(4^x) + 5 = 0$
$(3(4^x) - 1)(4^x - 5) = 0$
$4^x = \frac{1}{3}, 5$
$x = \frac{\lg \frac{1}{3}}{\lg 4}, \frac{\lg 5}{\lg 4} = -0.79, 1.16$

7 **a**



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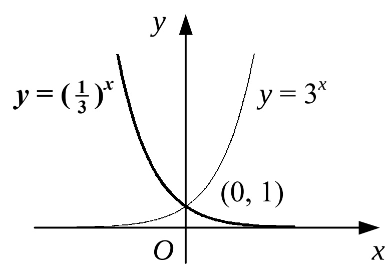

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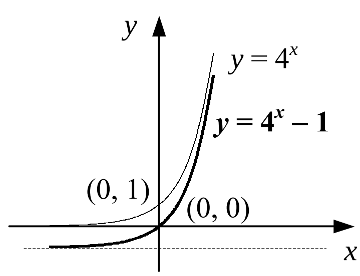
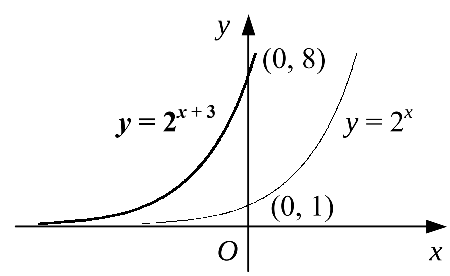
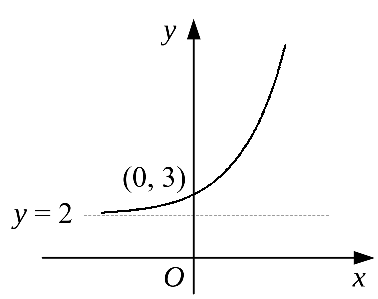


d



8 **a**



b $(3, 29) \Rightarrow 29 = 2 + a^3$
$a^3 = 27$
$a = 3$

9 $x = 0 \Rightarrow y = -4$
$y = 0 \Rightarrow 2^x = 5$
$x = \frac{\lg 5}{\lg 2}$
$AB^2 = 4^2 + (\frac{\lg 5}{\lg 2})^2 = 21.391$
$AB = 4.63$

1 **a** $= \log_{10} \frac{3}{2}$
$= \log_{10} 3 - \log_{10} 2$
$= b - a$
b $= \log_{10} (2^3 \times 3)$
$= 3 \log_{10} 2 + \log_{10} 3$
$= 3a + b$
c $= \log_{10} (1.5 \times 100)$
$= \log_{10} 1.5 + \log_{10} 100$
$= b - a + 2$

2 **a** $\log_3 x = \frac{5}{4}$
$x = 3^{\frac{5}{4}} = 3.95$ (3sf)
b $3 \log_3 x - 5 \log_3 x = 4$
$\log_3 x = -2$
$x = 3^{-2} = \frac{1}{9}$

3 **a** **i** $= \log_2 q^{\frac{1}{2}} = \frac{1}{2} \log_2 q = \frac{1}{2} p$
ii $= \log_2 8 + \log_2 q = 3 + p$
b $3 + p - \frac{1}{2} p = 2$
$p = \log_2 q = -2$
$\therefore\ q = 2^{-2} = \frac{1}{4}$

4 $2000 = 1000 \times 1.022^{4t}$
$2 = 1.022^{4t}$
$4t \lg 1.022 = \lg 2$
$t = \frac{\lg 2}{4 \lg 1.022} = 7.96$
$\therefore$ 8 years

5 **a** $(0, -3)$
b $k = -4$
c $(\frac{1}{3})^x - 4 = 0$
$(\frac{1}{3})^x = 4$
$x = \frac{\lg 4}{\lg \frac{1}{3}} = -1.26$ (3sf)

6 **a** $\log_3 \frac{x+1}{x-2} = 1$
$\frac{x+1}{x-2} = 3$
$x + 1 = 3x - 6$
$x = \frac{7}{2}$
b $(2x + 1) \lg 3 = (x - 4) \lg 2$
$x (\lg 2 - 2 \lg 3) = \lg 3 + 4 \lg 2$
$x = \frac{\lg 3 + 4 \lg 2}{\lg 2 - 2 \lg 3}$

7 **a** **i** $= 2^{-1}(2^x) = \frac{1}{2} t$
ii $= 2(2^{2x}) = 2(2^x)^2 = 2t^2$
b $2t^2 - 7t + 6 = 0$
$(2t - 3)(t - 2) = 0$
$t = 2^x = \frac{3}{2}, 2$
$x = \frac{\lg \frac{3}{2}}{\lg 2}, 1 = 0.585$ (3sf), 1

8 **a** $\log_2 (3x + 5) + 3 = 7$
$3x + 5 = 2^4 = 16$
$x = \frac{11}{3}$
b $\log_2 (x + 1) + \log_2 (3x - 1) = 5$
$(x + 1)(3x - 1) = 2^5 = 32$
$3x^2 + 2x - 33 = 0$
$(3x + 11)(x - 3) = 0$
$x = -\frac{11}{3}, 3$
for real $\log_2 (3x - 1)$, $x > \frac{1}{3}$ $\quad \therefore x = 3$

9 **a** $x + 4 = \frac{5}{4}x$

$x = 16$

b $y + 2 = \dfrac{12}{y+1}$

$(y + 2)(y + 1) = 12$

$y^2 + 3y - 10 = 0$

$(y + 5)(y - 2) = 0$

$y > 0 \quad \therefore\ y = 2$

c $\log_y x = \log_2 16 = 4$

10 **a** $t = 0 \ \Rightarrow\ n = 2000$

b $3600 = \dfrac{18000}{1+8c^{-3}}$

$1 + 8c^{-3} = 5$

$c^{-3} = \frac{1}{2}$

$c^3 = 2$

$c = \sqrt[3]{2}$

c $4000 = \dfrac{18000}{1+8c^{-t}}$

$1 + 8c^{-t} = \frac{9}{2}$

$c^{-t} = \frac{7}{16}$

$-t = \dfrac{\lg \frac{7}{16}}{\lg \sqrt[3]{2}}$

$t = 3.578$ weeks $= 25$ days

11 **a** **i** $\log_8 x^2 = 2\log_8 x = 2y$

ii $y = \log_8 x \ \Rightarrow\ x = 8^y = 2^{3y}$

$\therefore\ \log_2 x = 3y$

b $3(2y) + 3y = 6$

$y = \log_8 x = \frac{2}{3}$

$\therefore\ x = 8^{\frac{2}{3}} = 4$

12 $\log_2 y - \log_2 (3 - 2x) = 1 \ \Rightarrow\ \dfrac{y}{3-2x} = 2$

$\Rightarrow\ y = 6 - 4x$

$\log_4 xy = \frac{1}{2} \ \Rightarrow\ xy = 4^{\frac{1}{2}} = 2$

sub. $x(6 - 4x) = 2$

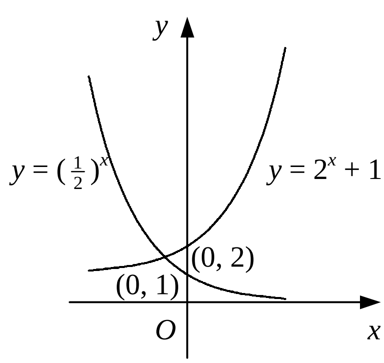
$2x^2 - 3x + 1 = 0$

$(2x - 1)(x - 1) = 0$

$x = \frac{1}{2}, 1$

$\therefore\ x = \frac{1}{2}, y = 4$ or $x = 1, y = 2$

13 **a**



b at A, $2^x + 1 = (\frac{1}{2})^x$

$(2^x)^2 + 2^x = 1$

$2^{2x} + 2^x - 1 = 0$

c $2^x = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$2^x = \dfrac{-1-\sqrt{5}}{2}$ [no sols] or $\dfrac{-1+\sqrt{5}}{2}$

$\therefore\ 2^x = \frac{1}{2}\sqrt{5} - \frac{1}{2}$

$\therefore\ y = (\frac{1}{2}\sqrt{5} - \frac{1}{2}) + 1 = \frac{1}{2}(\sqrt{5} + 1)$

14 **a** when $x = 1$,

LHS $= 8 - 4(4) + 2 + 6 = 0$

$\therefore\ x = 1$ is a solution

b $2^{3x} = (2^x)^3 = u^3$

$2^{2x} = (2^x)^2 = u^2$

$\therefore$ (I) $\Rightarrow\ u^3 - 4u^2 + u + 6 = 0$

c $x = 1 \ \Rightarrow\ u = 2 \quad \therefore\ (u - 2)$ is a factor

$$
\begin{array}{r|rrrr}
 & u^2 & -\ 2u & -\ 3 & \\
u-2 & u^3 - 4u^2 & +\ u & +\ 6 & \\
 & u^3 - 2u^2 & & & \\
 & -\ 2u^2 & +\ u & & \\
 & -\ 2u^2 & +\ 4u & & \\
 & & -\ 3u & +\ 6 & \\
 & & -\ 3u & +\ 6 & \\
\end{array}
$$

$(u - 2)(u^2 - 2u - 3) = 0$

$(u - 2)(u - 3)(u + 1) = 0$

$u = 2^x = -1$ [no sols], 2 or 3

$x = 1$ (given) or $\dfrac{\lg 3}{\lg 2} = 1.58$