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C2 COORDINATE GEOMETRY Worksheet A

1 Write down an equation of the circle with the given centre and radius in each case.

a centre (0, 0) radius 5 **b** centre (1, 3) radius 2 **c** centre (4, –6) radius 1

d centre (–1, –8) radius 3 **e** centre $(-\frac{1}{2}, \frac{1}{2})$ radius $\frac{1}{2}$ **f** centre (–3, 9) radius $2\sqrt{3}$

2 Write down the coordinates of the centre and the radius of each of the following circles.

a $x^2 + y^2 = 16$ **b** $(x - 6)^2 + (y - 1)^2 = 81$ **c** $(x + 1)^2 + (y - 4)^2 = 121$

d $(x - 7)^2 + y^2 = 0.09$ **e** $(x + 2)^2 + (y + 5)^2 = 32$ **f** $(x - 8)^2 + (y + 9)^2 = 108$

3 Find the coordinates of the centre and the radius of each of the following circles.

a $x^2 + y^2 - 4y + 3 = 0$ **b** $x^2 + y^2 - 2x - 10y - 23 = 0$

c $x^2 + y^2 + 12x - 8y + 36 = 0$ **d** $x^2 + y^2 - 2x + 16y = 35$

e $x^2 + y^2 = 8x - 6y$ **f** $x^2 + y^2 + 10x - 2y - 19 = 0$

g $4x^2 + 4y^2 - 4x - 24y + 1 = 0$ **h** $9x^2 + 9y^2 + 6x - 24y + 8 = 0$

4 Find an equation of the circle

a with centre (1, –2) which passes through the point (4, 2),

b with centre (–5, 7) which passes through the point (0, 5).

5 Find an equation of the circle in which AB is a diameter in each case.

a A (1, –2) B (3, –2) **b** A (–7, 2) B (1, 8) **c** A (1, 1) B (4, 0)

6 The points P (0, 1), Q (3, 10) and R (6, 9) all lie on circle C.

a Show that $\angle PQR$ is a right-angle.

b Hence, show that C has the equation $x^2 + y^2 - 6x - 10y + 9 = 0$.

7 Find in each case whether the given point lies inside, outside or on the given circle.

a (0, –9) $x^2 + y^2 = 64$ **b** (4, 7) $x^2 + y^2 - 2x - 6y - 26 = 0$

c (7, –3) $x^2 + y^2 + 10x - 4y = 140$ **d** (–4, 1) $x^2 + y^2 + 2x + 8y - 13 = 0$

8 The point P lies on the circle with equation $x^2 + y^2 + 12x - 6y + 27 = 0$ and the point Q has coordinates (8, 1). Find the minimum length of PQ giving your answer in the form $k\sqrt{2}$.

9 Find an equation of the circle which crosses the x-axis at the points (2, 0) and (8, 0) and touches the y-axis at the point (0, 4).

10 Given that the circle with equation $x^2 + y^2 + 8x - 12y + k = 0$ does not touch or cross either of the coordinate axes, find the set of possible values of the constant k.

11 The circle C passes through the points P, Q and R with coordinates (–2, –2), (2, –4) and (7, 1) respectively.

a Find an equation of the perpendicular bisector of the points P and Q.

b Find the coordinates of the centre of C.

c Find an equation of C.

12 The circle C has the equation $x^2 + y^2 - 4x - 4y - 28 = 0$.

a Find the distance of the point A (10, 8) from the centre of C.

The tangent to C at the point B passes through A.

b Find the length AB.

13 A circle has the equation $x^2 + y^2 + 6x - 2y = 0$ and passes through the point P.

Given that the tangent to the circle at P passes through the point Q (2, 6), find the exact length PQ in its simplest form.

14 The circle C has the equation $x^2 + y^2 - 6x - 10y + 16 = 0$ and passes through the point A (6, 2).

a Find the coordinates of the centre of C.

b Find the gradient of the normal to the circle at A.

c Find an equation of the normal to the circle at A.

15 Find an equation of

a the normal to the circle with equation $x^2 + y^2 + 4x = 13$ at the point (–1, 4),

b the tangent to the circle with equation $x^2 + y^2 + 2x + 4y - 40 = 0$ at the point (5, 1),

c the tangent to the circle with equation $x^2 + y^2 - 10x + 4y + 4 = 0$ at the point (2, 2).

16 Find the coordinates of the points where the circle with equation $x^2 + y^2 - 6x + 6y - 16 = 0$ intersects the coordinate axes.

17 Find in each case the coordinates of the points where the line l intersects the circle C.

a $l : y = x - 4$ $\quad C : x^2 + y^2 = 10$

b $l : 3x + y = 17$ $\quad C : x^2 + y^2 - 4x - 2y - 15 = 0$

c $l : y = 2x + 2$ $\quad C : 4x^2 + 4y^2 + 4x - 8y - 15 = 0$

18 The line with equation $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B.

Find the length of the chord AB, giving your answer in the form $k\sqrt{2}$.

19 Show that the line with equation $y = 2x + 1$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 8y + 27 = 0$ and find the coordinates of the point where they touch.

20 The line with equation $y = x + k$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 8y + 17 = 0$.

Find the two possible values of k.

21 The line with equation $y = mx$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 16y + 72 = 0$.

Find the two possible values of m.

22 The line with equation $2x + 3y = k$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y = 0$.

Find the two possible values of k.

23 The circle with equation $x^2 + y^2 - 4x - 6y = 7$ crosses the y-axis at the points A and B.

a Find the coordinates of the points A and B.

b Find the coordinates of the point where the tangent to the circle at A intersects the tangent to the circle at B.

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C2 COORDINATE GEOMETRY Worksheet B

1 The circle C has centre $(3, -2)$ and radius 5.

a Write down an equation of C in cartesian form.

The line $y = 2x - 3$ intersects C at the points A and B.

b Show that $AB = 4\sqrt{5}$.

2 The line AB is a diameter of circle C.

Given that A has coordinates $(-5, 6)$ and B has coordinates $(3, 8)$, find

a the coordinates of the centre of C,

b a cartesian equation for C,

c an equation of the tangent to C at A.

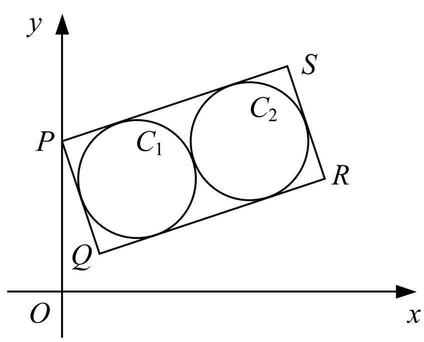
3 The circle C has equation $x^2 + y^2 + 8x - 16y + 62 = 0$.

a Find the coordinates of the centre of C and the exact radius of C.

The line l has equation $y = 2x + 1$.

b Show that the minimum distance between l and C is $3(\sqrt{5} - \sqrt{2})$.

4



The diagram shows rectangle $PQRS$ and circles C_1 and C_2.

Each circle touches the other circle and three sides of the rectangle. The coordinates of the corners of the rectangle are P (0, 4), Q (1, 1), R (7, 3) and S (6, 6).

a Find the radius of C_1.

b Find the coordinates of the point where the two circles touch.

c Show that C_1 has equation $2x^2 + 2y^2 - 8x - 12y + 21 = 0$.

5 The circle C touches the y-axis at the point A (0, 3) and passes through the point B (2, 7).

a Find an equation of the perpendicular bisector of AB.

b Find an equation for C.

c Show that the tangent to C at B has equation

$$3x - 4y + 22 = 0.$$

6 The point P (x, y) moves such that its distance from the point A $(-3, 4)$ is twice its distance from the point B $(0, -2)$.

Show that the locus of P is a circle and find the coordinates of the centre and the exact radius of this circle.

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7 The points $P(-4, 9)$ and $Q(-2, -5)$ are such that PQ is a diameter of circle C.

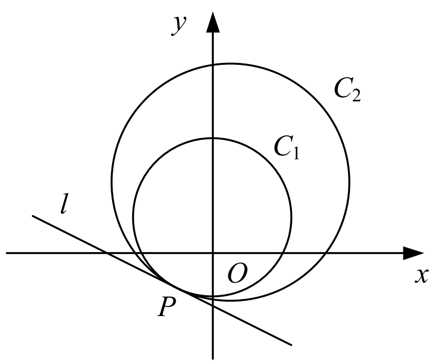
a Find the coordinates of the centre of C.

b Find an equation for C.

c Show that the point $R(2, 7)$ lies on C.

d Hence, state the size of $\angle PRQ$, giving a reason for your answer.

8



The diagram shows circles C_1 and C_2, which both pass through the point P, and the common tangent to the circles at P, the line l.

Circle C_1 has the equation $x^2 + y^2 - 4y - 16 = 0$.

a Find the coordinates of the centre of C_1.

Circle C_2 has the equation $x^2 + y^2 - 2x - 8y - 60 = 0$.

b Find an equation of the straight line passing through the centre of C_1 and the centre of C_2.

c Find an equation of line l.

9 The circle C has equation $x^2 + y^2 - 8x + 4y + 12 = 0$.

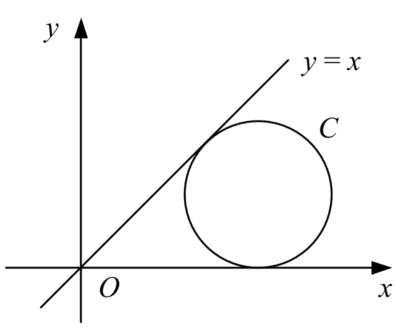
a Find the coordinates of the centre of C and the radius of C.

The point P has coordinates $(3, 5)$ and the point Q lies on C.

b Find the largest and smallest values of the length PQ, giving your answers in the form $k\sqrt{2}$.

c Find the length of PQ correct to 3 significant figures when the line PQ is a tangent to C.

10



The diagram shows the circle C and the line $y = x$.

Given that circle C has centre (a, b), where a and b are positive constants, and that C touches the x-axis,

a find a cartesian equation for C in terms of a and b.

Given also that the line $y = x$ is a tangent to C,

b show that $a = (1 + \sqrt{2})b$.

C2 COORDINATE GEOMETRY Worksheet C

1 A circle has the equation $x^2 + y^2 - 8x + 7 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Find the radius of the circle. **(2)**

2 A circle has the equation $x^2 + y^2 - 6x + 2y - 15 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Find the radius of the circle. **(1)**

c Show that the tangent to the circle at the point (7, 2) has equation

$$4x + 3y - 34 = 0.$$ **(4)**

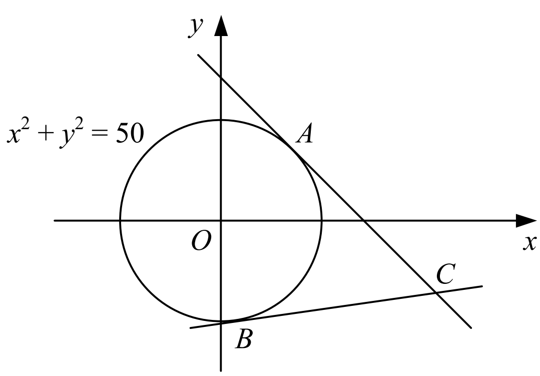
3 A circle has the equation $x^2 + y^2 + 6x - 8y + 21 = 0$.

a Find the coordinates of the centre and the radius of the circle. **(3)**

The point P lies on the circle.

b Find the greatest distance of P from the origin. **(2)**

4



The diagram shows the circle with equation $x^2 + y^2 = 50$ and the tangents to the circle at the points A (5, 5) and B (1, −7).

a Find an equation of the tangent to the circle at A. **(3)**

b Show that the tangent to the circle at B has the equation

$$x - 7y - 50 = 0.$$ **(3)**

c Find the coordinates of the point C where the tangents to the circle at A and B intersect. **(2)**

5 Circle C_1 has the equation $x^2 + y^2 - 2ay = 0$, where a is a positive constant.

a Find the coordinates of the centre and the radius of C_1. **(4)**

Circle C_2 has the equation $x^2 + y^2 - 2bx = 0$, where b is a constant and $b > a$.

b Sketch C_1 and C_2 on the same diagram. **(4)**

6 The circle C has the equation $x^2 + y^2 + 2x - 14y + 30 = 0$.

a Find the coordinates of the centre of C. **(2)**

b Find the radius of C, giving your answer in the form $k\sqrt{5}$. **(2)**

c Show that the line $y = 2x - 1$ is a tangent to C and find the coordinates of the point of contact. **(4)**

7 The circle C has equation $x^2 + y^2 - 6x - 12y + 28 = 0$.

a Find the coordinates of the centre of C. **(2)**

The line $y = x - 2$ intersects C at the points A and B.

b Find the length AB in the form $k\sqrt{2}$. **(6)**

8 The circle C has centre $(8, -1)$ and passes through the point $(4, 1)$.

a Find an equation for C. **(3)**

b Show that the line with equation $x + 2y + 4 = 0$ is a tangent to C. **(3)**

9 The points $P\ (-10, 2)$, $Q\ (8, 14)$ and $R\ (-2, -10)$ all lie on circle C.

a Show that PR is perpendicular to PQ. **(2)**

b Hence, show that C has the equation $x^2 + y^2 - 6x - 4y - 156 = 0$. **(5)**

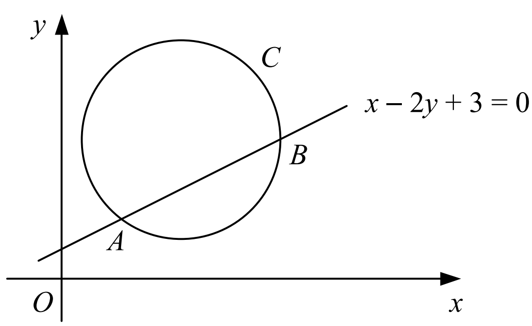
10 A circle has the equation $x^2 + y^2 - 2x - 7y - 16 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Show that the radius of the circle is $k\sqrt{13}$, where k is an exact fraction to be found. **(2)**

c Find an equation of the tangent to the circle at the point $(4, 8)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

11



The line with equation $x - 2y + 3 = 0$ intersects the circle C at the points A and B as shown in the diagram above. Given that the centre of C has coordinates $(6, 7)$,

a find the coordinates of the mid-point of the chord AB. **(6)**

Given also that the x-coordinate of the point A is 3,

b find the coordinates of the point B, **(3)**

c find an equation for C. **(2)**

12 The circle C has equation $x^2 + y^2 - 8x - 16y + 72 = 0$.

a Find the coordinates of the centre and the radius of C. **(3)**

b Find the distance of the centre of C from the origin in the form $k\sqrt{5}$. **(2)**

The point A lies on C and the tangent to C at A passes through the origin O.

c Show that $OA = 6\sqrt{2}$. **(3)**

13 The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.

a Show that l passes through the centre of C. **(3)**

b Find an equation for each tangent to C that is parallel to l. **(6)**

C2 COORDINATE GEOMETRY — Answers - Worksheet A

1 **a** $x^2 + y^2 = 25$ **b** $(x-1)^2 + (y-3)^2 = 4$ **c** $(x-4)^2 + (y+6)^2 = 1$

d $(x+1)^2 + (y+8)^2 = 9$ **e** $(x+\frac{1}{2})^2 + (y-\frac{1}{2})^2 = \frac{1}{4}$ **f** $(x+3)^2 + (y-9)^2 = 12$

2 **a** centre (0, 0) radius 4 **b** centre (6, 1) radius 9 **c** centre (–1, 4) radius 11

d centre (7, 0) radius 0.3 **e** centre (–2, –5) radius $4\sqrt{2}$ **f** centre (8, –9) radius $6\sqrt{3}$

3 **a** $x^2 + (y-2)^2 - 4 + 3 = 0$
$x^2 + (y-2)^2 = 1$
centre (0, 2) radius 1

b $(x-1)^2 - 1 + (y-5)^2 - 25 - 23 = 0$
$(x-1)^2 + (y-5)^2 = 49$
centre (1, 5) radius 7

c $(x+6)^2 - 36 + (y-4)^2 - 16 + 36 = 0$
$(x+6)^2 + (y-4)^2 = 16$
centre (–6, 4) radius 4

d $(x-1)^2 - 1 + (y+8)^2 - 64 = 35$
$(x-1)^2 + (y+8)^2 = 100$
centre (1, –8) radius 10

e $(x-4)^2 - 16 + (y+3)^2 - 9 = 0$
$(x-4)^2 + (y+3)^2 = 25$
centre (4, –3) radius 5

f $(x+5)^2 - 25 + (y-1)^2 - 1 - 19 = 0$
$(x+5)^2 + (y-1)^2 = 45$
centre (–5, 1) radius $3\sqrt{5}$

g $x^2 + y^2 - x - 6y + \frac{1}{4} = 0$
$(x-\frac{1}{2})^2 - \frac{1}{4} + (y-3)^2 - 9 + \frac{1}{4} = 0$
$(x-\frac{1}{2})^2 + (y-3)^2 = 9$
centre $(\frac{1}{2}, 3)$ radius 3

h $x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$
$(x+\frac{1}{3})^2 - \frac{1}{9} + (y-\frac{4}{3})^2 - \frac{16}{9} + \frac{8}{9} = 0$
$(x+\frac{1}{3})^2 + (y-\frac{4}{3})^2 = 1$
centre $(-\frac{1}{3}, \frac{4}{3})$ radius 1

4 **a** radius = $\sqrt{9+16} = 5$ $\therefore (x-1)^2 + (y+2)^2 = 25$

b radius = $\sqrt{25+4} = \sqrt{29}$ $\therefore (x+5)^2 + (y-7)^2 = 29$

5 **a** centre $(\frac{1+3}{2}, -2) = (2, -2)$
radius = 1
$\therefore (x-2)^2 + (y+2)^2 = 1$

b centre $(\frac{-7+1}{2}, \frac{2+8}{2}) = (-3, 5)$
radius = $\sqrt{16+9} = 5$
$\therefore (x+3)^2 + (y-5)^2 = 25$

c centre $(\frac{1+4}{2}, \frac{1+0}{2}) = (\frac{5}{2}, \frac{1}{2})$
radius = $\sqrt{\frac{9}{4}+\frac{1}{4}} = \sqrt{\frac{5}{2}}$
$\therefore (x-\frac{5}{2})^2 + (y-\frac{1}{2})^2 = \frac{5}{2}$

6 **a** grad $PQ = \frac{10-1}{3-0} = 3$, grad $QR = \frac{9-10}{6-3} = -\frac{1}{3}$
grad $PQ \times$ grad $QR = 3 \times (-\frac{1}{3}) = -1$
$\therefore$ PQ and QR are perpendicular
$\therefore$ $\angle PQR$ is a right-angle

b $\angle PQR$ is a right-angle $\therefore$ PR is a diameter of C
$\therefore$ centre is $(\frac{0+6}{2}, \frac{1+9}{2}) = (3, 5)$
radius = 5
$\therefore (x-3)^2 + (y-5)^2 = 25$
$x^2 - 6x + 9 + y^2 - 10y + 25 - 25 = 0$
$x^2 + y^2 - 6x - 10y + 9 = 0$

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7 **a** centre (0, 0) radius 8
dist. pt to centre = 9
$\therefore$ outside circle

b $(x-1)^2 - 1 + (y-3)^2 - 9 - 26 = 0$
$(x-1)^2 + (y-3)^2 = 36$
centre (1, 3) radius 6
dist. pt to centre = $\sqrt{9+16} = 5$
$\therefore$ inside circle

c $(x+5)^2 - 25 + (y-2)^2 - 4 = 140$
$(x+5)^2 + (y-2)^2 = 169$
centre (–5, 2) radius 13
dist. pt to centre = $\sqrt{144+25} = 13$
$\therefore$ on circle

d $(x+1)^2 - 1 + (y+4)^2 - 16 - 13 = 0$
$(x+1)^2 + (y+4)^2 = 30$
centre (–1, –4) radius $\sqrt{30}$
dist. pt to centre = $\sqrt{9+25} = \sqrt{34}$
$\therefore$ outside circle

8 $(x+6)^2 - 36 + (y-3)^2 - 9 + 27 = 0$
$(x+6)^2 + (y-3)^2 = 18$
centre (–6, 3) radius $3\sqrt{2}$
dist. Q to centre = $\sqrt{196+4} = 10\sqrt{2}$
min. $PQ = 10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$

9 x-coord of centre = $\frac{2+8}{2} = 5$
y-coord of centre = 4 $\therefore$ centre (5, 4)
radius = dist. (0, 4) to (5, 4) = 5
$\therefore$ $(x-5)^2 + (y-4)^2 = 25$

10 $(x+4)^2 - 16 + (y-6)^2 - 36 + k = 0$
$(x+4)^2 + (y-6)^2 = 52 - k$
centre (–4, 6) $r^2 = 52 - k$
$r > 0$ $\therefore$ $k < 52$
also require $r < 4$
$\therefore$ $52 - k < 16$
$k > 36$
$\therefore$ $36 < k < 52$

11 **a** mid-point $PQ = (\frac{-2+2}{2}, \frac{-2+(-4)}{2}) = (0, -3)$
grad $PQ = \frac{-4+2}{2+2} = -\frac{1}{2}$
perp. grad = 2
$\therefore$ $y = 2x - 3$

b mid-point $PR = (\frac{-2+7}{2}, \frac{-2+1}{2}) = (\frac{5}{2}, -\frac{1}{2})$
grad $PR = \frac{1+2}{7+2} = \frac{1}{3}$
perp. grad = –3
perp. bisector $y + \frac{1}{2} = -3(x - \frac{5}{2})$
$y = 7 - 3x$
centre where intersect $2x - 3 = 7 - 3x$
$x = 2$ $\therefore$ (2, 1)

c radius = dist. (2, 1) to (7, 1) = 5
$\therefore$ $(x-2)^2 + (y-1)^2 = 25$

12 **a** $(x-2)^2 - 4 + (y-2)^2 - 4 - 28 = 0$
$(x-2)^2 + (y-2)^2 = 36$
centre (2, 2) radius 6
dist. = $\sqrt{64+36} = 10$

b tangent perp to radius
$\therefore$ $AB^2 = 10^2 - 6^2 = 64$
$AB = 8$

13 $(x+3)^2 - 9 + (y-1)^2 - 1 = 0$
$(x+3)^2 + (y-1)^2 = 10$
centre (–3, 1) radius $\sqrt{10}$
dist. centre to (2, 6) = $\sqrt{25+25} = \sqrt{50}$
$PQ^2 = (\sqrt{50})^2 - (\sqrt{10})^2 = 40$
$PQ = \sqrt{40} = 2\sqrt{10}$

14 **a** $(x-3)^2 - 9 + (y-5)^2 - 25 + 16 = 0$

$\therefore$ centre $(3, 5)$

b grad $= \frac{5-2}{3-6} = -1$

c $y - 2 = -(x - 6)$ $\quad [\, y = 8 - x \,]$

15 **a** $(x+2)^2 - 4 + y^2 = 13$

$\therefore$ centre $(-2, 0)$

grad $= \frac{0-4}{-2+1} = 4$

$\therefore$ $y - 4 = 4(x + 1)$ $\quad [\, y = 4x + 8 \,]$

b $(x+1)^2 - 1 + (y+2)^2 - 4 - 40 = 0$

$\therefore$ centre $(-1, -2)$

grad normal $= \frac{-2-1}{-1-5} = \frac{1}{2}$

$\therefore$ grad tangent $= -2$

$\therefore$ $y - 1 = -2(x - 5)$ $\quad [\, y = 11 - 2x \,]$

c $(x-5)^2 - 25 + (y+2)^2 - 4 + 4 = 0$

$\therefore$ centre $(5, -2)$

grad normal $= \frac{-2-2}{5-2} = -\frac{4}{3}$

$\therefore$ grad tangent $= \frac{3}{4}$

$\therefore$ $y - 2 = \frac{3}{4}(x - 2)$ $\quad [\, 3x - 4y + 2 = 0 \,]$

16 $x = 0 \Rightarrow y^2 + 6y - 16 = 0$

$(y + 8)(y - 2) = 0$

$y = -8, 2$

$y = 0 \Rightarrow x^2 - 6x - 16 = 0$

$(x + 2)(x - 8) = 0$

$x = -2, 8$

$\therefore$ $(0, -8)$, $(0, 2)$, $(-2, 0)$ and $(8, 0)$

17 **a** sub. $x^2 + (x-4)^2 = 10$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1, 3$

$\therefore$ $(1, -3)$ and $(3, -1)$

b sub. $y = 17 - 3x$

$x^2 + (17 - 3x)^2 - 4x - 2(17 - 3x) - 15 = 0$

$x^2 - 10x + 24 = 0$

$(x - 4)(x - 6) = 0$

$x = 4, 6$

$\therefore$ $(4, 5)$ and $(6, -1)$

c sub.

$4x^2 + 4(2x + 2)^2 + 4x - 8(2x + 2) - 15 = 0$

$4x^2 + 4x - 3 = 0$

$(2x + 3)(2x - 1) = 0$

$x = -\frac{3}{2}, \frac{1}{2}$

$\therefore$ $(-\frac{3}{2}, -1)$ and $(\frac{1}{2}, 3)$

18 sub.

$x^2 + (1 - x)^2 + 6x + 2(1 - x) = 27$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4, 3$

$\therefore$ $(-4, 5)$ and $(3, -2)$

$AB = \sqrt{49 + 49} = 7\sqrt{2}$

19 sub.

$x^2 + (2x + 1)^2 - 8x - 8(2x + 1) + 27 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

repeated root $\therefore$ tangent

touch when $x = 2$ $\therefore$ at $(2, 5)$

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20 sub.
$x^2 + (x + k)^2 + 6x - 8(x + k) + 17 = 0$
$2x^2 + (2k - 2)x + k^2 - 8k + 17 = 0$
tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$
$\Rightarrow \quad (2k - 2)^2 - 8(k^2 - 8k + 17) = 0$
$k^2 - 14k + 33 = 0$
$(k - 3)(k - 11) = 0$
$\therefore$ $k = 3$ or 11

21 sub.
$x^2 + m^2x^2 - 8x - 16mx + 72 = 0$
$(1 + m^2)x^2 - (8 + 16m)x + 72 = 0$
tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$
$\Rightarrow \quad (8 + 16m)^2 - 288(1 + m^2) = 0$
$m^2 - 8m + 7 = 0$
$(m - 1)(m - 7) = 0$
$\therefore$ $m = 1, 7$

22 sub. $x = \frac{k-3y}{2}$
$(\frac{k-3y}{2})^2 + y^2 + 6(\frac{k-3y}{2}) + 4y = 0$
$(k - 3y)^2 + 4y^2 + 12(k - 3y) + 16y = 0$
$13y^2 - (6k + 20)y + k^2 + 12k = 0$
tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$
$\Rightarrow \quad (6k + 20)^2 - 52(k^2 + 12k) = 0$
$k^2 + 24k - 25 = 0$
$(k + 25)(k - 1) = 0$
$\therefore$ $k = -25, 1$

23 **a** $x = 0 \Rightarrow y^2 - 6y - 7 = 0$
$(y + 1)(y - 7) = 0$
$y = -1, 7$
$\therefore$ $(0, -1)$ and $(0, 7)$

b $(x - 2)^2 - 4 + (y - 3)^2 - 9 = 7$
$\therefore$ centre $(2, 3)$
grad normal at $(0, -1) = \frac{3+1}{2-0} = 2$
$\therefore$ grad tangent at $(0, -1) = -\frac{1}{2}$
$\therefore$ $y = -\frac{1}{2}x - 1$
grad normal at $(0, 7) = \frac{3-7}{2-0} = -2$
$\therefore$ grad tangent at $(0, 7) = \frac{1}{2}$
$\therefore$ $y = \frac{1}{2}x + 7$
intersect when $-\frac{1}{2}x - 1 = \frac{1}{2}x + 7$
$x = -8$
$\therefore$ $(-8, 3)$

C2 COORDINATE GEOMETRY — Answers - Worksheet B

1 **a** $(x-3)^2+(y+2)^2=25$

b sub. $(x-3)^2+[(2x-3)+2]^2=25$

$(x-3)^2+(2x-1)^2=25$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x=-1,\ 3$

$\therefore$ $(-1,-5)$ and $(3, 3)$

$AB^2=4^2+8^2=80$

$AB=\sqrt{80}=\sqrt{16\times5}=4\sqrt{5}$

2 **a** $=(\frac{-5+3}{2},\ \frac{6+8}{2})=(-1, 7)$

b radius $=\sqrt{16+1}=\sqrt{17}$

$\therefore$ $(x+1)^2+(y-7)^2=17$

c grad of radius $=\dfrac{7-6}{-1-(-5)}=\frac{1}{4}$

$\therefore$ grad of tangent $=-4$

$\therefore$ $y-6=-4(x+5)$

$[\ y=-4x-14\ ]$

3 **a** $(x+4)^2-16+(y-8)^2-64+62=0$

$(x+4)^2+(y-8)^2=18$

$\therefore$ centre $(-4, 8)$ radius $3\sqrt{2}$

b grad of $l=2$ $\therefore$ grad of perp. $=-\frac{1}{2}$

eqn. of line perp to l through centre:

$y-8=-\frac{1}{2}(x+4)$

$y=6-\frac{1}{2}x$

intersects l when:

$2x+1=6-\frac{1}{2}x$

$x=2$ $\therefore$ $(2, 5)$ is closest point

dist. $(2, 5)$ to centre

$=\sqrt{36+9}=\sqrt{45}=3\sqrt{5}$

min. dist. $=3\sqrt{5}-3\sqrt{2}=3(\sqrt{5}-\sqrt{2})$

4 **a** $PQ=\sqrt{1+9}=\sqrt{10}$

radius $=\frac{1}{2}PQ=\frac{1}{2}\sqrt{10}$

b $=$ midpoint of PR

$=(\frac{0+7}{2},\ \frac{4+3}{2})=(\frac{7}{2},\ \frac{7}{2})$

c midpoint of $PQ=(\frac{0+1}{2},\ \frac{4+1}{2})=(\frac{1}{2},\ \frac{5}{2})$

centre of C_1 = midpoint of $(\frac{1}{2},\ \frac{5}{2})$ and $(\frac{7}{2},\ \frac{7}{2})$

$=(\frac{\frac{1}{2}+\frac{7}{2}}{2},\ \frac{\frac{5}{2}+\frac{7}{2}}{2})=(2, 3)$

$\therefore$ eqn. of C_1:

$(x-2)^2+(y-3)^2=(\frac{1}{2}\sqrt{10})^2$

$x^2-4x+4+y^2-6y+9=\frac{5}{2}$

$2x^2-8x+8+2y^2-12y+18=5$

$2x^2+2y^2-8x-12y+21=0$

5 **a** midpoint $AB=(\frac{0+2}{2},\ \frac{3+7}{2})=(1, 5)$

grad $AB=\dfrac{7-3}{2-0}=2$

$\therefore$ perp. grad $=-\frac{1}{2}$

$\therefore$ $y-5=-\frac{1}{2}(x-1)$

$[\ y=\frac{11}{2}-\frac{1}{2}x\ ]$

b circle touches y-axis at $(0, 3)$

$\therefore$ y-coord of centre $=3$

sub. $3=\frac{11}{2}-\frac{1}{2}x$

$x=5$

$\therefore$ centre $(5, 3)$ radius 5

$\therefore$ $(x-5)^2+(y-3)^2=25$

c grad of radius $=\dfrac{7-3}{2-5}=-\frac{4}{3}$

$\therefore$ grad of tangent $=\frac{3}{4}$

$\therefore$ $y-7=\frac{3}{4}(x-2)$

$4y-28=3x-6$

$3x-4y+22=0$

6 $AP^2=(x+3)^2+(y-4)^2$

$BP^2=x^2+(y+2)^2$

$AP=2BP$ $\therefore$ $AP^2=4BP^2$

$\therefore$ $(x+3)^2+(y-4)^2=4[x^2+(y+2)^2]$

$x^2+6x+9+y^2-8y+16=4x^2+4y^2+16y+16$

$x^2-2x+y^2+8y-3=0$

$(x-1)^2-1+(y+4)^2-16-3=0$

$(x-1)^2+(y+4)^2=20$

in form $(x-a)^2+(y-b)^2=r^2$ $\therefore$ circle

centre $(1, -4)$ radius $2\sqrt{5}$

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7 **a** $= (\frac{-4+(-2)}{2}, \frac{9+(-5)}{2}) = (-3, 2)$

b radius $= \sqrt{1+49} = \sqrt{50}$

$\therefore\ (x+3)^2 + (y-2)^2 = 50$

c sub. (2, 7) into eqn of C:

$(2+3)^2 + (7-2)^2 = 50$

$25 + 25 = 50$

true $\therefore$ R lies on C

d 90°

PQ is a diameter

$\therefore$ $\angle PRQ$ is the angle in a semicircle

8 **a** $x^2 + (y-2)^2 - 4 - 16 = 0$

$\therefore$ centre (0, 2)

b C_2: $(x-1)^2 - 1 + (y-4)^2 - 16 - 60 = 0$

$\therefore$ centre (1, 4)

grad $= \frac{4-2}{1-0} = 2$

$\therefore\ y = 2x + 2$

c sub. into eqn of C_1:

$x^2 + [(2x+2) - 2]^2 - 20 = 0$

$x^2 + (2x)^2 - 20 = 0$

$x^2 = 4$

$x = \pm 2$

from diagram, $x = -2$ at P

$\therefore$ $P\,(-2, -2)$

l perp to line through centres

$\therefore$ grad $= -\frac{1}{2}$

$\therefore\ y + 2 = -\frac{1}{2}(x+2)$

$[\ y = -\frac{1}{2}x - 3\ ]$

9 **a** $(x-4)^2 - 16 + (y+2)^2 - 4 + 12 = 0$

$(x-4)^2 + (y+2)^2 = 8$

centre (4, −2) radius $2\sqrt{2}$

b dist. P to centre

$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

$\therefore$ max. $PQ = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

min. $PQ = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$

c tangent perp. to radius

$PQ^2 = (5\sqrt{2})^2 - (2\sqrt{2})^2 = 50 - 8 = 42$

$PQ = \sqrt{42} = 6.48$

10 **a** radius $= b$

$\therefore\ (x-a)^2 + (y-b)^2 = b^2$

b sub. $y = x$ into eqn

$(x-a)^2 + (x-b)^2 = b^2$

$x^2 - 2ax + a^2 + x^2 - 2bx + b^2 = b^2$

$2x^2 - 2(a+b)x + a^2 = 0$

tangent $\therefore$ repeated root

$\therefore$ "$b^2 - 4ac$" $= 0$

$4(a+b)^2 - 8a^2 = 0$

$a^2 - 2ab - b^2 = 0$

$a = \frac{2b \pm \sqrt{4b^2 + 4b^2}}{2} = b \pm \sqrt{2}\,b$

$a > 0, b > 0$ $\therefore$ $a = (1 + \sqrt{2})b$

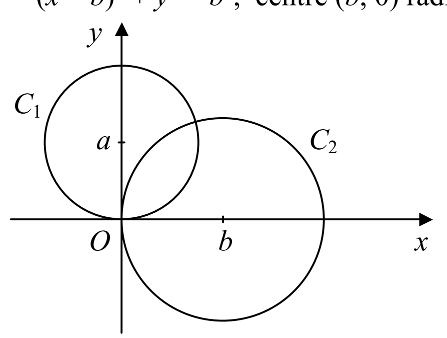
1 **a** $(x-4)^2 - 16 + y^2 + 7 = 0$
$\therefore$ centre (4, 0)
b $(x-4)^2 + y^2 = 9$
$\therefore$ radius = 3

2 **a** $(x-3)^2 - 9 + (y+1)^2 - 1 - 15 = 0$
$\therefore$ centre (3, −1)
b $(x-3)^2 + (y+1)^2 = 25$
$\therefore$ radius = 5
c grad of radius = $\frac{2-(-1)}{7-3} = \frac{3}{4}$
$\therefore$ grad of tangent = $-\frac{4}{3}$
$\therefore y - 2 = -\frac{4}{3}(x-7)$
$3y - 6 = -4x + 28$
$4x + 3y - 34 = 0$

3 **a** $(x+3)^2 - 9 + (y-4)^2 - 16 + 21 = 0$
$(x+3)^2 + (y-4)^2 = 4$
$\therefore$ centre (−3, 4) radius 2
b dist. of centre from $O = \sqrt{9+16} = 5$
$\therefore$ max. dist. of P from O
$= 5 + 2 = 7$

4 **a** centre (0, 0) $\therefore$ grad of radius = 1
$\therefore$ grad of tangent = −1
$\therefore y - 5 = -(x-5)$ [$y = 10 - x$]
b grad of radius = −7
$\therefore$ grad of tangent = $\frac{1}{7}$
$\therefore y + 7 = \frac{1}{7}(x-1)$
$7y + 49 = x - 1$
$x - 7y - 50 = 0$
c sub. $x - 7(10 - x) - 50 = 0$
$x = 15$
$\therefore$ (15, −5)

5 **a** $x^2 + (y-a)^2 - a^2 = 0$
$x^2 + (y-a)^2 = a^2$
$\therefore$ centre $(0, a)$ radius a
b C_2: $(x-b)^2 - b^2 + y^2 = 0$
$(x-b)^2 + y^2 = b^2$, centre $(b, 0)$ radius b



6 **a** $(x+1)^2 - 1 + (y-7)^2 - 49 + 30 = 0$
$\therefore$ centre (−1, 7)
b $(x+1)^2 + (y-7)^2 = 20$
$\therefore$ radius = $\sqrt{20} = 2\sqrt{5}$
c sub. $y = 2x - 1$ into eqn. of circle
$x^2 + (2x-1)^2 + 2x - 14(2x-1) + 30 = 0$
$x^2 - 6x + 9 = 0$
$(x-3)^2 = 0$
repeated root $\therefore$ tangent
point of contact (3, 5)

7 **a** $(x-3)^2 - 9 + (y-6)^2 - 36 + 28 = 0$
$\therefore$ centre (3, 6)
b sub.
$x^2 + (x-2)^2 - 6x - 12(x-2) + 28 = 0$
$x^2 - 11x + 28 = 0$
$(x-4)(x-7) = 0$
$x = 4, 7$
$\therefore A$ (4, 2), B (7, 5)
$\therefore AB = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

8 **a** radius = $\sqrt{16+4} = \sqrt{20}$
$\therefore (x-8)^2 + (y+1)^2 = 20$
b sub. $x = -2y - 4$ into eqn. of circle:
$(-2y-12)^2 + (y+1)^2 = 20$
$4y^2 + 48y + 144 + y^2 + 2y + 1 = 20$
$y^2 + 10y + 25 = 0$
$(y+5)^2 = 0$
repeated root $\therefore$ tangent

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9 **a** grad $PQ = \frac{14-2}{8+10} = \frac{2}{3}$

grad $PR = \frac{-10-2}{-2+10} = -\frac{3}{2}$

grad $PR \times$ grad $PQ = -\frac{3}{2} \times \frac{2}{3} = -1$

$\therefore$ PR is perpendicular to PQ

b $\angle QPR = 90° \;\therefore\; QR$ is a diameter of the circle

$\therefore$ centre of circle is mid-point of QR

$= (\frac{8-2}{2}, \frac{14-10}{2}) = (3, 2)$

radius $= \sqrt{25+144} = 13$

$\therefore\; (x-3)^2 + (y-2)^2 = 169$

$x^2 - 6x + 9 + y^2 - 4y + 4 - 169 = 0$

$x^2 + y^2 - 6x - 4y - 156 = 0$

10 **a** $(x-1)^2 - 1 + (y - \frac{7}{2})^2 - \frac{49}{4} - 16 = 0$

$\therefore$ centre $(1, \frac{7}{2})$

b $(x-1)^2 + (y - \frac{7}{2})^2 = \frac{117}{4}$

$\therefore$ radius $= \sqrt{\frac{117}{4}} = \sqrt{\frac{9\times13}{4}} = \frac{3}{2}\sqrt{13}$ $[k = \frac{3}{2}]$

c grad of radius $= \frac{8-\frac{7}{2}}{4-1} = \frac{3}{2}$

$\therefore$ grad of tangent $= -\frac{2}{3}$

$\therefore\; y - 8 = -\frac{2}{3}(x-4)$

$3y - 24 = -2x + 8$

$2x + 3y - 32 = 0$

11 **a** grad of $x - 2y + 3 = 0$ is $\frac{1}{2}$

$\therefore$ grad of perp bisector $= -2$

passes through centre of circle

$\therefore\; y - 7 = -2(x-6)$

$y = -2x + 19$

mid-point of chord where intersect

$x - 2(-2x + 19) + 3 = 0$

$x = 7 \;\therefore\; (7, 5)$

b $3 - 2y + 3 = 0$

$\therefore y = 3 \;\therefore\; A\,(3, 3)$

let B be (p, q)

$\therefore\; (\frac{3+p}{2}, \frac{3+q}{2}) = (7, 5)$

$p = 11, q = 7 \;\therefore\; B\,(11, 7)$

c radius $= \sqrt{9+16} = 5$

$\therefore\; (x-6)^2 + (y-7)^2 = 25$

12 **a** $(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$

$(x-4)^2 + (y-8)^2 = 8$

$\therefore$ centre $(4, 8)$ radius $2\sqrt{2}$

b $= \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

c tangent perp. to radius

$\therefore\; OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$

$OA = \sqrt{72} = \sqrt{36\times2} = 6\sqrt{2}$

13 **a** C: $(x-2)^2 - 4 + y^2 - 6 = 0$

$\therefore$ centre $(2, 0)$

l: when $x = 2,\; y = 3(2) - 6 = 0$

$\therefore\; l$ passes through centre of C

b eqn. of tangent: $y = 3x + k$

sub. into eqn. of circle:

$x^2 + (3x+k)^2 - 4x - 6 = 0$

$10x^2 + (6k-4)x + k^2 - 6 = 0$

tangent $\therefore$ repeated root $\therefore\; b^2 - 4ac = 0$

$(6k-4)^2 - 40(k^2 - 6) = 0$

$k^2 + 12k - 64 = 0$

$(k+16)(k-4) = 0$

$k = -16, 4$

$\therefore\; y = 3x - 16$ and $y = 3x + 4$