C3 EXPONENTIALS AND LOGARITHMS Worksheet A

1 Find, to 3 significant figures, the value of

a e^3 **b** e^{-2} **c** $5e$ **d** $\ln 0.55$ **e** $\frac{3}{7}\ln 100$ **f** $\log_{10} e$

2 Without using a calculator, find the value of

a $e^{\ln 4}$ **b** $e^{\frac{1}{2}\ln 9}$ **c** $2e^{-\ln 6}$ **d** $\ln e^7$ **e** $\ln \frac{1}{e}$ **f** $5\ln e^{-0.1}$

3 Find the value of x in each case.

a $e^{\ln x} = 4$ **b** $\ln e^x = 17$ **c** $e^{2\ln x} = 25$ **d** $e^{-\ln x} = \frac{1}{3}$

4 Solve each equation, giving your answers in terms of e.

a $\ln x = 15$ **b** $\frac{1}{2}\ln t - 3 = 0$ **c** $\ln (x - 4) = 7$

d $17 - \ln 5y = 9$ **e** $\ln (\frac{1}{2}x + 3) = 2.5$ **f** $\ln (4 - 3x) - 11 = 0$

5 Solve each equation, giving your answers in terms of natural logarithms.

a $e^x = 0.7$ **b** $9 - 2e^y = 5$ **c** $e^{5x} - 3 = 0$

d $e^{4t+1} = 12$ **e** $\frac{1}{2}e^{2x-3} - 7 = 0$ **f** $2e^{4-5x} + 9 = 16$

6 Solve each equation, giving your answers to 2 decimal places.

a $\frac{1}{3}e^x = 4$ **b** $\ln (15x - 7) = 4$ **c** $4e^{\frac{1}{2}y+3} = 11$

d $\frac{3}{7}\ln (5 - 2x) - 1 = 0$ **e** $\ln (10 - 3y) - e = 0$ **f** $\ln x^2 + \ln x^3 = 19$

g $e^{2x} = 3e^{-\frac{1}{4}x}$ **h** $e^{5t} = 4e^{2t+1}$ **i** $\ln (2x - 5) - \ln x = \frac{1}{4}$

7 Find, in exact form, the solutions to the equation

$$2e^{2x} + 12 = 11e^x.$$

8 **a** Simplify

$$\frac{3x^2 - 10x + 8}{x^2 - 5x + 6}.$$

b Hence, solve the equation

$$\ln (3x^2 - 10x + 8) - \ln (x^2 - 5x + 6) = \ln 2x.$$

9 Solve the following simultaneous equations, giving your answers to 2 decimal places.

$$e^{5y} - x = 0$$
$$\ln x^4 = 7 - y$$

10 Sketch each pair of curves on the same diagram, showing the coordinates of any points of intersection with the coordinate axes.

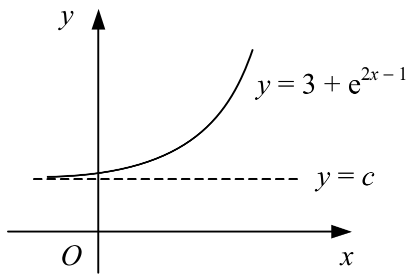
a $y = e^x$, $y = e^{-2x}$ **b** $y = 2e^x$, $y = e^{x-1}$ **c** $y = 2 + e^x$, $y = e^{2x+1}$

d $y = e^x$, $y = \ln x$ **e** $y = -\ln x$, $y = 2 + \ln x$ **f** $y = \ln (x - 2)$, $y = \ln 3x$

11 **a** Sketch on the same diagram the curves $y = \ln(x + 1)$ and $y = 1 + \ln x$.

b Show that the x-coordinate of the point where the two curves intersect is $\frac{1}{e-1}$.

12



The diagram shows the curve with the equation $y = 3 + e^{2x-1}$ and the asymptote of the curve which has the equation $y = c$.

a State the value of the constant c.

b Find the exact coordinates of the point where the curve crosses the y-axis.

c Find the x-coordinate of the point on the curve where $y = 7$, giving your answer in the form $a + \ln b$, where a is rational and b is an integer.

13 A quantity N is decreasing such that at time t

$$N = 50e^{-0.2t}.$$

a Find the value of N when $t = 10$.

b Find the value of t when $N = 3$.

14 A radioactive substance is decaying such that its mass, m grams, at a time t years after initial observation is given by

$$m = 240e^{kt},$$

where k is a constant.

Given that when $t = 180$, $m = 160$, find

a the value of k,

b the time it takes for the mass of the substance to be halved.

15 A quantity N is increasing such that at time t

$$N = 20e^{0.04t}.$$

a Find the value of N when $t = 15$.

b Find, in terms of the constant k, expressions for the value of t when

i $N = k$,

ii $N = 2k$.

c Hence, show that the time it takes for the value of N to double is constant.

16 A quantity N is decreasing such that at time t

$$N = N_0e^{kt}.$$

Given that at time $t = 10$, $N = 300$ and that at time $t = 20$, $N = 225$, find

a the values of the constants N_0 and k,

b the value of t when $N = 150$.

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C3 EXPONENTIALS AND LOGARITHMS Worksheet B

1 A radioactive substance is decaying such that its mass, m grams, at a time t years after initial observation is given by

$$m = 60e^{kt},$$

where k is a constant.

Given that when $t = 100$, $m = 42$,

a find the value of k, **(3)**

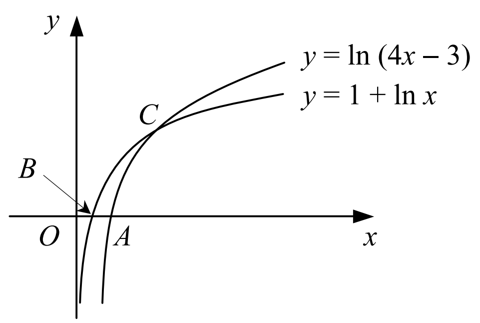
b find the value of t when $m = 30$. **(2)**

2 Solve each equation, giving your answers correct to 2 decimal places.

a $e^{2x} - 5.7e^{-x} = 0$ **(3)**

b $\ln x - \ln (x - 1) = \frac{1}{2}$ **(4)**

3



The diagram shows the curves $y = \ln (4x - 3)$ and $y = 1 + \ln x$ which cross the x-axis at the points A and B respectively.

a Find the coordinates of A and B. **(4)**

The two curves intersect at the point C.

b Find the exact x-coordinate of C, giving your answer in terms of e. **(4)**

4 Find, as natural logarithms, the roots of the equation

$$2e^{x} + 3e^{-x} = 7.$$ **(5)**

5 A scientist carries out an experiment to investigate the growth of a population of flies. She introduces a colony of flies into a closed environment and uses the model that after t days the number of flies in the environment, N, is given by

$$N = 800e^{0.01t}.$$

Find, according to this model,

a the number of flies introduced into the environment, **(1)**

b the size of the population after 20 days, **(2)**

c the least number of days after which the population will exceed 2000. **(3)**

6 $$f(x) = 1 + e^{2x + 1}.$$

a Solve the equation $f(x) = 10$, giving your answer in the form $a + \ln b$ where a is rational and b is an integer. **(3)**

b Find, to 3 significant figures, the x-coordinate of the point where the curve $y = f(x)$ intersects the curve $y = 3 - e^{x}$. **(5)**

7 Giving your answers in exact form, solve the equations

a $\ln(4x - 1) = 2$, **(3)**

b $7 - e^{1-3y} = 0$. **(3)**

8 At time $t = 0$, there are 800 bacteria present in a culture. The number of bacteria present at time t hours is modelled by the continuous variable N and the relationship

$$N = ae^{bt},$$

where a and b are constants.

a Write down the value of a. **(1)**

Given that when $t = 2$, $N = 7200$,

b find the value of b in the form $\ln k$, **(3)**

c find, to the nearest minute, how long it takes for the number of bacteria present to double. **(4)**

9 a Simplify

$$\frac{x^2 - 4x + 3}{x^2 + x - 2}.$$ **(3)**

b Solve the equation

$$\ln(x^2 - 4x + 3) = 1 + \ln(x^2 + x - 2),$$

giving your answer in terms of e. **(4)**

10 Giving your answers to an appropriate degree of accuracy, solve the simultaneous equations

$$e^y + 5 - 9x = 0$$

$$y - \ln(x + 4) = 2$$ **(7)**

11 a Describe fully the single transformation which maps the graph of $y = e^x$ onto the graph of $y = e^{-x}$. **(1)**

b Sketch the graphs of $y = e^{-x}$ and $y = e^{3x+1}$ on the same diagram, showing the coordinates of any points of intersection with the coordinate axes. **(4)**

c Find the exact coordinates of the point of intersection of the two graphs. **(3)**

12 a Given that $t = \ln x$, find expressions in terms of t for

i $\ln \sqrt{x}$,

ii $\ln(e^2x)$. **(4)**

b Hence, or otherwise, solve the equation

$$5 + \ln \sqrt{x} = \ln(e^2x).$$ **(3)**

13 A bead is projected vertically upwards in a jar of liquid with a velocity of 13 m s^{-1}. Its velocity, v m s^{-1}, at time t seconds after projection, is given by

$$v = ce^{-kt} - 2.$$

a Find the value of c. **(2)**

Given that the bead has a velocity of 7 m s^{-1} after 5.1 seconds, find

b the value of k correct to 4 decimal places, **(3)**

c the time taken for its velocity to decrease from 10 m s^{-1} to 4 m s^{-1}. **(5)**

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1 **a** 20.1 **b** 0.135 **c** 13.6 **d** −0.598 **e** 1.97 **f** 0.434

2 **a** $= 4$ **b** $= e^{\ln 3} = 3$ **c** $= 2e^{\ln \frac{1}{6}} = \frac{1}{3}$ **d** $= 7$ **e** $= \ln e^{-1} = -1$ **f** $= -0.5$

3 **a** $x = 4$ **b** $x = 17$ **c** $x^2 = 25$, $x > 0 \therefore x = 5$ **d** $\frac{1}{x} = \frac{1}{3}$, $x = 3$

4 **a** $x = e^{15}$

b $\ln t = 6$
$t = e^6$

c $x - 4 = e^7$
$x = e^7 + 4$

d $\ln 5y = 8$
$5y = e^8$
$y = \frac{1}{5}e^8$

e $\frac{1}{2}x + 3 = e^{2.5}$
$\frac{1}{2}x = e^{2.5} - 3$
$x = 2e^{2.5} - 6$

f $4 - 3x = e^{11}$
$3x = 4 - e^{11}$
$x = \frac{1}{3}(4 - e^{11})$

5 **a** $x = \ln 0.7$

b $e^y = 2$
$y = \ln 2$

c $5x = \ln 3$
$x = \frac{1}{5}\ln 3$

d $4t + 1 = \ln 12$
$t = \frac{1}{4}(\ln 12 - 1)$

e $e^{2x-3} = 14$
$2x - 3 = \ln 14$
$x = \frac{1}{2}(\ln 14 + 3)$

f $e^{4-5x} = \frac{7}{2}$
$4 - 5x = \ln \frac{7}{2}$
$x = \frac{1}{5}(4 - \ln \frac{7}{2})$

6 **a** $e^x = 12$
$x = \ln 12 = 2.48$

b $15x - 7 = e^4$
$x = \frac{1}{15}(e^4 + 7) = 4.11$

c $e^{\frac{1}{2}y+3} = \frac{11}{4}$
$\frac{1}{2}y + 3 = \ln \frac{11}{4}$
$y = 2(\ln \frac{11}{4} - 3) = -3.98$

d $\ln(5 - 2x) = \frac{7}{3}$
$5 - 2x = e^{\frac{7}{3}}$
$x = \frac{1}{2}(5 - e^{\frac{7}{3}}) = -2.66$

e $10 - 3y = e^e$
$y = \frac{1}{3}(10 - e^e) = -1.72$

f $2\ln x + 3\ln x = 19$
$\ln x = \frac{19}{5}$
$x = e^{\frac{19}{5}} = 44.70$

g $e^{\frac{9}{4}x} = 3$
$\frac{9}{4}x = \ln 3$
$x = \frac{4}{9}\ln 3 = 0.49$

h $e^{3t-1} = 4$
$3t - 1 = \ln 4$
$t = \frac{1}{3}(\ln 4 + 1) = 0.80$

i $\ln \frac{2x-5}{x} = \frac{1}{4}$
$2x - 5 = e^{\frac{1}{4}}x$
$(2 - e^{\frac{1}{4}})x = 5$
$x = \frac{5}{2 - e^{\frac{1}{4}}} = 6.98$

7 $2e^{2x} - 11e^x + 12 = 0$
$(2e^x - 3)(e^x - 4) = 0$
$e^x = \frac{3}{2}, 4$
$x = \ln \frac{3}{2}, \ln 4$

8 **a** $= \dfrac{(3x-4)(x-2)}{(x-2)(x-3)} = \dfrac{3x-4}{x-3}$

b $\ln \dfrac{3x^2-10x+8}{x^2-5x+6} = \ln 2x$

$\dfrac{3x^2-10x+8}{x^2-5x+6} = 2x$

$\dfrac{3x-4}{x-3} = 2x$

$3x - 4 = 2x(x - 3)$

$2x^2 - 9x + 4 = 0$

$(2x - 1)(x - 4) = 0$

$x = \tfrac{1}{2}, 4$

9 $e^{5y} - x = 0 \quad \Rightarrow \quad 5y = \ln x$

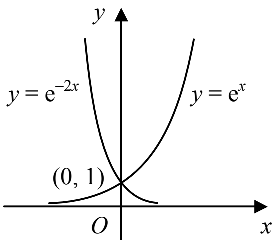
$\ln x^4 = 7 - y \quad \Rightarrow \quad 4 \ln x = 7 - y$

sub. $20y = 7 - y$

$y = \tfrac{1}{3}$

$\therefore\ x = e^{\frac{5}{3}} = 5.29,\ y = 0.33$

10 **a**



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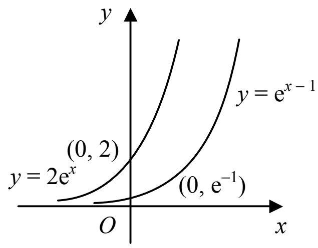

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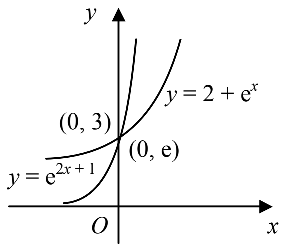

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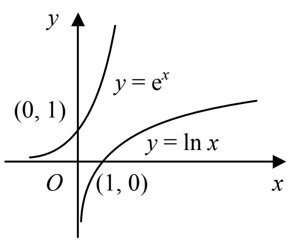

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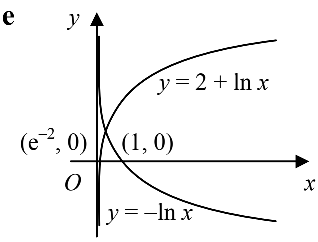

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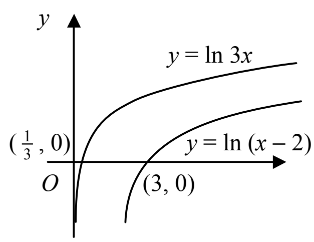
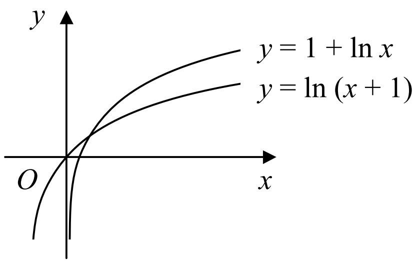


11 **a**



b $\ln (x + 1) = 1 + \ln x$

$\ln (x + 1) - \ln x = 1$

$\ln \dfrac{x+1}{x} = 1$

$\dfrac{x+1}{x} = e$

$x + 1 = ex$

$1 = x(e - 1)$

$x = \dfrac{1}{e-1}$

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12 **a** 3

b $x = 0 \ \therefore\ y = 3 + e^{-1}$

$\therefore\ (0, 3 + e^{-1})$

c $3 + e^{2x-1} = 7$

$e^{2x-1} = 4$

$2x - 1 = \ln 4$

$x = \frac{1}{2}(1 + \ln 4)$

$x = \frac{1}{2} + \ln 2$

13 **a** $t = 10,\ N = 50e^{-2} = 6.77$ (3sf)

b $3 = 50e^{-0.2t}$

$t = -5 \ln \frac{3}{50} = 14.1$ (3sf)

14 **a** $160 = 240e^{180k}$

$k = \frac{1}{180} \ln \frac{2}{3} = -0.00225$ (3sf)

b $m = 240e^{-0.002253t}$

$120 = 240e^{-0.002253t}$

$t = \frac{-1}{0.002253} \ln \frac{1}{2} = 308$ years (3sf)

15 **a** $t = 15,\ N = 20e^{0.6} = 36.4$ (3sf)

b **i** $k = 20e^{0.04t}$

$t = \frac{\ln(\frac{k}{20})}{0.04} = 25 \ln \frac{k}{20}$

ii $2k = 20e^{0.04t}$

$t = \frac{\ln(\frac{k}{10})}{0.04} = 25 \ln \frac{k}{10}$

c time for N to increase from k to $2k$

$= 25 \ln \frac{k}{10} - 25 \ln \frac{k}{20}$

$= 25 \ln \frac{(\frac{k}{10})}{(\frac{k}{20})}$

$= 25 \ln 2$

$\therefore$ time for N to double is constant

16 **a** $300 = N_0 e^{10k} \ \Rightarrow\ N_0 = \frac{300}{e^{10k}}$

$225 = N_0 e^{20k}$

$\therefore\ 225 = \frac{300}{e^{10k}} \times e^{20k}$

$e^{10k} = \frac{3}{4}$

$k = \frac{1}{10} \ln \frac{3}{4} = -0.0288$ (3sf)

$N_0 = \frac{300}{\frac{3}{4}} = 400$

b $N = 400e^{-0.02877t}$

$150 = 400e^{-0.02877t}$

$t = \frac{-1}{0.02877} \ln \frac{3}{8} = 34.1$ (3sf)

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C3 EXPONENTIALS AND LOGARITHMS — Answers - Worksheet B

1 **a** $42 = 60e^{100k}$

$100k = \ln 0.7$

$k = \frac{1}{100}\ln 0.7 = -0.00357$ (3sf)

b $30 = 60e^{kt}$

$kt = \ln 0.5$

$t = \frac{100\ln 0.5}{\ln 0.7} = 194$ (3sf)

2 **a** $e^{3x} = 5.7$

$x = \frac{1}{3}\ln 5.7 = 0.58$ (2dp)

b $\ln \frac{x}{x-1} = \frac{1}{2}$

$\frac{x}{x-1} = e^{\frac{1}{2}}$

$x = e^{\frac{1}{2}}(x - 1)$

$x(e^{\frac{1}{2}} - 1) = e^{\frac{1}{2}}$

$x = \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}} - 1} = 2.54$ (2dp)

3 **a** $\ln(4x - 3) = 0$

$4x - 3 = 1$

$x = 1 \quad \therefore A\,(1, 0)$

$1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1} \quad \therefore B\,(e^{-1}, 0)$

b $\ln(4x - 3) = 1 + \ln x$

$\ln(4x - 3) - \ln x = 1$

$\ln \frac{4x-3}{x} = 1$

$\frac{4x-3}{x} = e$

$4x - 3 = ex$

$x(4 - e) = 3$

$x = \frac{3}{4-e}$

4 $2e^{2x} - 7e^{x} + 3 = 0$

$(2e^{x} - 1)(e^{x} - 3) = 0$

$e^{x} = \frac{1}{2}, 3$

$x = \ln \frac{1}{2}, \ln 3$

5 **a** $t = 0 \Rightarrow N = 800$

b $t = 20 \Rightarrow N = 800e^{0.2}$

$= 977$ (nearest unit)

c $800e^{0.01t} > 2000$

$e^{0.01t} > 2.5$

$0.01t > \ln 2.5$

$t > 91.6 \quad \therefore$ 92 days

6 **a** $1 + e^{2x+1} = 10$

$e^{2x+1} = 9$

$2x + 1 = \ln 9$

$x = \frac{1}{2}(-1 + \ln 9)$

$x = -\frac{1}{2} + \ln 3$

b $1 + e^{2x+1} = 3 - e^{x}$

$e(e^{2x}) + e^{x} - 2 = 0$

$e^{x} = \frac{-1 \pm \sqrt{1+8e}}{2e}$

$x = \ln \frac{-1-\sqrt{1+8e}}{2e}$ (not real)

or $\ln \frac{-1+\sqrt{1+8e}}{2e}$

$\therefore x = -0.366$ (3sf)

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7 **a** $4x - 1 = e^2$

$x = \frac{1}{4}(e^2 + 1)$

b $7 = e^{1-3y}$

$1 - 3y = \ln 7$

$y = \frac{1}{3}(1 - \ln 7)$

8 **a** $a = 800$

b $7200 = 800e^{2b}$

$b = \frac{1}{2}\ln 9 = \ln 3$

c $1600 = 800e^{t\ln 3}$

$t = \frac{\ln 2}{\ln 3} = 0.631$ hours

$\therefore\ 60 \times 0.631 = 38$ minutes

9 **a** $= \frac{(x-1)(x-3)}{(x+2)(x-1)}$

$= \frac{x-3}{x+2}$

b $\ln(x^2 - 4x + 3) - \ln(x^2 + x - 2) = 1$

$\ln\frac{x^2-4x+3}{x^2+x-2} = \ln\frac{x-3}{x+2} = 1$

$\frac{x-3}{x+2} = e$

$x - 3 = e(x + 2)$

$x(1 - e) = 2e + 3$

$x = \frac{2e+3}{1-e}$

10 $e^y + 5 - 9x = 0 \Rightarrow y = \ln(9x - 5)$

sub. $\ln(9x - 5) - \ln(x + 4) = 2$

$\frac{9x-5}{x+4} = e^2$

$9x - 5 = e^2(x + 4)$

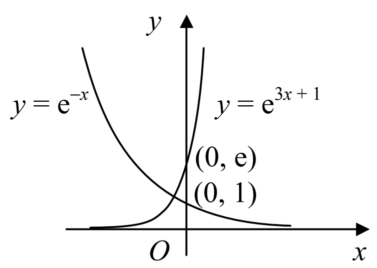
$x(9 - e^2) = 4e^2 + 5$

$x = \frac{4e^2+5}{9-e^2} = 21.4509$

$\therefore\ x = 21.5,\ y = 5.24$ (3sf)

11 **a** reflection in y-axis

b



c $e^{-x} = e^{3x+1}$

$1 = e^{4x+1}$

$4x + 1 = 0$

$x = -\frac{1}{4}$

$\therefore\ (-\frac{1}{4},\ e^{\frac{1}{4}})$

12 **a** **i** $= \ln x^{\frac{1}{2}} = \frac{1}{2}\ln x = \frac{1}{2}t$

ii $= \ln e^2 + \ln x = 2 + t$

b $5 + \frac{1}{2}t = 2 + t$

$t = \ln x = 6$

$x = e^6$

13 **a** when $t = 0$, $v = 13$

$\therefore\ 13 = c - 2$

$c = 15$

b $7 = 15e^{-5.1k} - 2$

$e^{-5.1k} = \frac{3}{5}$

$k = \frac{\ln\frac{3}{5}}{-5.1} = 0.1002$

c $10 = 15e^{-0.1002t} - 2, \quad 4 = 15e^{-0.1002T} - 2$

$t = \frac{\ln\frac{4}{5}}{-0.1002} = 2.2278, \quad T = \frac{\ln\frac{2}{5}}{-0.1002} = 9.1481$

$T - t = 6.92$ seconds (3sf)